PREDICTING AND IMPROVING THE DYNAMIC BEHAVIOR OF MULTISTAGE HIGH PERFORMANCE PUMPS

by

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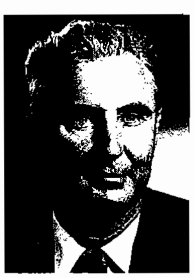

Dusan Florjancic graduated in 1950 from the Federal Institute of Technology in Zurich, Switzerland, with a degree in Mechanical Engineering. After nearly a year of scientific fellowship at the Institute of Aerodynamics in Zurich, he joined the Luwa Company in Zurich as Development Engineer in 1951. Two years later, he joined Sulzer and was first engaged as a Development and Design Engineer in the Gas Turbine Division. In 1958 he became Head of the Hydraulic Department of the Pump Division. In July 1970, he was awarded the Doctor of Engineering degree by the Federal Institute of Technology, Zurich, for a research thesis.

Dr. Florjancic was appointed Technical Manager of the Pump Division of Sulzer Brothers, Limited in 1974. A year later he was appointed to Assistant Vice President, and in 1980 to Deputy Vice President, Head of Technical Department, Pump Division.

ABSTRACT

Good rotordynamic behavior, implying sufficient stability margin and low sensitivity to unbalance forces, is mandatory for modern high pressure multistage pumps. Stiffness, damping and mass matrices describing the forces acting on a rotor at bearings, seals, balancing device and impeller is discussed. Comparisons are made between calculated natural frequencies from coupled damped models and "wet critical speeds" calculated from undamped uncoupled models. The effect of design changes on the stability limit at increased clearances is shown experimentally, and a simplified mathematical model is presented, showing the effect of the various coefficients on the stability limit. Finally, results are shown from experiments and calculations on coupled rotor casing pedestal systems.

INTRODUCTION

Modern boiler feed pumps or injection pumps are turbomachines with very high power concentrations. Typically the power per unit area of the impeller axial projection is 25 MW/m^2 (3200 hp/ft^2) [1]. In addition, high reliability, long life of the hydraulic parts, low sensitivity to transient conditions and good efficiency are required. Sound rotor-dynamic behavior is vital to achieve these aims. Since major design parameters of the rotor, such as bearing span and shaft diameter, are closely linked to the hydraulic design and must be optimized as a system, good predictive methods are necessary.

Historically, rotordynamic design calculations for pumps have evolved in three major steps. First, critical speeds were calculated without consideration of hydraulic effects of the internal clearances. This would still be proper today for the case of a dry-running pump. The second step came with the realization that small internal clearances would create a stiffening effect proportional to the pressure drop. Since the pressure drop is usually proportional to the square of the rotational speed, this stiffening effect takes the form of a negative mass, as first shown by Lomakin [2]. Recently, Gopalakrishnan elaborated experimentally and theoretically on the "Lomakin mass" [3]. Critical speeds could now be calculated including this effect. They are often called "wet critical speeds." In these calculations damping is neglected, and also the components of fluid forces not being in the direction of the displacement are neglected, i.e., the coupling between the vertical and horizontal deflections by the fluid forces is not considered. We call this simplified calculation method "undamped uncoupled." It gives no information on system damping (stability) and limited information on the unbalance response of rotors. Knowledge of these quantities, however, has become more important with the increase in power concentration. It is quite clear that major improvements of high performance pumps in the future can only be made on the basis of more complete dynamic models. This is the third step in rotordynamic calculations. All fluid forces acting on the rotor are considered in their linearized form—stiffness and damping matrices, including coupling terms, for bearings, seals, impeller-diffuser hydraulic inter-action, balance piston, etc. Computer programs to handle such problems were available. However, our knowledge of said stiffness and damping coefficients is still partially insufficient. These calculation methods are valuable design tools, but for exact predictions or the evaluation of bids, these calculations are still limited by the lack of quantitative information of some of the fluid forces acting on the rotor.

In this paper, the state of the art of modelling the fluid forces acting on the rotor is briefly reviewed, typical results of calculations including damping and coupling and their relationship to the classical "wet critical speeds" are discussed, the interaction of rotor- and casing vibrations is treated, and some thoughts on future design criteria for multistage high performance pumps are presented. This paper has been stimulated by an EPRI contract on feed pumps, work on which started in 1983 [4].

PREDICTING THE DYNAMIC BEHAVIOR OF A PUMP ROTOR

Mathematical Model

The mathematical model must be able to predict the following two points (including torsional vibrations which are not discussed here):

- The eigenvalues, consisting of the natural frequencies and damping, and the appropriate mode shapes
- The forced shaft response when the pump is run up from zero speed to operational speed

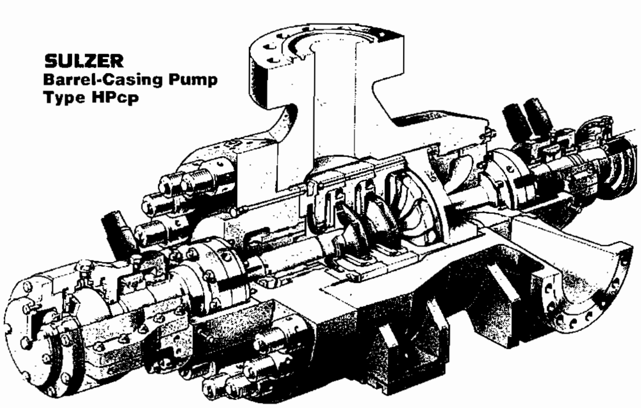

Figure 1. Cross Section Through a Typical Multistage High Performance Pump.

The mass and stiffness properties of the rotor are described either by transfer matrices or by the finite element method. For a full dynamic model, the interaction between rotor and casing vibrations should be considered. This was done here in a special section, whereas in the first part of this paper it was assumed that the casing does not move.

The radial forces acting on the rotor are of two kinds:

1. Forces due to rotor deflections: these forces arise from dynamic rotor deflections. They are usually linearized about the static equilibrium position and are then proportional to the rotor displacement (stiffness, velocity (damping), or acceleration (mass). These forces contribute to the system behavior eigenvalues, mode shapes). They are expressed as follows:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -\begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} - \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} - \begin{bmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \quad (1)$$

F_x, F_y = forces in the x,y-direction acting on the rotor
k_{ij} = stiffness coefficients
c_{ij} = damping coefficients
m_{ij} = mass coefficients

2. Excitation forces: these are forces which are always present, whether the shaft deflects or not. They can be broad band forces (i.e., due to turbulance), transient forces (i.e., due to seismic forces), harmonic forces at rotational frequency or other frequencies (i.e., mass unbalance, vane passing frequency). These forces are needed for forced response calculations. Static forces (i.e., weight, static radial thrust) also belong to this class, although they are not directly considered in dynamic analyses.

A cross section through a multistage high performance pump is shown in Figure 1. A schematic diagram indicating the major components generating radial forces is presented in Figure 2. In the following section the forces acting on the rotor are briefly reviewed.

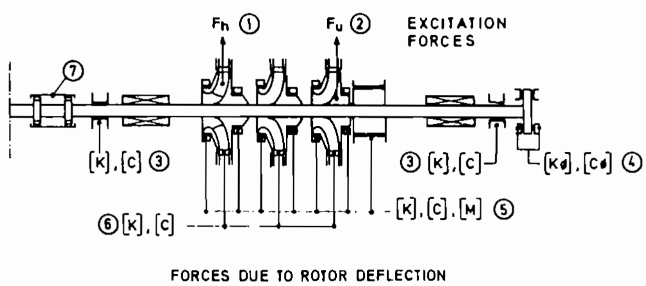

Figure 2. Schematic Diagram and Definition of Dynamical Forces Acting on a Pump Rotor (Linearized Model). ① Hydraulic Excitation Forces. ② Unbalance Excitation Forces. ③ Coefficients for Journal Bearings. ④ Coefficients for Thrust Bearings. ⑤ Coefficients for Sealing Gaps Including Balance Piston. ⑥ Coefficients for Impeller-diffuser interaction. ⑦ Coupling.

FORCES DUE TO ROTOR DEFLECTIONS

Forces in *journal bearings* can reasonably well be linearized for motions not exceeding about a third of the bearing clearance. Inertia forces are normally not significant, therefore the mass term in equation (1) is neglected. Stiffness and damping coefficients are well known from theoretical and experimental work, for example Orcutt [5], Schaffrath [6], Glienicke [7], Lund [8]. The coefficients depend on rotor speed and static load. The static load is the sum of the rotor weight and all static fluid forces. It may be flow dependent and cannot exactly be calculated. A vertical upward offset of the rotor is often employed in order to provide a more definite loading of the bearing.

To describe the forces in the sealing gaps and in the balance piston, the full matrices in Equation (1) may have to be considered. For short plain sealing gaps, the coefficients are reasonably well known from investigations, for example by Black [9], and Childs [10, 11]. Major parameters are pressure drop, rotational speed and inlet swirl (prerotation). The effect of serrations is known quantitatively to some degree [12, 13, 14, 15, 16], but the knowledge is not sufficient for good predictions on various designs, especially for long seals such as in balance pistons. We have started experimental work on test rigs for both impeller labyrinths and balance pistons, where different geometries are tested under full speed and pressure conditions. A number of other test rigs are in operation or are planned [17, 18, 19, 20, 21].

In the area of *impeller-diffuser interaction,* often simply called hydraulic interaction, an early experimental investigation was done by Hergt and Krieger [22]. They displaced the impeller statically within the diffuser and measured the static force due to this displacement. Black [23] adapted his theoretical investigations to these results. He gave semiempirical equations for stiffness and damping coefficients. Little is known on the influence of geometric parameters of the impeller and the diffuser on the coefficients. Experimental results on pumps concerning rotor stability indicate considerable destabilizing forces from hydraulic interaction at least for some hydraulic designs. We are presently designing a test stand for the measurement of the coefficients as part of the EPRI-contract [4]. A smaller test stand by Chamieh [24] was partly operational.

Both tilting stiffness and damping are present at the *thrust bearing*. The coefficients depend on axial thrust. For proper modelling of the rotor, it may be necessary to include these effects.

The *coupling* may have an effect on the system model. For short and fairly light couplings, it is usually admissible to decouple the model of the pump rotor from the model of the drive. Usually one-half or one-third of the coupling spacer mass is added to the inboard shaft end, depending on the expected deflection pattern of the coupling. For long coupling spacers, it may be necessary to include the entire shaft train in the model.

EXCITATION FORCES

The most common excitation force is *mechanical unbalance.* It originates from the residual unbalance of all parts fitted to the rotor and from residual shaft runout. Unbalance forces and their distribution and phase angle along the rotor are not predictable, but are held to within acceptable limits by balancing procedures and limits on residual shaft runout. Sometimes the coupling is a source of radial forces, especially if the shafts are not properly aligned.

More difficult to handle are *hydraulic dynamical forces.* They may be more or less broad band random forces, especially at minimum flow conditions [25, 26], or they may be periodic forces characterized by their frequency. Known are subsynchronous forces due to rotating stall at low flows, while synchronous forces, often called "hydraulic unbalance," are due mainly to impeller inaccuracies. Hydraulic dynamical forces may have a strong effect on forced shaft response. Some knowledge exists, for example [22, 25, 26, 27, 28], but a good prediction of these forces as a function of impeller and diffuser geometrical parameters and tolerances is not generally possible. Radial dynamic excitation forces at higher harmonics of the rotational frequency depend on blade numbers of impeller and diffuser. These forces do not generally influence the overall rotordynamic behavior.

COMPUTER PROGRAMS AND INTERPRETATION OF RESULTS

We are using a relatively new computer program which is based on the finite element approach, called MADYN [23]. It was especially developed for calculating rotor-foundation systems, but is usable also for general structures modelled by beam elements. It can handle unsymmetric stiffness and damping matrices, gyroscopic forces, harmonic- and transient excitation, unrestrained systems (i.e., torsional vibrations). Real or complex eigenvalues can be computed with six algorithms to choose from.

An example calculation of "wet critical speeds" calculated from an uncoupled, undamped model and complex eigenvalues calculated from a coupled, damped model is shown in Figure 3. Both calculations were done for the same 5-stage boiler feed pump. For the bearings, the stiffness and damping matrices were derived from Glienicke's [7] measurements, for the wear rings, seals and piston, Black's [9] theory was employed for plain seals. For the undamped, uncoupled case, only the coefficients k_{xx} of bearings, wear rings, seals and piston were used. Curve "a" shows the familiar decrease of the natural frequency with increasing clearance. The results from the coupled damped calculations are much more complex. Three natural frequencies were found in the range of the previous single one, two of which are strongly damped. With increasing clearance only one, mode d, loses damping. With a further increase in clearance, or with additional destabilizing forces, it is this mode which would tend to become instable. It is also the mode most closely approximated by the undamped uncoupled calculation. However, this may not be generally true. What are

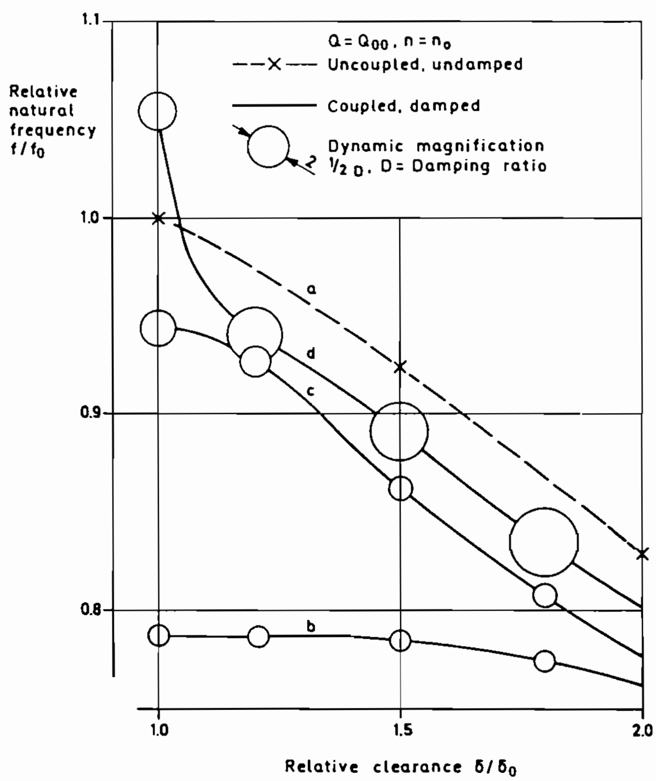

Figure 3. Calculated Natural Frequencies and Damping for a Five Stage Boiler Feed Pump as a Comparison Between "Wet Critical Speeds" (Uncoupled Undamped) and Results from Coupled Damped Calculations. Results Are Normalized to the "Wet Critical Speed" f_0 clearance $\delta/\delta_0 = 1$.

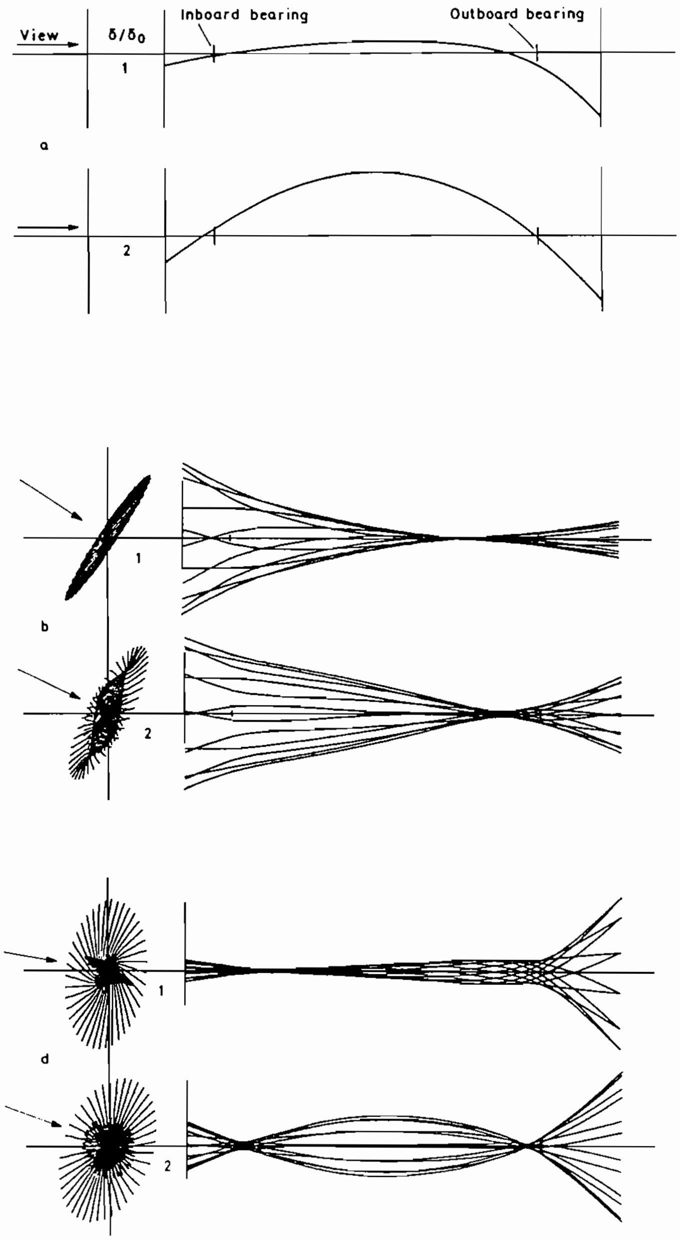

Figure 4. Mode Shapes for Modes a, b, d (Figure 3) for Nominal Clearance $\delta/\delta_0 = 1$ and Twice Nominal Clearance $\delta/\delta_0 = 2$.

the modes b, c, d? Why do we find only made d losing damping with increasing clearance? The answers are found in Figure 4, showing typical mode shapes. Mode b shows a dominant deflection of the inboard overhang, with a forward orbit. Mode c is not shown, as it is similar to mode b, but with its dominant deflection at the outboard overhang. For both modes, the displacements in the bearings are appreciable, even for increased clearances, and lead to high damping of these modes (four-lobe bearings are used in the calculations). Mode d shows both sides deflecting, and with increasing clearance, takes on the shape of the classical first bending mode. Its modes are almost at the bearings, leading to a nearly isotropic system (nearly circular orbit), with the bearings contributing little to system damping. Thus, the system damping is dominated by the hydraulic forces in the sealing gaps and by the hydraulic impeller diffuser interaction forces. System damping decreases strongly with increasing clearance. Mode shape d corresponds fairly well with mode shape a, the undamped uncoupled case, especially at larger clearances.

An unbalance response calculation for the same pump is shown in Figure 5. Clearly, the two strongly damped modes b and c cannot be seen as resonance peaks at all, and even mode d does not lead to much of a resonance—as long as the clearances are not too large. From both considerations, system damping and unbalance response calculations, it is quite clear that the modes b and c are not of much consequence dynamically. Again, this may not be generally the case, especially if the bearings are not properly loaded.

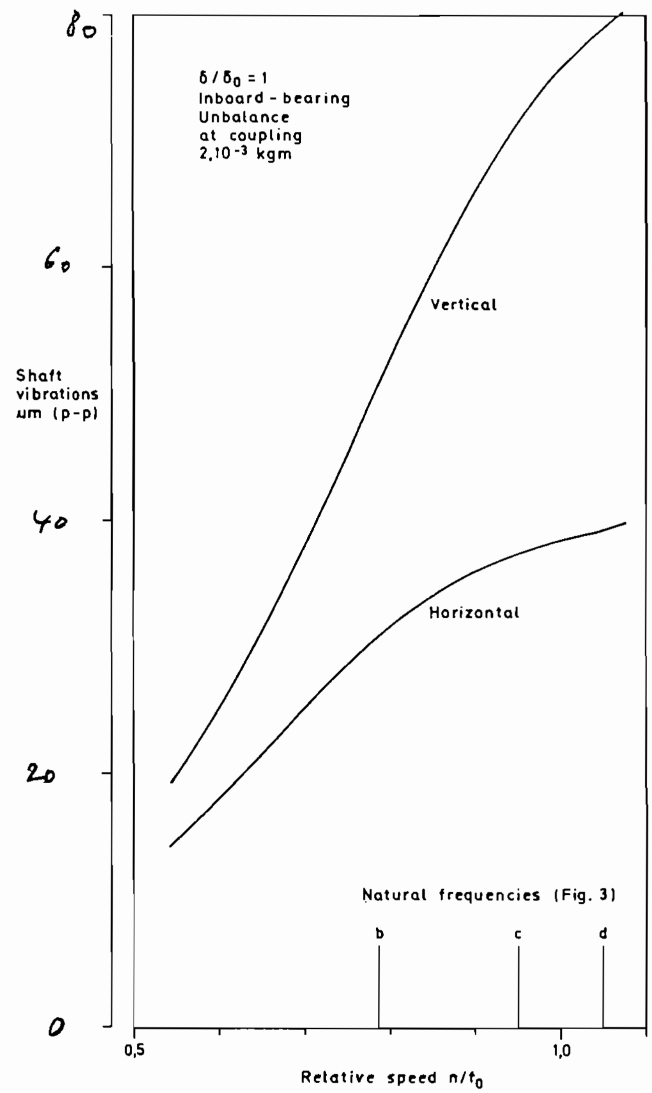

Figure 5. Unbalance Response Calculation for the Same Model as Treated in Figure 3. Operational Speed Normalized by the "Wet Critical Speed" f_0 at Nominal Clearance.

ROTOR STABILITY

The prediction of system damping, on which stability calculations are based, is the most difficult task in dynamic analysis, as it requires full knowledge of the hydraulic force matrices. The damping of the least damped mode calculated for a typical 5 stage boiler feed pump is shown in Figure 6. The model includes full bearing matrices and matrices for internal clear-

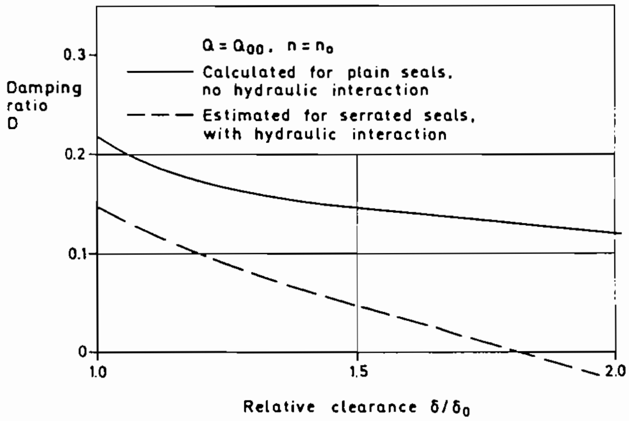

Figure 6. Damping Ratio for the Least Damped Mode for a Five Stage Boiler Feed Pump as a Function of Clearance.

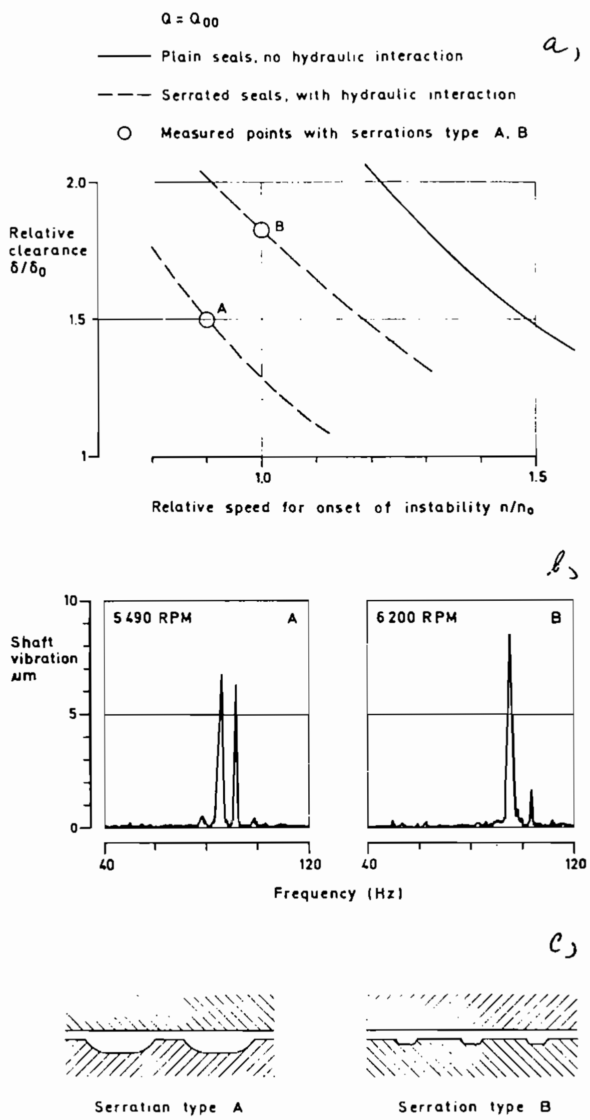

Figure 7. a) Dependence of the Onset Speed of Instability on Clearance, Hydraulic Impeller Diffuser Interaction, and Balance Piston Design (Schematically), With Measured Points. b) Observed Spectra of Shaft Vibrations at the Onset Speed. c) Serrations Type A and B of the Balance Piston, the Design of Impeller Seals Remaining the Same.

ances based on Black's Theory [9], for plain seals. The estimated damping of the actual machine with the serrated piston and hydraulic impeller-diffuser intraction is also indicated in Figure 6. Both contribute to a decrease in system damping which cannot exactly be predicted with today's knowledge of these effects. Some measured results for the stability limits for two different types of serration of the balance piston are presented in Figure 7. Serration type A is a design used for pumps with dry-running capability. The serrations are deep and wide to avoid seizure. Type B is a typical design used in pumps without dry-running capability. It provides for a considerably higher system stability margin. These experiments indicate that there is an additional destabilizing effect, probably from hydraulic interaction between impeller and diffuser, otherwise the instability frequency could not be so close to the running speed frequency.

From the previous discussion of the modeshapes (Figure 4), we know that for a low damped mode, the orbit is almost circular and the bearings do not contribute much to the dynamic behaviour. Under these conditions, a simplified model can give us additional insights on the effect of the various coefficients on stability. We consider a massless rotor carrying a single impeller which is subject to fluid forces, which are completely isotropic.

The equations of motion of such a system are:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c & c_c \\ -c_c & c \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} k & k_c \\ -k_c & k \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \qquad (2)$$

m = mass of the impeller plus fluid masses (coupling terms in the mass matrix neglected)

c = sum of all direct damping terms

c_c = coupling terms of damping

k = sum of all direct stiffness terms (including shaft-stiffness)

k_c = coupling terms of stiffness

For a circular orbit, we combine the two displacements to a complex quantity

$$r = x + jy$$

and can then write equation (2) in the form:

$$mr + cr - jc_cr = kr - jk_cr = 0 \qquad (3)$$

For the limit of stability we can assume a harmonic time function of the form

$$r = r_o e^{j\Omega t} \qquad (4)$$

Ω = natural frequency of the system

After inserting (4) in (3) and separating real and imaginary parts, we obtain the following two equations:

$$\Omega^2 - \Omega \frac{c_c}{m} - \frac{k}{m} = 0 \qquad (5)$$

$$\Omega c = k_c \qquad (6)$$

Equation (5) can be solved for the natural frequency Ω:

$$\Omega = \frac{1}{2} \frac{c_c}{m} + \sqrt{\left(\frac{1}{2} \frac{c_c}{m}\right)^2 + \frac{k}{m}} \qquad (7)$$

We see that the natural frequency is given by the term k/m as one would expect, modified by the coupling term of the damping. Equation (6) simply states that at the limit of stability, the forward driving force proportional to k_c is balanced by the backward driving damping force proportional to Ω_c. In equation (4) a negative value of Ω can also be postulated, representing the backward whirling mode. It can be shown that it always remains stable. The function of the various coefficients at the stability limit is thus quite clear:

m, k, c_c determine the natural frequency

k_c, c, Ω determine the system damping (zero at the stability limit)

We see that the damping force is proportional to the natural frequency, and thus is higher for a stiffer shaft, all other parameters remaining the same. A stiffer shaft is obtained by shorter bearing spans, therefore the trend to lower the number of stages was made. The advantages may, however, be offset by higher rotational speeds required, or larger, heavier impellers and by cavitation problems. The same is true for larger shaft diameters. Note that the rotational speed does not enter the equations at this stage. There is no inherent condition saying that the natural frequency must be so much below the rotational speed for instability to occur.

In principle, an instability could be supersynchronous, i.e., at a natural frequency higher than the rotational speed. The authors are not aware of such a case, but Figure 7 shows a case where the instability frequency is only about 10 percent below the rotational frequency. Clearly, then, for prediction of stability it is necessary to know the dependence of the coefficients on speed, flow, clearance and detailed design features.

THE EFFECT OF CASING VIBRATIONS

So far the casing including the bearings was assumed not to move. In practice, high casing vibrations can occur if one of the pedestal-casing natural frequencies corresponds to the running speed frequency. Field experience has shown that for modern feed pumps with a relatively small axial distance between the fixation points, especially the vertical and horizontal rotational modes (Figure 11), the natural frequencies may lie in the running speed range. Shaft and bearing vibrations measured on such a pump during run-up from 2100 to 6200 rpm are shown in Figure 8. The bearing vibrations, being representative for casing vibrations, show distinct peaks, while the shaft vibrations depend far less on speed. Without casing resonances, the bearing vibrations could easily be a factor of five lower. To show the interaction between rotor and casing vibrations, some calculations have been done on a coupled rotor-casing model. Calculated horizontal and vertical shaft and bearing vibrations are shown in Figure 9. In the model, the casing is undamped. Thus, the interaction effects are somewhat exaggerated. We see that the shaft vibrations are significantly influenced only in the immediate vicinity of the casing natural frequency. Further evaluation of such models is being done now to gain more insight into these coupled vibrations and to obtain design guidelines [4].

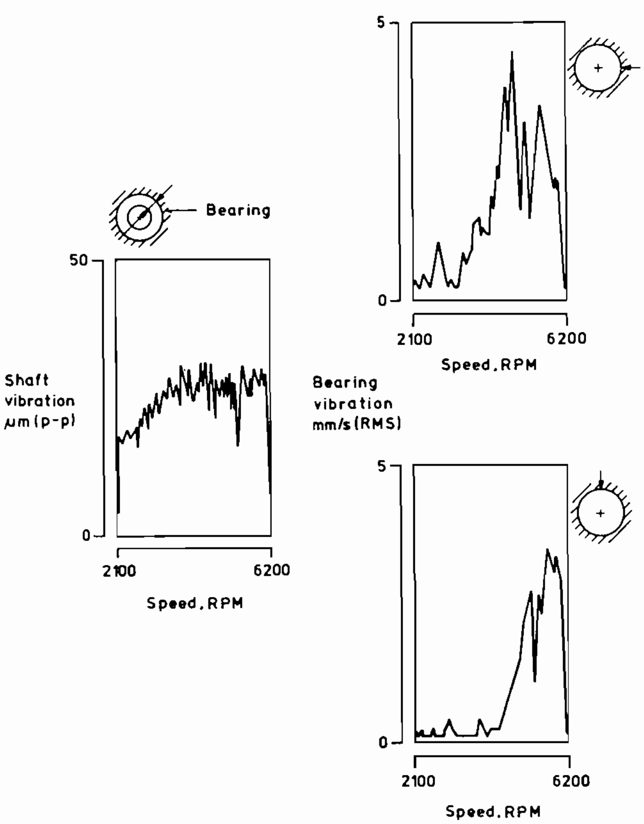

Figure 8. Measured Shaft and Bearing Vibrations on a Pump with Casing Natural Frequencies in the Running Speed Range.

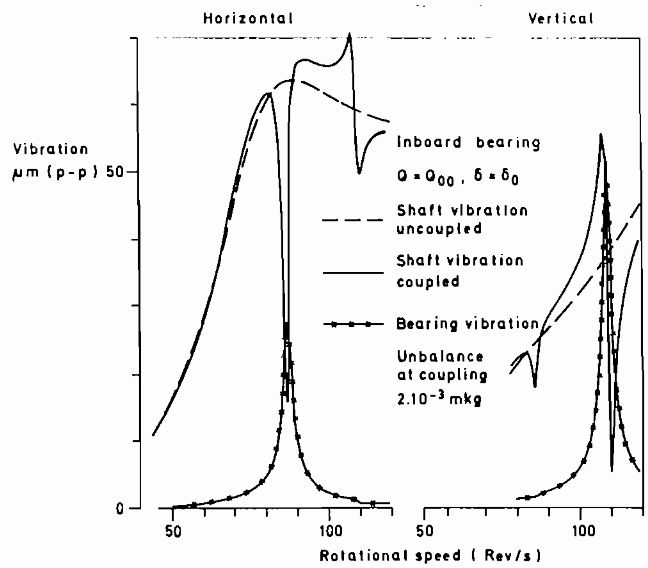

Figure 9. Calculated Vibrations for a Coupled Rotor Casing Pedestal Model.

A number of remedies for pumps in the field are known and have been applied with marginal success. Raising the natural frequencies by stiffening devices has sometimes been successful. It is, however, not always possible to increase the natural frequencies sufficiently, especially in the horizontal direction. In addition, such stiffening devices are cumbersome to install in plants. Another method is to lower the natural frequencies, for instance, by providing slots in the pedestal [3], inserting elastic elements between the base plate and the foundation [24], or providing for additional elasticities in the keys, keeping the pump aligned. In all cases, good dynamic behavior resulted. Again, these are not general solutions, as with the reduced stiffness of the pedestal, the deflection of the pump under the pipe forces and moments may cause alignment problems. A third solution looks quite promising, the use of a dynamic damper attached to the outboard bearing of the pump. The dynamic damper is a mass supported by special rubber elements tuned to the casing resonance frequencies. Such a damper has been successfully tested at full size in the laboratory.

The best solution, obviously, is the optimization of the pedestal in the design stage. The pedestal should be quite stiff in order to maintain the alignment under pipe forces. The significant natural frequencies should lie outside of the running speed range of the pump. Such calculations have been done using the NASTRAN finite element program. The model and also the deformed shapes for two typical modes are shown in Figure 10. Detailed investigations in plants have shown that the major difficulty in predicting the stiffness of the pedestal is the elasticity to be employed in the model at the baseplate-foundation interface. The result of an optimized bedplate is shown in Figure 10. Shown are the calculated natural frequencies for the six mode shapes in question. We see that especially the horizontal and vertical rotational modes have natural frequencies far above the operating range. The operating range is near the axial translational mode which is not excited by radial forces. The horizontal translational mode is below the operating speed range. Thus, a good dynamic behaviour is expected even for mounting conditions somewhat more elastic than assumed in the model.

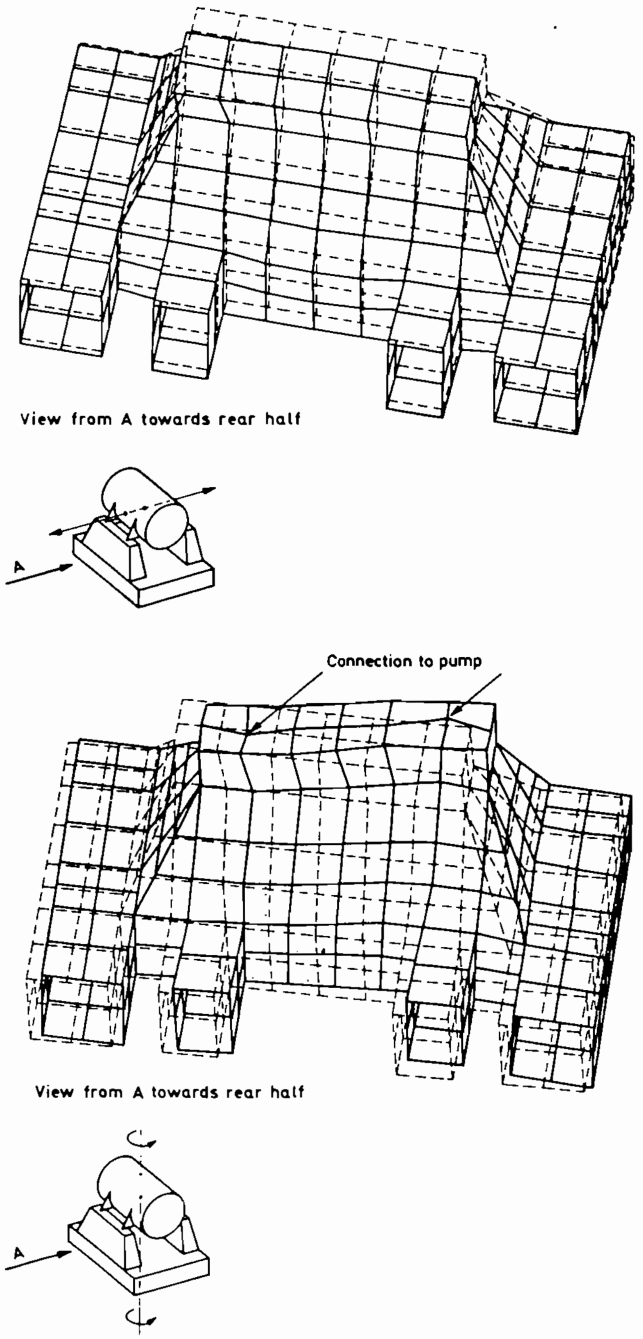

Figure 10. NASTRAN Model of One Half of a Fabricated Bedplate with Pedestals. Deformations Are Due to the Horizontal Translation and the Horizontal Rotation Models (Pump Casing Not Shown).

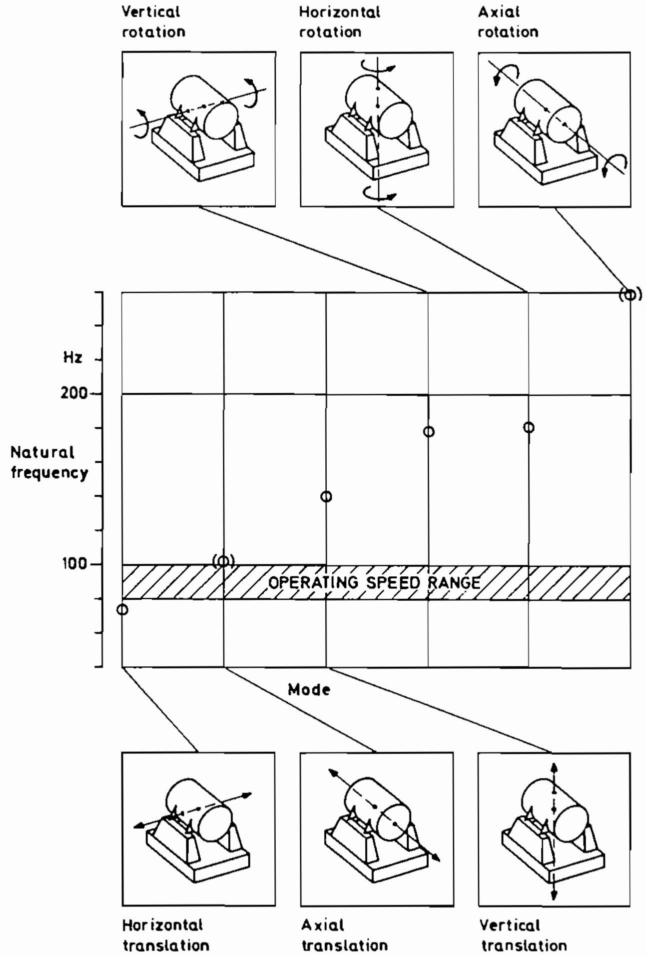

Figure 11. Calculated Natural Frequencies for the Base Plate Model Shown in Figure 10 Carrying a Four Stage Boiler Feed Pump.

CONCLUSIONS

Major improvements of high performance multistage pumps in the future can only be made with improved understanding and good predictive methods of rotor dynamics. Such methods are based on linearized models of all forces acting on the rotor, i.e., stiffness, damping and mass matrices for seals, balancing devices, bearings and hydraulic impeller diffuser interaction. Computer tools are available to calculate natural frequencies, damping complex mode shapes and forced response from such models. Experimental and theoretical investigations are under way to improve the knowledge of the stiffness damping and mass matrices.

Of major concern to the designer is the stability of the rotor and the sensitivity to unbalance forces. These are not critical at nominal clearances, but must be investigated for increased clearances at near end-of-life conditions. Calculations of so called "wet critical speeds" based on undamped uncoupled models are insufficient to judge stability and sensitivity to unbalance forces. In fact, it has been demonstrated that natural frequencies can exist which contribute very little to unbalance response and remain stable even for increased clearances, because of their large damping. Such modes could be tolerated even in the running speed range.

It seems that more weight should be given to actual demonstration of stability and low sensitivity to unbalance forces instead of a calculated margin between "wet critical speeds" and maximum operating speed.

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