Impact of metal prices on plant optimization

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ABSTRACT

The optimum economic operating conditions of a processing plant are highly dependent on the market price of the metal produced. Since the price of a metal product is determined by supply and demand relationships and is greatly affected by local and world political events, future metal prices are not explicitly defined. Apart from historical trends, such factors create a degree of speculation. The study provides a method to account for the volatility of future metal prices when optimizing fully integrated mineral processing plants. A model was developed for estimating the "certainty-equivalent price" throughout the project lifetime while taking into account the volatility of future metal prices. The certainty-equivalent price should be used as a substitute for the volatile spot price in any optimization process that depends on the realized metal price. Non-ferrous metals were considered in the investigation rather than iron-bearing and/or precious metals. A hypothetical copper project was provided as an example to illustrate the applicability of the model and investigate the advantages of the suggested procedures in improving the financial performance of the plant. © 2005 SDU. All rights reserved.

Keywords: Metal; Mineral processing; Mineral economics; Process optimization

1. INTRODUCTION

Metals are important resources for many industries on a global basis. The enormous growth of industrialization in the past century has led to a significant increase in the demand for metals amongst established nations and developing countries. To meet the increasing demand, the annual output of many metals, particularly base metals, has grown dramatically (Wills, 1992). The excessive consumption of metals has resulted in the exhaustion of most rich ores, and consequently the potential profits from mining and processing operations have decreased due to the low grade of the available ores and the worldwide decline of most metal prices. In order for the mineral commodities industry to face the challenges, a sound management policy should be adapted that takes into account the unstable economic environment. In a world of continuously changing economic conditions, no plant operates under static conditions. At any point in time at least one of the interlocking components, which control the actual operation, is subject to change. For instance, ore grades fluctuate, market conditions vary, and new technology emerges (Evans, 1980).

The goal set for any processing plant is the production of a concentrate of the valuable mineral with a grade as high as possible and at a cost as low as possible. This goal is to be achieved simultaneously with maximizing the recovery (Currie, 1973). In order for a plant to realize the maximum economic performance, the concentrate grade and the recovery should be optimized in order to maximize the profit per unit of ore processed. This profit depends mainly on the price of the metal produced. As explained by Wills (1992), the price of most metals is governed by the supply and demand relationships and the prices of many metals, particularly copper, have not kept pace with inflation. Since the market prices of metals are determined by unforeseen economic and technological conditions, these prices are highly volatile both in short and long runs (McMillan and Speight, 2001). Under these market conditions, managers of processing plants have to set the optimum operating conditions of plants on the basis of the information available at the project start-up. These plants will continue in production, under the operating conditions that have been decided at the project start-up, for long periods in the future, governed by the estimated tonnage of ore reserves (measured and indicated) and the scheduled production rates. During this long period, the economic conditions, based on which the operating conditions are optimized, change continuously. The continuous

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adjustment of the operating plants according to the continuously changing economic conditions is neither feasible nor practicable, and in some cases uneconomical. On the other hand, the changing economic conditions may result in inefficient plant operations and in the worst case may result in uneconomical operations. To avoid these problems, the stochastic behavior of future metal prices should be accounted for when optimizing the plant at the project start-up.

This study provides a method to account for the volatility of metal prices when optimizing processing plants. An example for the potential applications of the proposed method is provided. This example illustrates how the suggested procedures can be applied to optimize the concentrate grade and the recovery of a hypothetical copper plant. Also, the paper illustrates the contribution of the suggested procedures to the improvement of the financial performance of the project.

2. OPTIMIZING CONCENTRATE GRADE AND RECOVERY UNDER UNCERTAINTY

2.1. Basic model

A processing plant is operating at the optimum economic conditions when it realizes the maximum revenue per ton of ore treated. The revenue per ton of ore processed is the product of the net smelter return per short ton of concentrate "NSR", and the weight of concentrate produced from one ton of feed "C". The "NSR" depends, among other factors, on the concentrate grade and the unit price of the metal produced. The weight of concentrate produced from one ton of ore depends on the concentrate grade and the recovery. Since there is an inverse relationship between the concentrate grade and the recovery, managers should select the point, at the grade-recovery curve, at which the revenue per ton of ore is maximized. The "NSR" can be expressed by the following equation (Farrish, 1989; Segovia and Schena, 1993):

\[
NSR = 2000 \frac{C - u}{100} \left( \frac{G - u}{100} \right) (P - r_c) - T_c - X + Y - C,
\]

where:
- \(c_d\) percentage deduction,
- \(G\) concentrate grade,
- \(u\) unit deduction,
- \(P\) unit price of metal,
- \(r_c\) refining charge,
- \(T_c\) treatment charge, per short ton of concentrate,
- \(X\) penalties, per short ton of concentrate,
- \(Y\) by-product credit, per short ton of concentrate, and
- \(C\) realization cost, per short ton of concentrate.

The revenue per short ton of feed, \(F_r\), is:

\[
F_r = NSR_c C
\]

where "C" is the weight of concentrate produced from one ton of ore, which can be expressed as:

\[
C = R \frac{f}{G}
\]

where "R" is the percentage recovery and "f" is the feed grade.

Substituting from Equation (3) into Equation (2), then:

\[
F_r = NSR_c \frac{R_f}{G}
\]

The grade-recovery relationship can be expressed such as (Hall, 1971):

\[
G = M \left( \frac{M - f}{100} \right) \left( \frac{AR}{100 + A - R} \right)
\]

where "M" is the pure mineral value, %, and "A" is the liberation coefficient. Substituting from Equation (5) into Equation (4), then:

\[
F_r = \frac{NSR_c R_f}{M \left( \frac{M - f}{100} \right) \left( \frac{AR}{100 + A - R} \right)}
\]

where the "NSR_c" is obtained from Equation (1) after substituting for the value "G" from Equation (5). At definite values of the variables "M", "f" and "A", Equation (6) can be easily solved to find the recovery "R" at which "F_r" is maximized. Substituting for this optimum recovery in Equation (5), provides the optimum concentrate grade.
2.2. Introducing uncertainty

As indicated from Equations (1) to (6) inclusive, both the optimum recovery and the optimum concentrate grade depend on the market price of metal “$P$”, and some definitely known variables. The current metal price (the spot price at the decision time) is known, but future metal prices are unpredictable. They fluctuate over time, increasing and decreasing, depending on the behavior of future metal market. Using the current metal price in the optimization process is not appropriate. This implicitly assumes that the metal price is constant throughout the project life and definitely equals the current spot price. Such assumption ignores the uncertainty and risk associated with the future metal prices, which may make the underlying plant not operating at its maximum profitable potential. To overcome this problem, the risky future metal prices should be substituted by a constant certainty-equivalent price. This price represents the certainty-equivalent average price throughout the project life. Since this price is constant and risk-free, it could be inputted into Equations (1) to (6) to determine the optimum recovery and the optimum concentrate grade.

The first step in determining the certainty-equivalent price is to find an appropriate model describing the stochastic process of the metal price under study using the historical price data. The second step is to determine the certainty-equivalent price throughout the project life based on the price model.

2.2.1. Metal price modeling

The historical behavior of the majority of metal prices indicates that a stable linear trend does not exist to illustrate the major amount of metal prices. As explained by Krautkraemer (1998), metal price decreases by virtue of exploration and discovery, or technological change that lowers extraction cost. Eventually, the effect of increasing user cost outweighs the decrease in extraction cost, or exploration opportunities are exhausted, so that price begins to increase. Therefore, the metal prices are strongly related to long-run production costs. Pindyck (1991) argued that over the long-run, the price of a commodity like copper will follow a mean-reverting process, for which the mean reflects long-run marginal cost. Since the trend for metal prices is not stable, then metal prices can be modeled as mean-reverting process in which the real price of metal tends to revert back to a normal level. Dixit and Pindyck (1994) examined the historical price data of crude oil and copper using the unit root test and concluded that the prices are mean-reverting. It should be noted that the historical pricing employed in preparation of the calculations and resultant graphs is based upon American dollars. It is recognized that it was not practical to express currency in euros (€) as customary for the journal since this monetary system did not come into effect until the past decade.

In the mean-reverting process, the change in price “$dP$” over a small time interval “$dt$” can be represented as follows (Pindyck, 1991; Dixit and Pindyck, 1994; Dixit et al., 1999):

$$dP = \eta (\bar{P} - P) dt + \sigma dz$$  

(7)

where $\eta$ is the speed of reversion, $\bar{P}$ is the normal level of the price $P$, $\sigma$ is the standard deviation of price changes and $dz$ is the increment of a Wiener process ($dz = \epsilon_t dt^{0.5}$ where $\epsilon_t$ is normally distributed with zero mean and unit standard deviation).

If the value of metal price is currently “$P_0$”, then its expected value at any future time “$t$” is:

$$E[P_t] = \bar{P} + (P_0 - \bar{P}) e^{-\eta t}$$  

(8)

The expected price change from year “$t-1$” to year “$t$” can be expressed such as:

$$E[dP] = E[P_t] - P_{t-1} = \bar{P} + (P_{t-1} - \bar{P}) e^{-\eta t} - P_{t-1}$$  

(9)

Rearranging Equation (9):

$$P_t - P_{t-1} = \bar{P} \left(1 - e^{-\eta t}\right) + P_{t-1} \left(e^{-\eta t} - 1\right)$$  

(10)

Since both “$\bar{P}$” and “$\eta$” are constants for the metal price under study, then Equation (10) can be rewritten as:

$$P_t - P_{t-1} = a + bP_{t-1}$$  

(11)

Where

$$a = \bar{P} \left(1 - e^{-\eta t}\right)$$  

(12)

and

$$b = \left(e^{-\eta t} - 1\right)$$  

(13)

The parameters “$a$” and “$b$” can be determined by regression analysis using the historical data of metal prices. After determining “$a$” and “$b$”, the normal level of metal price “$\bar{P}$” and the speed of reversion “$\eta$” can be determined using Equations (12) and (13) as follows:
\[ P = -\frac{a}{b} \quad (14) \]

and

\[ \eta = -\ln(1 + b) \quad (15) \]

After determining \( P \) and \( \eta \) for the price of metal under study, then Equation (8) can be used to forecast the future values of metal price throughout the project life.

2.2.2. Certainty-equivalent price

Assume that the current spot price is \( P_0 \), then, the expected future price at any future time \( t \) is obtained from Equation (8). Since these expected future prices are risky, the present value \( PV_r \) of the stream of the risky prices over the project lifetime \( T \) is calculated using the risk-adjusted discount rate \( r \) as follows:

\[ PV_r = \int_0^T P e^{-rt} dt \quad (16) \]

Substituting for \( P_t \) from Equation (8), then:

\[ PV_r = \int_0^T \left( P + (P_0 - P) e^{-\eta t} \right) e^{-rt} dt \quad (17) \]

then

\[ PV_r = \frac{P}{r} \left( 1 - e^{-rt} \right) + \frac{(P_0 - P)}{\eta + r} \left( 1 - e^{-\left(\eta + r\right)t} \right) \quad (18) \]

Assume that the certainty-equivalent constant price throughout the project life is \( P^* \). Since this price is risk-free, the present value \( PV_c \) of the stream of the risk-free prices over the project life is calculated using the safe, risk-free, discount rate \( r_f \), as:

\[ PV_c = \int_0^T P^* e^{-r_ft} dt \quad (19) \]

then

\[ PV_c = \frac{P^*}{r_f} \left( 1 - e^{-r_ft} \right) \quad (20) \]

\( PV_c \) represents the risk-free amount of money that the investor is willing to accept as a substitute for the risky amount \( PV_r \). Since the price \( P^* \) represents the certainty-equivalent of risky future prices, the present value of the stream of the risk-free prices calculated at the risk-free discount rate must equal to the present value of the stream of the risky prices calculated at the risk-adjusted discount rate. Equating \( PV_c \) to \( PV_r \) produces:

\[ \frac{P^*}{r_f} \left( 1 - e^{-r_ft} \right) \left( 1 - e^{-rt} \right) + \frac{(P_0 - P)}{\eta + r} \left( 1 - e^{-\left(\eta + r\right)t} \right) = \frac{P}{r} \left( 1 - e^{-rt} \right) \quad (21) \]

Rearranging:

\[ P^* \left( 1 - e^{-r_ft} \right) + \frac{(P_0 - P)}{\eta + r} \left( 1 - e^{-\left(\eta + r\right)t} \right) = \frac{P}{r} \left( 1 - e^{-rt} \right) \quad (22) \]

where

\( P^* \) certainty-equivalent constant price throughout the project life,

\( P_0 \) current spot metal price at the decision time, year 0,

\( P \) normal level of metal price,

\( \eta \) speed of reversion,

\( T \) project lifetime, years,

\( r_f \) risk-free real discount rate, usually based on government bond rates (Smith, 1995), and

\( r \) risk-adjusted real discount rate.

The risk-adjusted real discount rate is determined using the capital asset pricing model as follows (Brealey and Meyers, 1996; Lumby and Jones, 1999):

\[ r = r_f + \beta (r - r_f) \quad (23) \]
where

\[ \beta \]  
project beta, which reflects the degree of responsiveness of the expected return on the project relative to movements in the expected return on the market. For example, \( \beta = 1.13 \) for base metals mining and 0.27 for gold mining (Smith, 1995). Since most of project risk comes from the volatility of metal prices, the beta factor is assumed to be reflecting the variability of the metal prices with respect to the market. 

\( r_m \)  
expected return on the market, and 

\( r_m - r_f \)  
market risk premium. 

The price "P*" obtained from Equation (22) is the constant average price throughout the project life. Hence, it should be inputted into any optimization process that depends on the realized metal price.

3. DISCUSSIONS

Consider a copper project in which the copper ore is mined out and processed, and the copper is refined and sold by a single company. For simplicity, assume that the project produces only copper cathodes and does not produce any other by-products. The economic performance of the processing plant is measured by the net smelter return per ton of feed "F". The plant is operating at the optimum economic conditions when "F" is maximized. As indicated from Equations (1) to (6), "F" depends mainly on the concentrate grade, the recovery and the copper price. Since the size of the project is small with respect to the whole market, the project is a price-taker rather than a price-maker. Therefore, the price of copper is tied to the project and the managers of the project can do nothing with the copper price. They can only select the operating conditions of the plant (e.g., the concentrate grade and the recovery) so as to maximize "F" at the given copper price. 

Table 1 lists the prices of copper cathodes in 1998 constant American dollars for the period from 1921 to 2000. *(The prices are obtained from U.S. Geological Survey Open-File Report 01-006 in $/metric tonne of copper cathodes and converted to USD/lb.)*

<table>
<thead>
<tr>
<th>Year</th>
<th>Price, USD/lb</th>
<th>Year</th>
<th>Price, USD/lb</th>
<th>Year</th>
<th>Price, USD/lb</th>
<th>Year</th>
<th>Price, USD/lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1924</td>
<td>1.266</td>
<td>1944</td>
<td>1.112</td>
<td>1964</td>
<td>1.702</td>
<td>1984</td>
<td>1.050</td>
</tr>
<tr>
<td>1925</td>
<td>1.325</td>
<td>1945</td>
<td>1.094</td>
<td>1965</td>
<td>1.824</td>
<td>1985</td>
<td>1.015</td>
</tr>
<tr>
<td>1926</td>
<td>1.289</td>
<td>1946</td>
<td>1.171</td>
<td>1966</td>
<td>1.810</td>
<td>1986</td>
<td>0.982</td>
</tr>
<tr>
<td>1927</td>
<td>1.221</td>
<td>1947</td>
<td>1.552</td>
<td>1967</td>
<td>1.860</td>
<td>1987</td>
<td>1.184</td>
</tr>
<tr>
<td>1931</td>
<td>0.898</td>
<td>1951</td>
<td>1.543</td>
<td>1971</td>
<td>2.098</td>
<td>1991</td>
<td>1.309</td>
</tr>
<tr>
<td>1932</td>
<td>0.690</td>
<td>1952</td>
<td>1.502</td>
<td>1972</td>
<td>2.005</td>
<td>1992</td>
<td>1.248</td>
</tr>
<tr>
<td>1933</td>
<td>0.917</td>
<td>1953</td>
<td>1.774</td>
<td>1973</td>
<td>2.183</td>
<td>1993</td>
<td>1.034</td>
</tr>
<tr>
<td>1936</td>
<td>1.143</td>
<td>1956</td>
<td>2.514</td>
<td>1976</td>
<td>1.993</td>
<td>1996</td>
<td>1.133</td>
</tr>
<tr>
<td>1937</td>
<td>1.520</td>
<td>1957</td>
<td>1.742</td>
<td>1977</td>
<td>1.797</td>
<td>1997</td>
<td>1.087</td>
</tr>
<tr>
<td>1938</td>
<td>1.184</td>
<td>1958</td>
<td>1.488</td>
<td>1978</td>
<td>1.645</td>
<td>1998</td>
<td>0.787</td>
</tr>
<tr>
<td>1939</td>
<td>1.316</td>
<td>1959</td>
<td>1.733</td>
<td>1979</td>
<td>2.071</td>
<td>1999</td>
<td>0.743</td>
</tr>
<tr>
<td>1940</td>
<td>1.343</td>
<td>1960</td>
<td>1.779</td>
<td>1980</td>
<td>2.005</td>
<td>2000</td>
<td>0.835</td>
</tr>
</tbody>
</table>

It is well known that there is an inverse relationship between the concentrate grade and the recovery, which is called the grade-recovery curve. Figure 1 shows the grade-recovery curve for a copper ore with a liberation coefficient of 23.0 and a pure mineral value of 47.37% [data obtained from Hall (1971)]. In order to maximize "F", the managers can move through the curve to the point that gives the maximum "F". Assume that the project is optimized in 1980 when the copper price was USD2.0/lb and the production commenced in 1981 at a capacity of 20,000 ton per day throughout the project life which is assumed to be 20 years. Also, assume that the refining cost was USD0.1/lb and the smelting cost was USD100/ton (Farrish, 1989). Based on these assumptions, the optimum recovery and the optimum concentrate grade that maximize "F" have been found to be 90.1% and 18.36%, respectively, as indicated in Figure 2. These optimum operating conditions depend on the copper price used in the analysis. If the copper price is
changed, the optimum conditions will change. As illustrated in Figure 3, both the optimum recovery and the optimum concentrate grade are sensitive to the copper price used in the optimization process. As the copper price increases, the optimum recovery increases and the optimum concentrate grade decreases.

![Figure 1. Grade-recovery relationship](image1)

![Figure 2. The optimum recovery and the optimum concentrate grade](image2)

![Figure 3. Dependence of both the optimum recovery and the optimum concentrate grade on the copper price](image3)
As shown in Figure 4, the copper price is highly volatile. It fluctuates from year to another due to unpredictable economic conditions. Therefore, managers of the plant should take into account such behavior of future copper prices when designing the plant. In this respect, the certainty-equivalent price throughout the project life should be estimated. This price represents the constant copper price upon which the metallurgical performance of the plant should be optimized.

![Figure 4. Copper price from 1921 to 2000 in 1998 constant US dollars](image)

Based on the copper prices listed in Table 1, the normal level to which the copper price tends to revert and the speed of reversion have been found to be USD1.44/lb and 0.17 respectively. The certainty-equivalent price for our example project, using Equation (22) with a real-risk-free interest rate of 2.5%, a "β" factor of 1.13 and a market risk premium of 5% (Smith, 1995), has been found to be USD1.046/lb. Feeding it into the optimization process, the new optimum recovery and the optimum concentrate grade have been found to be 85.4% and 23.31% respectively. Figure 5 illustrates how the optimum recovery is changed when accounting for the fluctuation of future copper prices.

![Figure 5. The change in the optimum recovery when accounting for the price volatility](image)

It is important now to investigate the contribution of the procedures developed in this study in improving the financial performance of the project. Table 2 lists the actual copper prices throughout the life of the hypothetical project that is assumed to be optimized in the year 1980 and commenced production in 1981. The project is assumed to process 20,000 tons of copper ore per day throughout its 20 year life.
The annual revenues of the project are estimated based on the actual copper prices in the two cases, that is when optimizing the plant using the certainty-equivalent price and when optimizing the plant using the copper price prevailing at the project start-up (year 1980). Comparing the last two columns of Table 2, it is clear that the annual revenues generated when optimizing the plant using the certainty-equivalent price are mostly greater than the annual revenues generated when optimizing the plant using the price prevailing at the optimization time (year 1980). More definitely, optimizing the plant using the certainty-equivalent price has generated a USD38.44 million more revenue throughout the project life. Accordingly, it could be concluded that, optimizing the recovery and the concentrate grade using the certainty-equivalent price can improve the economic performance of the heavy industrial project.

Table 2
Comparison between annual revenues

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual copper price, USD/lb (1998 constant dollars)</th>
<th>Annual revenue when optimizing the plant using the certainty-equivalent price USDmillion</th>
<th>Annual revenue upon optimizing the plant using the 1980 price USDmillion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>1.510</td>
<td>187.958</td>
<td>188.710</td>
</tr>
<tr>
<td>1982</td>
<td>1.230</td>
<td>143.936</td>
<td>142.265</td>
</tr>
<tr>
<td>1983</td>
<td>1.253</td>
<td>147.505</td>
<td>146.030</td>
</tr>
<tr>
<td>1984</td>
<td>1.050</td>
<td>115.558</td>
<td>112.325</td>
</tr>
<tr>
<td>1985</td>
<td>1.015</td>
<td>110.197</td>
<td>106.669</td>
</tr>
<tr>
<td>1986</td>
<td>0.982</td>
<td>104.992</td>
<td>101.178</td>
</tr>
<tr>
<td>1987</td>
<td>1.184</td>
<td>136.673</td>
<td>134.602</td>
</tr>
<tr>
<td>1988</td>
<td>1.662</td>
<td>211.777</td>
<td>213.840</td>
</tr>
<tr>
<td>1989</td>
<td>1.723</td>
<td>221.415</td>
<td>224.008</td>
</tr>
<tr>
<td>1990</td>
<td>1.535</td>
<td>191.873</td>
<td>192.840</td>
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<td>1991</td>
<td>1.309</td>
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<td>1993</td>
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<tr>
<td>1994</td>
<td>1.221</td>
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<tr>
<td>1995</td>
<td>1.480</td>
<td>183.242</td>
<td>183.734</td>
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<tr>
<td>1996</td>
<td>1.133</td>
<td>128.749</td>
<td>126.242</td>
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<tr>
<td>1997</td>
<td>1.087</td>
<td>121.406</td>
<td>118.495</td>
</tr>
<tr>
<td>1998</td>
<td>0.787</td>
<td>74.256</td>
<td>68.750</td>
</tr>
<tr>
<td>1999</td>
<td>0.743</td>
<td>67.401</td>
<td>61.518</td>
</tr>
<tr>
<td>2000</td>
<td>0.835</td>
<td>81.881</td>
<td>76.795</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

The optimum economic operating conditions of a processing plant are highly dependent on the price of the metal produced. The metal price based on which the plant is optimized is more likely to be changed because future metal prices are uncertain and that a long span of time exists between project commissioning and closing. Optimizing the plant on the basis of the metal price prevailing at the project start-up may result in inefficient plant operations.

The results indicated that both the optimum recovery and the optimum concentrate grade have been changed significantly when considering the volatility of future copper prices. Also, it has been found that the annual revenues generated from the project increase when optimizing the plant based on the certainty-equivalent price ($P^*$) rather than the spot metal price prevailing at the optimization time.

REFERENCES

Currie, J.M., Unit operations in mineral processing. 1973, Published by Colorado School of Mines, 332 p.


**NOMENCLATURE**

A liberation coefficient
C weight of concentrate produced from one ton of feed
C_d percentage deduction
C_r realization cost, per ton of concentrate
f feed grade
F_r revenue per ton of feed
G concentrate grade
M pure mineral value
NSR_c net smelter return per ton of concentrate
P unit price of metal
P^c certainty-equivalent constant price throughout the project lifetime
P_0 normal level to which the metal price tends to revert
P_o current spot price of metal, at the project start-up, year 0
PV_c present value of a stream of certain prices over the project lifetime
PV_r the present value of a stream of uncertain prices over the project lifetime
R percentage recovery
r risk-adjusted discount rate
r_e refining charge
r_f safe risk-free discount rate
r_m expected return on the market
r_m-r_f market risk premium.
T project lifetime
T_c treatment charge, per short ton of concentrate
u unit deduction
USD United States dollars
X penalties, per short ton of concentrate
Y by-product credit, per ton short of concentrate

Greek letters
\( \beta \) the project beta, which reflects the degree of responsiveness of the expected return on the project relative to movements in the expected return on the market
\( \eta \) the speed of reversion