APPLICATION OF MONTE CARLO SIMULATIONS TO SYSTEM RELIABILITY ANALYSIS

by

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ABSTRACT

Monte Carlo analysis is a powerful tool for modeling the reliability of systems. Proper application of this tool requires an understanding of its underlying principles. In this paper, Monte Carlo analysis is explained at a fundamental level with special emphasis on its application in estimating the reliability of pump systems. The first part of the paper explains step-by-step how to perform a Monte Carlo simulation. In the second part of the paper, examples are given to demonstrate the analysis techniques, what inputs are required, and how to use the information obtained from the analysis.

INTRODUCTION

Over the last decade, the use of statistical analysis techniques has become more and more widespread in the field of reliability engineering. Decisions that were once made based on intuition are now driven by a scientific analysis of maintenance data. The question of "How good is our data?" seems to always surface during the migration to this methodology of decision making. But the fact is that, so long as the size of the population of data is adequate, decisions based on statistical analysis are still better than the "gut-feel" approach.

There are a wide variety of statistical analysis tools available to the modern reliability specialist. Weibull analysis, in particular, has proven to be a highly useful tool for equipment failure analysis because of its ease of use and robust results. With the aid of any one of several commercially available software packages, reliability specialists can examine equipment failure modes without having to wade through complex mathematics.

Another statistical analysis technique, known as a Monte Carlo simulation, can also be extremely useful in analyzing system reliability. Monte Carlo simulation was developed in the 1940s as part of the atomic bomb program. Scientists at the Los Alamos National Laboratory originally used it to model the random diffusion of neutrons. The scientists who developed this simulation technique gave it the name "Monte Carlo" after the city in Monaco and its many casinos. Today, Monte Carlo simulations are used in a wide array of applications, including physics, finance, and system reliability.

When applied to reliability analysis, Monte Carlo simulations use the failure and repair statistical distributions of individual equipment units to model the system behavior over time. The simulation results can then be used to make more accurate decisions for improving the reliability of the system. Step-by-step procedures for performing Monte Carlo simulations are explained, then demonstrated in three example applications.

FUNDAMENTALS OF MONTE CARLO SIMULATION

Monte Carlo analysis uses statistics to mathematically model a real-life process and then estimate the likelihood of possible outcomes. Before performing a Monte Carlo simulation, the statistical distributions of the failure and repair processes must be determined. Most failure processes are best modeled using the Weibull distribution, while the lognormal distribution is usually best for modeling repair processes. Some exceptions worthy of noting are electronic circuit components, which tend to have exponentially distributed times to failure (TTF), and modular type repairs (i.e., swapping out an entire unit rather than just the failed component), which commonly have exponentially distributed times to repair (TTR). Companies with mature reliability programs will already have the statistical distributions determined for much of their equipment. However, when this information is not readily available, the appropriate distribution can easily be determined by curve fitting the TTFs and TTRs with any math or statistical software package. Often, reasonable accuracy is obtainable using as few as four or five data points. TTF and TTR data points can be taken from an individual piece of equipment over time or from a group of units provided they are sufficiently similar.

Once the failure and repair distributions are known, a Monte Carlo simulation can be performed to model the process over time. To perform the simulation, a random number (RN) between 0 and 1 is generated to represent the probability of an event occurring at a given time. From the probability, a corresponding event-time can be calculated using the statistical distribution chosen to model the process. For example, if a random number of 0.7785 were generated and assigned to a failure process, the Monte Carlo simulation would then determine the TTF corresponding to a 77.85 percent probability. If the failures in this scenario were Weibull distributed with parameters of $\alpha = 2$ and $\beta = 10{,}000$ hours, then there would a 77.85 percent chance of a failure by time 12,275 hours (assuming the simulation began at time 0). In effect, Monte Carlo simulations work backwards from the outcome probability to determine the event-time.

By including a similar model for repairs, the failure-repair cycle can be simulated for any time duration desired. The next probability generated by the random number stream would yield a repair time for the failed equipment. The calendar is then updated and the simulation continues by calculating the next TTF from the third random number as previously prescribed. Therefore if the simulated repair time were 12 hours and the subsequent failure time were 8900 hours, the simulation calendar would be at t = 12,275 + 12 + 8900 = 21,187 hours. The simulation would continue if the specified running time were greater than the current calendar time of 21,187 hours. Otherwise, it would be stopped.

Combining individual units to form larger systems can simulate entire plants. When economic considerations such as lost profits and repair costs are included in the model, Monte Carlo simulations can be a powerful tool for optimizing maintenance policies. Smaller system simulations can be performed using any spreadsheet software, such as Microsoft® Excel, with the necessary statistical distribution functions built in. Larger systems can quickly become too difficult to manage without the aid of simulation software. The following application examples illustrate how Monte Carlo simulations are applied to system reliability analysis. The second example is performed using RAPTOR 4.0, a software program developed by the U.S. Air Force in the 1980s. It can be downloaded, free of charge, from several Internet sites. Example 3 demonstrates how to simulate a pump as a system of components.

EXAMPLES

Example 1

Using Monte Carlo analysis, estimate the predicted availability of a steam turbine over the next 10 years. How many failures should be expected during that time period? The failure and repair distributions over the past 10 years are:

$$\Pr_f(t) = 1 - \exp\left[-\left(\frac{TTF}{1600\ days}\right)^{3.2}\right] \tag{1}$$

$$\Pr_r(t) = Lognormal\{\mu = \ln(6.2), \sigma = \ln(1.6)\} \tag{2}$$

Since the lognormal distribution has no closed-form solution for the cumulative distribution function, standardized tables have been developed from numerically generated solutions. The standardized tables are used by first computing the parameter z as follows:

$$z = \frac{TTR - \ln(6.2)}{\ln(1.6)} \tag{3}$$

Solution

1. First a RN stream for each process (failure and repair) must be generated. The simplest way to accomplish this is using a random number function that is built into a spreadsheet or a math software package. The following RN streams will be used for this example:

- Stream #1 = 0.647552, 0.420674, 0.435368
- Stream #2 = 0.034957, 0.773936, 0.719203

2. Calculate the first time to failure (TTF). To do so, Equation (1) must be solved for the TTF:

$$TTF = \left(-\ln(1 - 0.647553)\right)^{\frac{1}{3.2}} * 1600\ days = 1621.1\ days \tag{4}$$

3. Calculate the subsequent repair time. Just as before, Equation (3) must be solved for TTR. Using the standardized normal distribution tables (or software with the tables preprogrammed) at a probability of 0.034957, the corresponding z = −1.085. Therefore,

$$TTF = \exp\left[-1.085 * \ln(1.6) + \ln(6.2)\right] = 2.6\ days \tag{5}$$

4. Next, the calendar is updated to reflect total elapsed time.

$$Now = 1621.1 + 2.6 = 1623.7\ days \tag{6}$$

5. Repeating step 2 with the second RN from Stream #1 yields:

$$TTF = \left(-\ln(1 - 0.420674)\right)^{\frac{1}{3.2}} * 1600\ days = 1324.2\ days \tag{7}$$

6. Repeating step 3 with the second RN from Stream #2 yields:

$$TTR = \exp\left[0.751783 * \ln(1.6) + \ln(6.2)\right] = 8.8\ days \tag{8}$$

7. Updating the calendar:

$$Now = 1623.7 + 1324.2 + 8.8 = 2956.7\ days \tag{9}$$

8. This process is continued until the stopping criterion of 10 years (3650 days) is met or exceeded. The next TTF calculated is 1343 days, which will bring the calendar beyond the 10 year stopping criterion. Therefore, the expected number of failures over the next 10 years is two. The expected availability is:

$$A = \frac{TTF}{TTF + TTR} = \frac{(1621.1 + 1324.2)}{(1621.1 + 1324.2) + (8.8 + 2.6)} = 0.99614 \tag{10}$$

Example 2

In order to keep their process operational, a chemical plant requires four out of five cooling water pumps to be running. Net revenues from product sales are approximately $250,000/day. The TTRs for all pumps are lognormal distributed with μ = 600, σ = 25. The average hourly repair cost for all the pumps is $150/hour. The TTFs for four pumps are known to have Weibull distributions with α = 0.8 and β = 24,350 hours. The fifth pump is much older and its failure distribution is unknown, but maintenance records over the past 10 years show that failures occurred on the following dates: 11/13/91, 4/2/94, 8/19/95, 10/20/97, 1/3/99. Management feels that the current pump system is not reliable enough and that an upgrade is needed. Adding a sixth pump identical to the newer four pumps would cost $500,000. However due to savings associated with reusing existing auxiliaries, just replacing the older pump with a newer model would cost 20 percent less. Based on a 20 year projection, which option should be selected? Is an upgrade even justified?

Solution

1. Determine the TTFs for each failure experienced by the fifth pump. (One simple way to accomplish this is to put the failure dates into one column of a Microsoft® Excel spreadsheet. In the next column of the spreadsheet, subtract the adjacent cell from the one above it, and then convert that column format to "Number.") The results are given below.

- Pump #5 TTFs (hours): 20,904, 12,096, 19,032, 10,560

2. Determine the failure distribution that best fits the TTF data. (This is easily done in any math or statistical software program. The data in this example were fitted to the model below using STATGRAPHICS®.)

$$F(t) = 1 - \exp\left[-\left(\frac{t}{17321}\right)^{4.11}\right] \tag{11}$$

3. Run system simulation using Monte Carlo analysis.

a. First simulate the existing system. A reliability block diagram (RBD) of the system is shown in Figure 1. Figure 2 is the RAPTOR input window used to define the model. Further sophistication can be built into the simulation using the "Maintenance Information" and "Advanced" tabs, but they are not needed for this exercise. Finally, the simulation output is shown in Figure 3.

b. Review the results. The output of RAPTOR gives us a great deal of information about the system, but the statistic we need for this exercise is Availability, A_o. Availability is defined as:

$$A = \frac{TTF}{TTF + TTR} \tag{12}$$

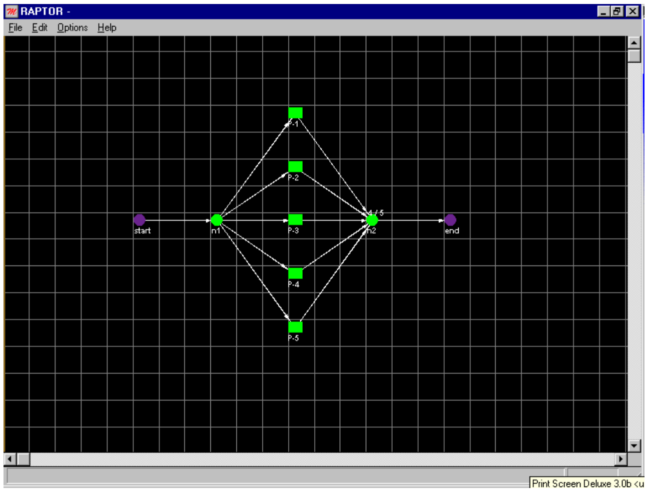


Figure 1. Cooling Tower Pump System Reliability Block Diagram.

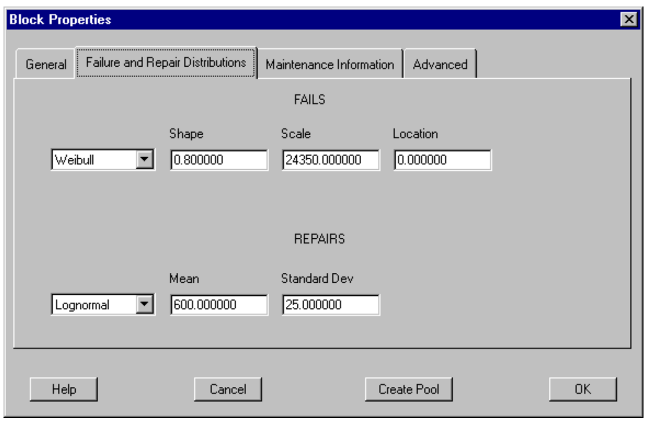
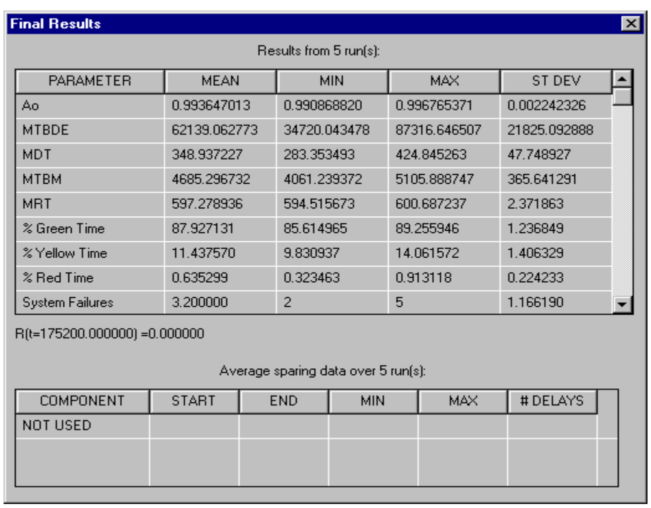


Figure 2. Component Failure and Repair Distribution Entry Form.


Final Results

Results from 5 run(s):

PARAMETER	MEAN	MIN	MAX	ST DEV
Ao	0.993647013	0.990868820	0.996765371	0.002242326
MTBDE	62139.062773	34720.043478	87316.646507	21825.092888
MDT	348.937227	283.353493	424.845263	47.748927
MTBM	4685.296732	4061.239372	5105.888747	365.641291
MRT	597.278936	594.515673	600.687237	2.371863
% Green Time	87.927131	85.614965	89.255946	1.236849
% Yellow Time	11.437570	9.830937	14.061572	1.406329
% Red Time	0.635299	0.323463	0.913118	0.224233
System Failures	3.200000	2	5	1.166190

R(t=175200.000000) =0.000000

Average sparing data over 5 run(s):

COMPONENT	START	END	MIN	MAX	# DELAYS
NOT USED					

Figure 3. Results of Cooling Tower Pump System Simulation.

At this point, it is worth noting that five simulations of the system were run rather than just one. The reason for this is best understood by recalling that the beginning point of the Monte Carlo simulation is a random number stream. Since these numbers are selected at random, there is always a possibility that extreme numbers may be chosen (i.e., 0.00001). The effects of any extremities in the random number stream are greatly reduced by running multiple simulations and averaging the statistics. Therefore, to model the existing system availability, we will use the average (or mean) availability of the five simulations—0.9936.

3. Estimate total downtime cost.

$$TDC = (1 - A_o)\left[\frac{\$250{,}000}{day}\right]\left[\frac{365\ days}{yr}\right]\left[20\ yrs\right] = \$11.68\ million \quad (13)$$

An even more accurate model could be built by using an investment rate of return and the pump failure times to calculate the net present value of the downtime cost. However, since the failures will mostly occur in the same general time period, the differences between each case will not change in magnitude. The cost of repairs can also be excluded from the model for decision-making purposes, since it is not a function of which pump is being repaired. If the total life-cycle cost is being investigated, these effects should be included.

4. Repeat steps 2 and 3 for the two other cases considered. The results are summarized in Table 1. Clearly, the most economical alternative is to add a new pump to the existing system. This example demonstrates how much impact even small improvements in reliability can have. Until it is coupled with economics, 99.36 percent reliability seems great!

Table 1. Economic Evaluation of Cooling Tower Pump System Reliability.

Case Description	20 yr. A_o	Total Downtime Cost ($M)	Initial Cost ($M)	Total Cost ($M)
Existing System	0.9936	$11.86	$0	$11.86
Replace older pump	0.9980	$3.65	$0.4	$4.05
Add new pump to existing system	1.0	$0	$0.75	$0.75

Example 3

A centrifugal condensate pump has components with the following Weibull failure distributions:

- Mechanical seal (α = 0.75, β = 967 days)
- Two bearings (α = 0.52, β = 2701 days)
- Casing (α = 0.60, β = 6095 days)
- Shaft (α = 0.43, β = 7280 days)

Assuming repairs are lognormal (μ = 0.5 days, σ = 0.2 days), what is the expected availability of the system over a 10 year period?

Solution

1. Model the pump as a system of components in series (Figure 4) with the distributions given.

2. Run the simulations using RAPTOR 4.0, and analyze the results (Figure 5). (RAPTOR or an equivalent software package will automatically simulate the system using Monte Carlo techniques.)

3. The mean availability of the 10 simulations run was 0.9991. Over 10 years, that equates to approximately 3.5 days that the pump will be out of operation.

CONCLUSIONS

Fundamental techniques for performing a Monte Carlo simulation have been explained and demonstrated. This tool can easily be applied to any system that comprises smaller components with a known, or at least determinable, failure distribution. The procedure for performing a Monte Carlo simulation is mathematically simple to use, and therefore lends itself well to spreadsheet calculations. The results of the analysis can then be used to provide economic justification for reliability improvements to existing equipment or to purchase new equipment for the system.

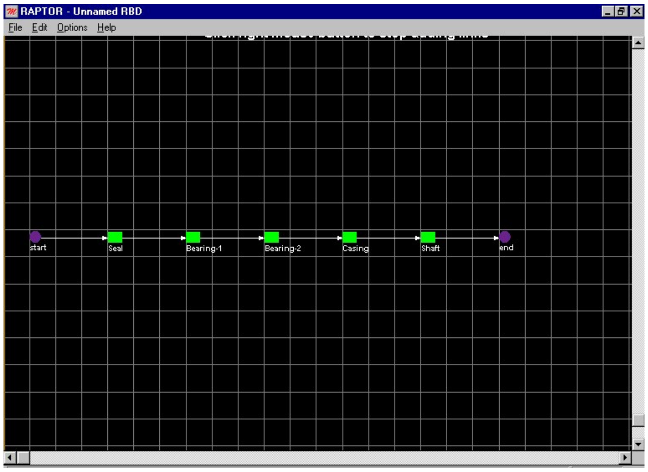


Figure 4. RBD for Example 3.

Final Results

Results from 10 run(s):

PARAMETER	MEAN	MIN	MAX	ST DEV
Ao	0.999098401	0.998207003	0.999700735	0.000476993
MTBDE	640.881427	260.246826	912.226921	229.091782
MDT	0.475716	0.273079	0.614971	0.099397
MTBM	640.881427	260.246826	912.226921	229.091782
MRT	0.475716	0.273079	0.614971	0.099397
% Green Time	99.909840	99.820700	99.970074	0.047699
% Yellow Time	0.000000	0.000000	0.000000	0.000000
% Red Time	0.090160	0.029926	0.179300	0.047699
System Failures	6.800000	4	14	3.310589

R(t=3650.000000) =0.000000

Figure 5. Simulation Results for Example 3.

NOMENCLATURE

A	[-]	Availability
α	[-]	Shape parameter
β	[days *or* hours]	Characteristic life
F	[-]	Cumulative failure probability
Pr	[-]	Cumulative probability
t	[days *or* hours]	Time
TDC	[$]	Total downtime cost
TTF	[days *or* hours]	Time-to-failure
TTR	[days *or* hours]	Time-to-repair
z	[-]	Standardized normal variate

Subscripts

f = Failure
o = Original
r = Repair

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ACKNOWLEDGEMENT

The author wishes to thank ExxonMobil Chemical for permission to publish the presented data, and ARINC products and services for permission to present data from the RAPTOR software.