SLURRY PIPELINE DESIGN FOR OPERATION WITH CENTRIFUGAL PUMPS

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HYDRAULIC DESIGN OF THE SLURRY PIPELINE

Introduction

The following covers what the writer sees as the main considerations involved in the design of a slurry pipeline. It is broken down into sections. This section is concerned mainly with the slurry and design of the hydraulics of the pipeline. The Slurry Pipeline Design and Operation section is concerned with the driving centrifugal pumps and associated equipment.

Slurry pipeline technology is extremely complex and by no means a fully understood science, so the writer makes no claim in this document to cover all details. It is hoped, however, that this will enable a better understanding and provide the reader with sufficient information to lay out the design of a working system.

Settling and Nonsettling Slurries

In nonsettling or “slow settling” slurries, the solids particles are sufficiently fine, light, or concentrated that they have little tendency to settle out from the carrier liquid. The slurry can then be treated for design purposes as if it were a single phase. In settling slurries, the tendency of the solids to settle out from the carrier liquid is sufficiently marked that the design procedure must treat the liquid and solid as distinct phases. The fundamental distinction is reflected by different variations of head loss with mixture’s velocity for the two types of (constant concentration) slurry, as shown schematically in Figure 1. For a nonsettling slurry, typically, the frictional head loss increases continuously with mixture velocity, although there is a gradual discontinuity at a laminar to turbulent critical velocity $V_\text{C}$. The curve for a settling slurry normally shows a shallow minimum around velocity $V_m$. As a convenient guide (Figure 1), a slurry may usually be treated as nonsettling, if the hindered settling velocity of the solids is less than 0.6 mm/s and settling, if the hindered settling velocity exceeds 1.5 mm/s. The interval between these limits reflects the arbitrary nature of the distinction. Aude [1] has given a figure for rapid indication of whether a slurry will show settling behavior, (Figure 2), which generally shows nonsettling behavior persisting to slightly larger particle sizes than indicated by the hindered settling criterion.

![Figure 1. Logarithmic Plot of $\Delta P = \phi (V_m)$](image1.png)

![Figure 2. Demarcation between Settling & Non-Settling Slurries](image2.png)
Nonsettling Slurries

Nonsettling slurries are formed by a mixture of fine solid particles and water in which the solid particles will settle very slowly. Clay water slurries in which the particles are very fine, d < 4.0 μm, are truly nonsettling. Clay water slurries constitute a major disposal problem in many mining operations because of the difficulty in separating the solids and water. The second type of nonsettling slurry is manufactured in that minerals or mineral waxes are ground to the silt-size range, 4.0 μm < d < 62 μm. In this case, the solid particles may settle to the bottom of a still pond or shutdown pipeline in a matter of minutes or hours; but if the flow is turbulent in the pipeline, the particle concentration will be uniform across the pipe and the velocity distribution will be axisymmetric. A third type of slurry that falls into the nonsettling classification is one with clay size particles, plus larger particles, even up into the sand sizes. In the third type, the clay water slurry is so viscous that the larger particles settle very slowly.

Inasmuch as all nonsettling slurries flowing in a pipeline exhibit a uniform distribution of particles across the flow section and an axisymmetric velocity distribution, flow of nonsettling slurry in a pipe is analyzed as a pseudofluid having the specific gravity, S_m. Unfortunately, from the standpoint of the pipeline designer, these pseudofluids (fine particles and water) exhibit non-Newtonian characteristics. In all fluids, internal shear stress, τ, is a function of the rate of strain. In uniform flow in a pipe, the rate of strain is the velocity gradient, -dv/dr, in which v = velocity at a point and r = radius. A Newtonian fluid, such as water, is one in which the shear stress is proportional to the rate of strain. The constant of proportionality relating the shear stress to the rate of strain is called dynamic viscosity, μ.

For a Newtonian fluid flowing in a pipe,

\[ \tau = - \mu \frac{dv}{dr} \tag{1} \]

For a non-Newtonian pseudofluid flowing uniformly in a pipe,

\[ \tau = \phi \left( -\frac{dv}{dr}, \text{makeup of the slurry, particle concentration temperature} \right) \tag{2} \]

The functional relationship, Equation (2), is called the rheological equation that has to be determined by experimental means for each slurry. Non-Newtonian slurries are classified by the nature of the rheograms as shown in Figure 3.

With slurries containing some larger particles, the accepted design practice is to design for turbulent flow with the design flow being slightly greater than the flow at which transition from laminar to turbulent flow occurs. The rationale begins that because laminar flow has no pickup mechanism, turbulent eddies, the larger particles will eventually settle on the bottom of the pipe and form a permanent bed. For slurries of ground particles and water, the designer attempts to obtain the most economical combination of particle size, flowrate, and concentration. The flow conditions at the transition from laminar to turbulent flow will be decided by the laboratory tests either by observation of transition in a pipe flow or by prediction from the experimentally determined function, Equation (2).

There are several types of viscometers by means of which the functional relationship, Equation (2) can be determined experimentally in the laboratory. In the following, only the tube-type viscometer will be discussed.

From the linear momentum equation it can be established that the shear stress distribution is linear varying from a maximum at the boundary to zero at the centerline of the pipe, that is,

\[ \tau = - \frac{dp}{dx} \frac{r}{2} \tag{3} \]

in which dp/dx = the pressure gradient along the axis of a horizontal pipe.

Laminar Flow

Laminar flow is characterized by molecular diffusion of linear momentum and matter. If dye is injected through a small hypodermic needle into laminar flow of water, the dye stain will spread slightly by the mechanism of molecular diffusion in the downstream direction requiring hundreds of diameters of pipe length before the dye spreads across the flow. On the other hand, with turbulent flow diffusion is affected by the turbulent eddies at a much more rapid rate. The coefficient of turbulent-eddy diffusivity is always several orders of magnitude greater than the coefficient of molecular diffusivity.

Newtonian Fluid

Kinematic viscosity, ν, of a Newtonian fluid is a coefficient of molecular diffusivity of linear momentum as evidenced by the following equation

\[ \tau = - \nu \frac{d(\rho v)}{dr} \tag{4} \]

in which ν is the linear momentum per unit volume.

Using the above, noting that νv = μ and v = 0 at the pipe wall where r = r_0, the following is derived

\[ \nu = \frac{1}{r_0^2 \pi} \int_{0}^{r_0} \nu 2\pi dr = \frac{1}{8\mu} \left( \frac{-dp}{dx} \right) r_0^2 \tag{5} \]

which is known as the Hagen-Poiseuille equation. Substituting Equation (3) and r_0 = D/2 into Equation (5) and solving for the boundary shear stress for steady laminar flow of a Newtonian fluid in a pipe

\[ \tau_0 = \mu \left( \frac{8
}{D} \right) \tag{6} \]

Non-Newtonian Fluid

Rabinowitsch and Mooney [3, 4] have shown that the rate of strain of a fluid particle adjacent to the pipe wall can be expressed as

\[ \left( \frac{-dv}{dr} \right) = \tau_0 \left( \frac{1 + 3\eta}{4n} \right) \frac{8
}{D} \tag{7} \]
for all laminar flows in a pipe. In Equation (7)

\[
n = \frac{d(\ln r_\infty)}{d(\ln 8V / D)}
\] (8)

The importance of the Rubiniowitsch and Mooney proof is that of establishing the scaling law

\[
\tau_\infty = \frac{8V}{D}
\] (9)

For all steady uniform laminar flows in a pipe. In other words, both \( \tau_\infty \) and \( 8V / D \) can be determined from experiment with the result that the experimentally determined function, Equation (9), can be applied to other pipe sizes and/or velocities provided that the same non-Newtonian fluid is involved in both the test and the design.

For illustration, the results of a test, Test 126-78, involving the flow of a Florida clay water slurry in an 203 mm steel pipe will be analyzed in the following. Columns 2.0 through 5.0, inclusive to Table 1, are the experimentally determined values of interest.

**Table 1. Test 126-78. Data for Phosphate Slimes Slurry in 203 MM Pipe.**

<table>
<thead>
<tr>
<th>Run</th>
<th>( V_\infty ) (m/s)</th>
<th>( 8V / D ) (s²)</th>
<th>( i_a )</th>
<th>( \tau_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.53</td>
<td>21.0</td>
<td>0.1004</td>
<td>49.9</td>
</tr>
<tr>
<td>2</td>
<td>1.52</td>
<td>60.1</td>
<td>0.1130</td>
<td>56.2</td>
</tr>
<tr>
<td>3</td>
<td>2.00</td>
<td>78.8</td>
<td>0.1150</td>
<td>57.2</td>
</tr>
<tr>
<td>4</td>
<td>2.59</td>
<td>102.1</td>
<td>0.1189</td>
<td>59.1</td>
</tr>
<tr>
<td>5</td>
<td>3.24</td>
<td>127.9</td>
<td>0.1218</td>
<td>66.1</td>
</tr>
<tr>
<td>6</td>
<td>3.81</td>
<td>150.3</td>
<td>0.1237</td>
<td>61.5</td>
</tr>
<tr>
<td>7</td>
<td>4.43</td>
<td>174.7</td>
<td>0.1273</td>
<td>63.4</td>
</tr>
<tr>
<td>8</td>
<td>5.12</td>
<td>202.0</td>
<td>0.1348</td>
<td>67.0</td>
</tr>
<tr>
<td>9</td>
<td>5.64</td>
<td>222.6</td>
<td>0.1472</td>
<td>73.2</td>
</tr>
</tbody>
</table>

In accordance with Equation (9), values \( \tau_\infty \) are plotted in Figure 4 as a function of \( 8V / D \). Runs 1-6, inclusive, are laminar flow runs for which the scaling law is applicable. For an example of the use of the scaling law, compute the head required to pump this slurry through 701 m of 305 mm steel pipe at a mean velocity 2.44 m/sec, that is,

\[
\frac{8V}{D} = \frac{8(2.44)}{(305)} = 64 \text{ sec}^{-1}
\] (10)

From Figure 4 at \( 8V / D = 64 \text{ sec}^{-1} \)

\[
\tau_\infty \approx 56.23 \text{ Pa}
\] (11)

or

\[
\frac{dp}{da} = \frac{4 \tau_\infty}{D} = \frac{4 \times 36.12}{305} = 0.736 \text{ Pa/m pipe}
\]

and the total pressure drop

\[
\frac{0.73518 \times 701}{1.13} = 46.74 \text{ m sly}
\] (13)

**Turbulent Flow**

Nonsettling slurries are analyzed as single-phase fluids or pseudofluids. For single-phase fluids, the friction pressure gradient, \( \Delta P / L \), in turbulent flow is calculated by means of the Weisbach equation. A reasonable approximation is to utilize the Weisbach equation for turbulent flow of nonsettling slurries also, that is,

\[
\Delta P / L = \frac{f_m}{D} \frac{\rho V^2}{2}
\] (14)

in which

\[
f_m = \text{Darcy-Weisbach boundary-drag coefficient of the pseudofluid; and}
\]

\[
r_m = \text{mean density of the slurry.}
\]

Continuing the analogy of nonsettling slurries with single-phase Newtonian fluids, the Colebrook function is used to determine \( f_m \), that is,

\[
\frac{1}{f_m} = 1.14 - 2 \log \left( \frac{k}{D} \right) + \frac{9.35}{Re_m f_m}
\] (15)

in which

\[
Re_m = \frac{VD_\text{pm}}{\mu_0}
\] (16)

For more details on this, the reader should refer to Wilson, et al. [2].

**Different Types of Settling Slurries**

As has been distinguished between settling and nonsettling slurries, it is necessary also to distinguish between (1) very large size solids slurries, and (2) those called heterogeneous. Two other less common types, the so called carried liquid type, where large solids are carried in a fine nonsettling carrier and the dense phase plug flow type, where concentrations of solids by volume are 40 to 50 percent or more are not discussed here because of limits of space and time. Those interested in these may refer to Wilson, et al. [2].

The following is also restricted to horizontal pipeline transport and the special case of inclined pipes has been excluded. Readers needing information on either vertical or inclined pipeline operation are referred again to Wilson, et.al. [2].

In rough terms, the heterogeneous slurry is carried at least partially by the turbulent eddies and comprises slurries of solids size of approximately 100 micron in size up to 2.0 or so millimeters. The very large size slurries are those transported by rolling of sliding of the solids and may include particles up to 300 mm and more. Before getting into the head loss of different settling slurries concentration, and deposit must be considered.

**Solids Concentration**

Solids content of a slurry is normally expressed as the fraction of solids in the mixture fed to or received from the conveying line, that is, the **delivered concentration**. Concentration may be
expressed on a volume basis \((C_v)\) or on a weight basis \((C_w)\), and
the relationships between these concentrations and the mixture
specific gravity, \(S_m\), are summarized in Table 2. For most particu-
late solids which are deposited without vibration, the maximum
volume concentration will be about 60 percent. In this state, the
particles are in contact from the bottom of the pipe to the top. For
a slurry flow, the concentration will have to be 10 to 15 percent
lower. In order to preclude plugging with a settling solid, the
maximum value of \(C_v\) will be about 35 percent and will usually be
less.

<table>
<thead>
<tr>
<th>Table 2. Solids Concentration Equivalents.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expressions for (C_v)</td>
</tr>
<tr>
<td>(C_v) = volume concentration of solids, (C_w) = weight concentration of solids, (S_m) = mean specific gravity of solids, (S_f) = specific gravity of liquid</td>
</tr>
<tr>
<td>(\frac{S_f}{S_m - S_f} )</td>
</tr>
<tr>
<td>(\frac{S_f}{S_m - S_f} )</td>
</tr>
</tbody>
</table>

Especially for settling slurries, the mean velocity of the solids in
a conveying line is different from that of the mixture. Consider,
for example, a slurry being conveyed up a vertical pipe with mean
velocity \(V_m\). It can be shown [5], that the solids 'slip' \(\alpha\) relative to
the mixture velocity at their hindered settling velocity, \(V_s\). Thus, the
true solids velocity \(V_{s\alpha}\). Continuity then demands that the
volume concentration in the delivered mixture, \(C_{vd}\), is related to the
in situ concentration \(C_v\), by

\[ V_m C_{vd} = (V_m + V_s) C_v \]  

showing that \(C_{vd} < C_v\). Conversely, in a slurry moving vertically
downwards, the true solids velocity is \((V_m - V_s)\) so that

\[ V_m C_{vd} = (V_m - V_s) C_v \]  

and \(C_{vd} > C_v\). Consider now the flow of the settling slurry in a
horizontal pipe. It can be seen then that the solids concentration is
generally higher in the slower moving mixture near the bottom of
the pipe. Thus, the in situ solids concentration is less than \(V_m\). In effect,
the solids are 'slipping' \(\beta\) relative to the mixture so that, just as in \(V_m\)
slowing moving upwards, \(C_{vd} < C_v\). In practice, it is difficult to measure \(C_v\)
without very sophisticated instrumentation, so that design and scale-up
procedures are based on the delivered concentration, \(C_{vd}\). Even so, the distinction between
in situ and delivered concentrations is important. It explains the
discrepancy often found between instruments which measure \(C_{vd}\)
e.g., horizontally mounted gamma-ray density scanners) and techniques
which indicate \(C_{vd}\) e.g., inverted U-loops, or flow sampling
methods. It also explains why tests carried out in fixed-inventory
flow loops, where \(C_{vd}\) is effectively fixed by the quantity of solids
present in the loop, normally show values of \(C_{vd}\) that depend upon
the mixture velocity.

**Deposition in Horizontal Conveying**

At sufficiently high mixture velocity, the solids in a settling
slurry are conveyed by suspension in the carrier liquid. As the
mean mixture velocity, \(V_{m\alpha}\) is reduced, the particles tend to settle
towards the pipe invert, so that some of the solids are conveyed by
saltation (i.e., bouncing along the pipe wall) or as a sliding bed. As
the proportion of solids conveyed with continuous or intermittent
contact with the pipe increases, the mixture pressure drop diverges
more and more from the value for water alone. Thus, the pressure
drop may pass through a minimum, as shown in Figure 1. On
reducing the mixture velocity to \(V_{mp}^\star\) termed the *critical deposit
velocity*, the solids start to form a stationary deposit in the pipe. It
is normally essential to ensure that deposition does not occur. In
designing an installation for given solids throughput and concentra-
tion, the pipe must, therefore, be sized so that \(V > V_{mp}^\star\). The
deposit velocity can be estimated from a nomograph developed by
Wilson [6], which is reproduced as Figure 5. The left half of the
nomograph pertains to slurries in which \(S_f < S_m\). The right half of
the nomograph pertains to slurries in which \(S_f \geq S_m\).

**Figure 5. Nomographic Chart for Maximum Velocity at Limit of
Stationary Deposition, From Wilson (1979).**

To use Figure 5, a straight edge is placed on the pipe size (left
ordinate) with the straight line passing through the particle
diameter (central curved scale) with the value of \(V_{mp}^\star\) with \(S_f < 2.65\) being at the intersection of the straight line and the central
ordinate. If \(S_f \geq 2.65\), a second straight line is established on the
right half of the nomograph that passes through the correct value of
\(S_f\). The value of \(V_{mp}^\star\) is at the intersection of this second straight
line and the right ordinate.

The particle diameter scale is curved to account for the
immersion of the particle in the boundary layer of the pipe flow.
For example, \(V_{mp}^\star \approx 1.77 m/s\) for both \(d = 2.5 m\) (very fine
gravel) and \(d = 0.15 m\) (fine sand) in a 203 mm pipe. Assuming
that the 203 mm pipe is smooth, the velocity at the top of the 0.15
mm particle would be 0.85 m/sec and at the top of the 2.5 mm
particle would be 1.37 m/sec when the mean velocity is 1.77 m/sec.
The smaller particle would be submerged in the laminar sublayer,
whereas almost all of the larger particle would protrude into the
interior turbulent-flow region. Because of the different conditions
to which the two particles are exposed both the 0.15mm and the
2.5mm particles have the same deposit velocity, \(V_{mp}^\star \approx 1.77 m/sec\).

The deposit velocity actually varies with concentration. Figure 5
is drawn for the concentration at which particles first settle. In
practice, concentrations vary (even if only at startup), so this is still
the best guide to follow. For consistently operated operated higher
concentration case, lower values may be used.

In this case, the value of the deposit velocity, \(V_{mp}^\star\), at the specific
concentration compared to the velocity, \(V_{mr}\), at the upper limit
velocity can be obtained from Wilson [7], first by using Equation
19 to find the relative solids concentration, \(C_{mr}\), at the maximum
deposit velocity.

\[ C_{mr} = 0.16D^{0.40} d^{-0.50} [(S_f - S_f)/1.63]^{0.17} \]  

where \(D\) is the pipe diameter. After finding \(C_{mr}\), the deposit velocity
is obtained from the nomograph.
where the coefficient of 0.16 applies only for pipe diameter, D, in meters and particle diameter d in mm, and any result falling beyond the range of 0.85 to 0.66 must be reset to the appropriate boundary values, and where \( C_r = C_{v,0}/C_{v,b} \), i.e., delivered volumetric concentration/loose-poured volumetric solids fraction. \( C_{v,b} \) may be measured with bench-top apparatus; if a measured value is not available it is customary to assume a typical value of \( C_{v,b} = 0.60 \). The value of \( C_r \) which produces the maximum deposit velocity \( V_{sm} \) is denoted \( C_{rm} \), and either of Equations (3) and (5) as appropriate

\[
V_r/V_{sm} = 6.75 C_r G (1 - C_r)^2 \quad \text{(for } C_{rm} < 0.33 \text{)} \tag{20}
\]

where \( C_r = \ln(0.333)/\ln C_{rm} \), and

\[
V_r/V_{sm} = 6.75 (1-C_r)^2 [1-(1-C_r)^{0.33}] \quad \text{(for } C_{rm} > 0.33 \text{)} \tag{21}
\]

where \( \beta = \ln(0.666)/\ln(1-C_{rm}) \).

Figure 6 has been drawn using the above, and shows for a given pipe size the effect of both particle size and slurry concentration.

![Deposit Velocity in a 300-mm Pipe](image)

Figure 6.

Although the critical deposit velocity has often been considered crucially important for settling slurries, it is not always of great practical significance. The most economic conveying velocity is often well above \( V_{m,0} \). Moreover, for settling slurries with centrifugal pumps as prime movers, the conveying velocity is normally limited by operating stability to velocities well above \( V_{m,0} \). These considerations are examined later.

**Head Loss in Horizontal Conveying**

The head loss along a pipe conveying a settling slurry is conventionally expressed as head in meters (or feet) of carrier liquid per meter (or foot) of pipe, \( i_{m} \). The corresponding head loss for carrier liquid alone at the same mixture velocity will be denoted by \( i_{w} \). The excess head loss resulting from the presence of the solids is then \( (i_{m} - i_{w}) \). Empirical correlations commonly attempt to predict either \( i_{m} - i_{w} \) or the relative increase in head loss, \( (i_{m} - i_{w})/i_{w} \). Some of these correlations and their applications to slurries containing a wide range of particle sizes are explained by Wasp [7]. However, in the writer's experience, it is much more reliable to base design on tests, carried out on slurries representative of that to be pumped in practice.

**Heterogeneous Slurry Scaling Approach**

A promising approach to scaling up test results consists of distinguishing between different modes of solid transport, and assessing the contributions of the different modes to \( (i_{m} - i_{w}) \). This approach is derived from Wilson's development [8, 9] of early work on settling slurries by Newitt [10] and Clift [11]. It is known that at very high mixture velocities conveyed solids can be completely suspended by the turbulence of the conveying liquid. In this case, the solids concentration is nearly uniform across the pipe section, and the solids are said to be in pseudohomogeneous suspension. It should be noted that in this flow regime, slip of the solids relative to the mean mixture velocity is small, so that in situ and delivered concentrations are nearly equal. The head loss to fully suspended (pseudohomogeneous) flow will be denoted by \( i_{wh} \).

Turning now to low mixture velocities, it is known that settling solids tend to concentrate in the slower moving mixture towards the bottom of the pipe, i.e., flow becomes stratified at low velocity. The head loss in fully stratified flow will be denoted by \( i_{ms} \). At intermediate velocities, some of the conveyed solids will be stratified while the remainder will be in pseudohomogeneous suspension. It is therefore convenient to use the stratification ratio first defined by Shook [12]:

\[
R = \text{fraction of solids conveyed as stratified load or the ratio of the bed load to the total solids load.}
\]

The head loss for the slurry is now assessed from the contributions of the two modes of transport:

\[
i_{m} = Ri_{ws} + (1-R)i_{wh}
\tag{22}
\]

Slurry tests are then used to estimate \( i_{ws} \), \( i_{wh} \), and \( R \). It has been noted that solids become fully suspended at high mixture velocity, that is, as \( V_{m} \) becomes very large, \( R \to 0 \) and hence, \( i_{m} - i_{wh} \). Tests should, therefore, include measurements at high \( V_{m} \), higher than would normally be used for conveying, in order to determine \( i_{wh} \). It should also be noted that \( i_{wh} \) is frequently close to the head loss for liquid alone, \( i_{w} \). It is, therefore, essential to 'calibrate' the test section by careful determination of \( i_{w} \). Test results for lower velocities are then used to determine \( Ri_{ws} \). Unless the solids contain a very coarse fraction (see later), it is unnecessary to determine \( R \) and \( i_{ws} \) separately. However, or very coarse solids, it may be necessary to examine \( R \) and \( i_{ws} \) in detail.

For actual scaling of test results, Equation (23) is put in the form:

\[
i_{m} - i_{w} = \frac{A_{s}^{2}I}{S_{m} - 1} \left( \frac{V_{s}}{V_{m}} \right)^{M} + B \left( \frac{V_{s}}{V_{m}} \right)^{M}
\tag{23}
\]

as described by Clift [11], and further simplified to

\[
i_{m} - i_{w} = B' \left( \frac{V_{s}}{V_{m}} \right)^{M}
\tag{24}
\]

where \( B' \) and \( M \) are taken from a plot of the available test data as shown in Figure 5 from Clift [9] noting that the above only holds for the heterogeneous region of interest, and that the data used is restricted to this region.

Wilson [13], illustrates this in Figure 8 with results of several concentrations in two pipe diameters.

**Heterogeneous Slurry Estimate Without Test Data**

Experience has shown that for a large number of heterogeneous slurries without excess fines, the value of \( M \) in the above equation is around 1.7. Using this, and again, considering flow in the heterogeneous region of interest, the above may be simplified to

\[
i_{m} = i_{w} + (S_{md} - 1) \left( \frac{U_{s}}{V_{m}} \right)^{1.7}
\tag{25}
\]

as outlined by Addie [14]. Where the \( U_{s}' \) constant is shown in Figure 9 [14] plotted for different DS0 size slurries.
The form of Equation (25) is the expected inverted parabola shown in Figure 1. Where the minimum friction point is the lowest velocity $V_{\text{min stab}}$ for stable operation with a centrifugal pump noted earlier. The first derivative of Equation (25) provides a means of determining $V_{\text{min stab}}$ directly.

Figure 10 or Chart 5 [14] has been derived from the first derivative of Equation (9) and provides a means of estimating the lower limit of $V_m$ for stable operation directly in terms of a given size of typical slurry solids, concentration and pipe diameter for a smooth pipe.

**Accurate Heterogeneous Slurry Head Estimation**

The above takes no account of size distribution (or particle shape) which can cause significant variation in both the values and characteristic of the head curve.

For a more accurate estimate, the reader is referred to Wilson [2], that (Chapter 5) presents a method of calculating pipe friction that includes the effect of size distribution.

### Analysis of Very Coarse-Particle (Fully-Stratified) Transport

Where the coarse particles are large, fully-stratified flow occurs where almost all of the particles travel as contact load (i.e., fluid suspension is ineffective). The ratio of particle diameter to pipe diameter is of major importance in determining the presence of this flow type, which does not normally occur for $d/D$ ratios less than 0.015. Fully-stratified flow is less likely if the particles are broadly graded, especially if there is a significant homogeneous fraction (i.e., a significant fraction of particles smaller than 75 micron). Calculations made for narrow-graded slurries with water as a carrier fluid indicate fully-stratified behavior for values of $d/D$ above 0.018. For $d/D$ between 0.015 and 0.018, both types of behavior can occur, with fully-stratified flow more likely for larger values of $S_p$ and small values of $V_m$. In uncertain cases such as this, it is best to carry out analyses using both the method given here and that for heterogeneous flow.

Where virtually all the solids are larger than, say 0.018 D, fluid suspension of particles is not effective and the flow is fully stratified. The analysis presented later from Wilson [2] is proposed. Although it is less energy-efficient than heterogeneous flow, this transport mode can be economically attractive for pipelines of moderate length (an example might be dredging) as it obviates problems of head-end processing and subsequent separation problems at the tail end of the line.
For any given delivered solids concentration, the difference between the mixture pressure gradient and that for an equal flow of fluid alone depends not only on the relative concentration $C_r$ but also on the velocity ratio $V_r (V_m/V_{sn})$. The nature of this dependence can be seen from Figure 11 from Wilson [2]. On this figure, the excess pressure gradient or solids effect has been expressed in relative terms by dividing by the gradient required to move a dense particulate plug filling the pipe and the relative excess pressure gradient, denoted $\zeta$, is expressed as

$$\zeta = \frac{i_m - i_s}{i_{pg}}$$  \hspace{1cm} (26)$$

from Wilson [2] where the hydraulic gradient (m water/m pipe) required to set a dense phase slurry plug in motion, $i_{pg}$, is given by

$$i_{pg} = 2 \mu_s (S_t - S_p) C_{vb}$$  \hspace{1cm} (27)$$

from Wilson [2] where

$$\mu_s = 0.40 \text{ for gravel}$$
$$0.31 \text{ for clay balls}$$

$C_{vb}$ is typically 0.6.

Equation (28) is approached at high values of the velocity ratio $V_r$. At lower values of this ratio a further relation is required to account for the hold-up of solids, which produces a solids effect greater than that of the high-velocity asymptote, as shown on Figure 11. In this case, the fit function is written

$$\zeta = \zeta_m + (1 - \zeta_m)(1+V_r)^q$$  \hspace{1cm} (30)$$

from Wilson [2]

$$V_r = \frac{V_m}{V_{sn}}$$  \hspace{1cm} (31)$$

Noting that where the power $q$ is a function of $C_r$. If $C_r$ is greater than $C_{rm}$ as defined by Equation (19), noting that it requires pipe diameter, D, in meters and particle diameter, d, in mm then $q$ is calculated from the fit function

$$q = 3.6 - 5.2 \left(1 - C_r^{C_{rm}}\right) \quad \left(C_r > C_{rm}\right)$$  \hspace{1cm} (32)$$

from Wilson [2]

If $C_r$ is less than $C_{rm}$, $q$ is first calculated from Equation (31) using $C_{rm}$ in place of $C_r$, and the result is then multiplied by the ratio $C_{rm}/C_r$ to obtain the value needed for insertion into Equation (30) in order to calculate $\zeta$. With $\zeta$ calculated in this way (or read directly from Figure 11 if only a rough approximation is required), values of $i_m$ can readily be determined.

The values here are for coarse particles without any fines. The presence of fines will lower the above results. Tests at GIW Hydraulic Laboratory have shown that this reduction in head loss vary by as much as 50 percent and more where the fines content is significant.

**Specific Energy Consumption**

Irrespective of the flow conditions in the pipe, the most efficient slurry transport is achieved when the **specific-energy-consumption**, SEC, is a minimum. In dimensionless form

$$SEC = \frac{i_m}{S_s C_{vd}}$$  \hspace{1cm} (33)$$

in which

$$i_m = \text{friction pressure gradient in ft of water/ft pipe,}$$
$$S_s = \text{specific gravity of the solids, and}$$
$$C_{vd} = \text{delivered volume concentration. (decimal)}$$

To obtain a value in kWh/tonne-km, this ratio is multiplied by 2.73, and for horsepower-hr/ton-mile the factor is 5.33. If the power supplied to the pump is required, the expression must be divided by pump efficiency, and the efficiency of the drive train or motor can be taken into account in the same way.

Even though operation at SEC (min) is most efficient from the energy standpoint, cost of the pipeline, deposit velocity, or centrifugal pump characteristics will probably result in the selected operating velocity, $V_m$, being greater than $V_m(\text{minimum SEC})$. In any event, curves of $i_m = \phi (V_m)$ and SEC = $\phi (V_m)$ should be scrutinized by a designer before selecting pipeline size and operating conditions.

**Selection of Design Values**

In general, reducing the solids particles size of a settling slurry will reduce the head required to move the slurry. However, given a
certain sized particle slurry, the energy efficiency of a horizontal pipeline system will increase with increases of concentration up until dense phase or plug flow commences after which, it will decrease rapidly.

Where possible, the design concentration should be kept as high as practical, but with a suitable margin clear of dense phase or plug flow.

Too high a pipeline mean velocity design value, apart from not being energy efficient, will greatly increase wear to the pumps, the pipeline and auxiliary equipment.

Given a required throughput of dry solid and a slurry design concentration, a pipe diameter should be selected to allow the line to operate at a velocity just above the greater of the particle deposit velocity and the velocity at which the head loss is a minimum, plus some safety margin.

A value of 10 percent could be used to start with but this margin and the final design velocity should be selected based on an evaluation of the likely operational variations in concentration, and size of solids along with the characteristics of any pumping plant, the associated control system and the stability of the whole system resulting from these.

SLURRY PIPELINE DESIGN AND OPERATION—DRIVING PUMPS AND ASSOCIATED EQUIPMENT

Introduction

Slurry system components exhibit a great range of variation. The range is not merely in dimensions, though it is true that ocean-going hopper dredges are very large (impeller diameters up to 2.5 meters), while pumps of mine-tailing service can be very small.

The variations and associated design needs go beyond size alone.

The configurations of slurry systems for different applications can differ in almost all respects, including the geometry of pump suction piping, the mechanical arrangement and layout of pumping stations, and the ways in which the system is instrumented and controlled. For example, slurry transport systems which abstract material directly from the environment (such as harbor dredges or open-pit mining operations) differ greatly in the arrangements at the upstream (suction) end from the type of system that is fed with materials processed by an industrial operation.

Different types of systems also employ greatly different techniques for instrumentation and, especially, for control. A further dimension that complicates system interaction is line length, as long lines require multiple pumps located in a series of stations. This increases the problems of mechanical layout and instrumentation, and greatly complicates the control problems.

Solids Transportation

The purpose of a slurry pipeline is the transportation of a given quantity of solids in a certain period of time.

The pipeline design and normal concentration calculations were covered in HYDRAULIC DESIGN OF THE SLURRY PIPELINE. From the driving pump viewpoint, however, we can establish that the quantity of solids transported may be found using the following:

\[
\text{Transportation rate} = \frac{\text{flowrate (l/s)} \times \% \text{ solids} \times \text{SG}}{0.2775}
\]

where the above SG refers to the slurry SG and percent solids is by weight expressed as a decimal.

The concentration at which the solids are transported may be set by the process or may be varied to achieve minimum energy consumption. The pipeline diameter can be existing or may be set by capital cost, energy, head loss, or other requirements.

As distinct from a water or other single phase fluid pipeline, the energy efficiency of a slurry pipeline as defined in HYDRAULIC DESIGN OF THE SLURRY PIPELINE, is a function of the dry solids transported not the fluid flow that the pump sees.

Analysis of Pipeline Design

The determination of the specific pipe friction is covered in the first section. From the pump viewpoint, it can be said that most slurry pipelines involve transport over largely horizontal distances. Friction due to vertical or inclined sections, if it exists, is usually small in terms of the total head. Given the requirements and any fixed parameters, an analysis of the specific energy consumption, minimum recommended pumping velocity, and pipe friction needs to be made first from test data or other acceptable methods of modelling to establish the pipeline diameter, slurry concentration, mean velocity, and pipe friction per foot of pipeline before the pumps can be sized.

More than one combination of slurry concentration and pump selection are usually possible. It should be kept in mind that it is the total system costs that count, and that this configuration may not necessarily, in the overall interest, coincide with the lowest cost or highest efficiency pump selection.

Total System Resistance

Given the pipeline diameter, the slurry concentration, the mean mixture velocity, and the pipeline friction per foot of pipeline, the total system resistance, the total developed head (TDH) between an atmospheric pressure inlet and outlet can be determined using the following:

\[
\text{TDH} = H_{\text{dis static}} + H_{\text{friction}} + H_{\text{exit}} + H_{\text{fitting}}
\]

where

\[
H_{\text{dis static}} = \text{the difference in level between the pump centerline and liquid outlet level expressed in meters assuming positive is where the outlet is above the pump.}
\]

\[
H_{\text{friction}} = \text{the calculated loss per foot of horizontal pipe calculated as outlined in the previous section by the length of the pipeline}
\]

\[
H_{\text{exit}} = \text{an entrance loss incurred in the feed system where values may be estimated using}
\]

\[
H_{\text{exit}} = k \frac{V^2}{2g}
\]

and

\[
k = 0.05 \text{ for belmouth}
\]

\[
= 0.5 \text{ square edge inlet}
\]

\[
= 1.0 \text{ inward projecting pipe}
\]

\[
V = \text{mean mixture velocity in pipe in m/sec}
\]

\[
g = 9.8 \text{ m/sec}^2
\]

\[
H_{\text{exit}} = \text{the loss of velocity head at the end of the line and is equal to } \frac{V^2}{2g} \text{ using the units noted above.}
\]

\[
H_{\text{fitting}} = \text{the loss incurred in any bends or fitting = } k \frac{V^2}{2g}
\]

fittings are as noted above and k varies with the bend diameter radius and roughness or fitting details but usually lies in the range 0.1 to 0.6. For more details refer to the Hydraulic Institute Standards.

Pump Pipeline Interaction

A centrifugal slurry pump can only operate at a flow where the head it generates is in equilibrium with the pipeline resistance.

As noted earlier, this flow from the slurry pipeline point of view
must be sufficiently high to avoid system characteristic stability or deposit velocity limits while at the same time being as energy efficient as possible in regards to solids transport.

The slurry pipeline diameter provides at least the initial selection basis for a suitable pump. Variation of the pump rotational speed to achieve the necessary pipeline head may then be made and this speed then considered as to mechanical wear and other limitations. The pump efficiency at these conditions is the next criteria to consider with operation in the 70 to 100 percent range of the best efficiency point flow (BEPQ) being considered normal.

Unlike water pumps, performance adjustment by speed change is more common than impeller diameter change. This is because of the increased difficulty of trimming a hard metal impeller and the difficulty of determining the exact system head. For this reason, a lot of slurry pumps are v-belt driven so the speed can be adjusted easily.

**Pump Performance with Speed Change**

The output from a centrifugal pump varies with speed as is defined by the affinity laws namely,

\[
\frac{Q_1}{Q_2} = \left( \frac{N_1}{N_2} \right)^3 \quad \frac{H_1}{H_2} = \left( \frac{N_1}{N_2} \right)^3 \quad \frac{BHP_1}{BHP_2} = \left( \frac{N_1}{N_2} \right)^3
\]

(34)

where the above applies concurrently along what is referred to as an affinity line and where

\[ N \quad = \text{rotational speed} \]

\[ Q \quad = \text{flowrate} \]

\[ H \quad = \text{head produced} \]

\[ BHP \quad = \text{power absorbed} \]

1 = initial condition

2 = final condition

In a slurry pump, the above holds true except where at the extreme lower end speed ranges, where the constant friction and other fixes losses become a distorted proportion of the pump losses causing a lowering of efficiency and head.

**Pump Performance with Impeller Diameter Change**

Where the pump speed must be held at a given synchronous or other value, then the pump performance may be varied by reducing the impeller diameter.

In this case, the performance varies (within limits) as the affinity laws (noted earlier) but with the impeller diameter replacing the speed in the equations. Because most slurry pumps have large diameter impellers and are of low specific speed, up to a 20 percent reduction is possible before the slip and resultant head and efficiency loss becomes significant.

**Pump Selection**

Coarse slurries preclude the use of positive displacement pumps and centrifugal slurry pumps are used almost exclusively to drive coarse slurry pipelines. As noted earlier, pumps must be selected by matching their head-discharge performance to the requirements of the piping system. As shown in Figure 12, the intercept of the pump characteristic with the system characteristic defines the operating head and discharge. As noted earlier, however, that intercept will vary with system changes and any control system.

Once the operating conditions have been selected, pump selection in its simplest sense amounts to determining the specific performance of each available pump for the head and flow required, and selecting the one best suited to the duty. In general, the smaller the pump, the cheaper it will be. However, reducing pump size implies a higher speed for a given discharge. The shaft speed is limited by the wear life, which decreases as speed is increased. Therefore, slurry pumps are typically larger with lower rotational speed than water pumps for equivalent head and discharge. Actual wear data will seldom be available for a specific pump type, shaft speed, and slurry duty. However, the modelling techniques such as developed by Pegalthivarthi, et al., [15, 16, 17] can be used to transfer information on wear from one configuration to another.

Bearing this uncertainty in mind, Figure 13 and Table 3 give a rough guideline for pump selection. The duty is first classified as ‘light’, ‘medium’ or ‘heavy’ according to Figure 13. Table 3 is then used to define the range of operating conditions that should be considered. These values may be adjusted, depending on how abrasive the particles in the slurry. The recommended maximum discharge velocity enables pumps with suitable branch sizes to be identified. Different combinations of rotational speed and impeller diameter are then selected, within the impeller tip speed limit. As a final step, the efficiencies and power requirements of the possible pumps are compared. Limiting the impeller peripheral-speed according to the values in Table 3 forces selection towards lower speeds and larger pumps. Limited ranges of discharge around the best efficiency point are also recommended, indicated in Table 3 by the range of the ratio Q/Qe. This procedure clearly lends itself to a simple ‘expert system’ for pump selection, and this approach is used routinely by major pump suppliers.

**Figure 12. System and Pump Characteristics.**

Table 3. Recommended Operating Limits for Slurry Pumps.

<table>
<thead>
<tr>
<th></th>
<th>Light</th>
<th>Service Duty</th>
<th>Medium</th>
<th>Heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum discharge velocity: m³/s</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>25</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Maximum impeller peripheral speed: ft/min</td>
<td>45</td>
<td>36</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>(a) All metal pump: m/s</td>
<td>8500</td>
<td>7000</td>
<td>3500</td>
<td></td>
</tr>
<tr>
<td>(b) Rubbe-lined pump: m/s</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4500</td>
<td>4500</td>
<td>4500</td>
<td></td>
</tr>
<tr>
<td>Range of discharge: Q/Qe (%)</td>
<td>30-130</td>
<td>40-120</td>
<td>50-110</td>
<td></td>
</tr>
</tbody>
</table>

Note: The above values may be adjusted for unusually abrasive or benign slurries.

In addition, the pressure rating of each pump should be checked against the duty required. The size and capacity of the shaft and bearings must be checked, while the wear resistance of the wetted-end materials must be assessed using a modelling technique such as noted [15, 16, 17]. The NPSH available in the installation must be checked, to ensure that it exceeds the NPSH required by the pump. Some applications, notably dredging, involve special suction-side considerations. For pit pumps and some dredge
pumps, avoiding cavitation can limit pump speed to values below the wear limits in Table 3. It is sometimes necessary to change selection in favor of a slower-running pump or one with better suction characteristics.

It may sometimes occur that an existing pump or drive is to be used, with no possibility of free selection. Remedial action may then be needed. For example, if a drive of given rating is to be used with an existing pump, it may be necessary to trim the impeller in order to reduce the power required by the pump to match that available from the drive. The revised head-discharge performance of the pump must then be calculated as noted earlier to find the resulting operating point as defined by the intersection of the pump and system characteristics. The size of the pump branches must also be checked for any high and low flow limits. Although they need not be exactly the same diameter as the pipeline, the difference, however, should not be too great and allowance must be made for the head loss associated with flow through the expansion and contraction. If the branches are too large, solids deposition can occur. If they are too small, wear may be excessive. In general, the limits indicated by Table 3 should apply. It is not unusual for the suction branch of a centrifugal pump to be one nominal size larger than the discharge branch.

This configuration can cause difficulties when pumping settling slurries, as noted under Suction Piping Configurations.

For long lines, the total system head will be more than can be handled by a single pump. It is then necessary to use several pumps in series.

Suction Piping Configurations

In the design of suction piping, the prime consideration is to prevent cavitation by ensuring an adequate NPSH at the pump, and a secondary consideration is to locate the pump where it can easily be inspected or maintained. These considerations are of lesser interest in the case of booster pumps, for which the suction pressure is provided by the pump or pumps upstream in the series, and the pumping station layout usually provides space for maintenance.

It is highly undesirable to install valves in the suction piping. If such a valve must be installed, care should be taken to ensure that it remains fully open during pump operation, thus causing no restriction to flow. It is especially important to avoid pinch valves in the suction piping, because they may close in a rapid and uncontrolled fashion. Sometimes it is desirable to have a dropout spool-piece attached to the pump suction to provide for stripping the pump without excessive repiping, and for removal of roots and large particles that may block the impeller eye.

The concern for adequate NPSH at the pump at the upstream end of the system favors a low position for this pump. This consideration may conflict to some extent with the desire to locate the pump 'in the dry' where it can be accessed for inspection and maintenance. As noted above, arrangements differ considerably between implant installations where there is usually control over the consistency of the input slurry, and extractive applications such as dredges and open-pit mines where the slurry consistency can vary widely and often uncontrollably. In the implant cases, which will be considered first, it is usually possible to locate the pump above ground or in a dry well, and to provide it with positive suction from an open tank or sump. This configuration is typical of plants in the mining industry (except for drainage clean-up service), most tailings pipelines, and some booster stations where sumps open to air are provided.

Sump design varies with the type of slurry and the service. Generally, simple designs are preferred because they suffer least wear and require least maintenance. 'Sloughing off,' of solids accumulated in the sump is to be avoided, because it imposes a sudden load on the pump and can upset operation of the whole slurry system. Tapered and rounded sides can be used to minimize solids accumulation, but usually there is less likelihood of slough-off if solids are allowed to build up to their natural angle of repose. In this case, a flat-bottomed tank is best. It is desirable to maintain at least two meters of liquid level above the pump centerline, with a minimum volume in the sump equal to at least one minute's discharge. Maintaining an adequate liquid level also helps to prevent air entrainment, which is to be avoided, because if causes pump surging, increased wear, and shock loading on the shaft and bearings. To stop air from entering the pump, the flow into the sump should discharge below the surface, as distant as possible from the pump suction pipe. Adequate liquid depth also helps to prevent swirling, which can entrain air into the suction pipe. Baffles can be placed in the sump if entrainment is likely to be a particular problem, but should only be used if really necessary.

A generic sump layout shown in Figure 14 embodies the desirable features of a slurry system sump. The pump suction pipe passes through the side of the sump, and is entered through a downward-pointing bend with a short bell-mouth. A priming jet may be located facing into the bell-mouth entrance to assist in pump priming, and to eliminate blockages on startup. If the slurry has a high solids concentration, or if the solids are very coarse, the priming jet may be allowed to run continuously to prevent sporadic plugging of the inlet with resultant surging. The suction pipe between the tank and pump should be horizontal and as short as possible, with the same diameter as the pump discharge. In this way, solids deposition in the suction pipe can be avoided, and the pressure loss between the sump and the pump kept low. Water should be provided to the sump in sufficient quantity to fill the system and act as makeup water. For implant systems, tank level control is sometimes used to regulate the water flow.

Phosphate and other matrix-pumping systems normally have a pump that is located above ground level and draws slurry from a pit. Material is moved toward the pump by a dragline bucket and is then dropped into the pit, using high-pressure water guns, through some sort of grizzly screen to remove oversize tramp material, such as the fossil bones and Civil War cannon balls sometimes dragged up in U.S. phosphate deposits. The slurry density is controlled very crudely by an operator raising and lowering an inverted-L section of pipe joined to the pump suction by a flexible rubber connection. The flowrate can be varied by altering the speed of the feed pump. Along with the disadvantage of having the pump above ground, which tends to give difficulties with available NPSH, this type of suction system requires expensive high-pressure water guns to move the slurry to the pit. The arrangement
also limits the solids concentration entering the pump, and hence the concentration which can be delivered by the system. The result of using low solids concentration is usually high specific energy consumption.

Of the many types of dredge in routine service, the main ones using pumps and suction-piping systems are the cutter dredge and the trailing hopper dredge. A cutter dredge employs a 'ladder' that pivots off the bow of the vessel and usually carries some sort of rotating cutter or bucket wheel to loosen solids from the channel bottom and move them towards the suction pipe. The entry to the suction pipe is near the end of the ladder, and there may also be an underwater ladder pump. Operation of the cutter of bucket wheel affects the entrance losses, the solids concentration drawn into the suction pipe, and the solids size and shape. Increases in velocity, digging depth and solids concentration can eventually lead to the limiting suction condition at which the NPSH available to the onboard pump becomes insufficient to prevent cavitation. For liquids or, by extension, slurries which exhibit equivalent-fluid behavior, the onset of cavitation can be staved off by increasing the diameter of the suction piping, and it may be for this reason that pumps are often constructed with a suction branch larger than the discharge branch, as mentioned in the previous section. The effect of the larger suction is to diminish the mean velocity there, which will reduce the tendency for a liquid to cavitate. However, for a settling slurry this approach may well be counter-productive.

With settling slurries, any decrease in the mean velocity can promote deposition, specifically for particles of the 500 to 600 micron 'Murphian' size. If an increase in suction piping size results in deposition, the resulting uneven flow pattern and the increased frictional losses in the suction piping will increase the likelihood of cavitation rather than diminishing it. The situation is even less favorable if the suction piping is inclined, as in the ladder of a suction dredge. In this case, a submerged ladder pump is often required.

The calculation of NPSH for the general case is straightforward. However, some additional explanation is merited for a ladder pump. The ladder is shown in Figure 15 and defines the quantities required for NPSH calculations. The greater the submergence of the ladder pump, the greater NPSH available to it. Thus, the ladder pump will usually run without cavitation and will prevent cavitation in the next pump in the line, normally on board the areage. Capital and maintenance cost for the underwater ladder pump is, therefore, justified by increased production and reduced specific energy consumption due to operation at higher solids concentra-

![Figure 15. Schematic of Dredge Ladder Pump and Related Variables Determining NPSH.](image)

Reliable handling of most solids at volume concentrations up to or beyond 35 percent can be usually achieved with this configuration.

Trailing and hopper dredges employ a vacuum-cleaner type of suction in order to maximize the pickup of fine solids. In these designs, the suction pipe usually passes through the hull at a pivot point on the side of the dredge and is raised and lowered by winches located at the side or stern of the vessel. The pumps are located low in the hull and made large and slow-running to decrease the required NPSH. slurries with up to 35 percent solids by volume can also be handled by this type of dredge.

Where the suction is from a pipe or where a dredge is operated without a ladder pump, it may be necessary to accept moderate cavitation for some operating conditions. Any increase in concentration of solids brought into the pipeline tends to decrease specific energy consumption, and this effect may outweigh the loss in pump efficiency associated with moderate cavitation. In such cases, it is vital to have a pump of the best possible suction performance, not only with small required NPSH at incipient cavitation, but also with the ability to continue pumping when the suction pressure is low so that the head and efficiency are depressed by 10 percent or more. The skill of the pit operator or dredgemans is very important for this type of operation. If sudden suction blockages do occur, they may well induce water hammer in the pipe, with resulting overpressure failure of pumps or other equipment further down the line.

The land dredge is a concept now under development which bridges the technological gap between dredging operations and mine-pit pumping [18]. The land dredge design replaces the usual mine-pit feed arrangement with a dredge ladder, including a cutter and ladder pump. The action of the cutter reduces the need for high-pressure gun water, and the ladder pump obviates NPSH difficulties and makes it possible to increase delivered concentration consistently and reliably. As can be seen, control of slurry concentration can enable operation with reduced specific energy consumption. In addition to increased production and reduced energy cost, this design is also seen as a way of eliminating hydraulic transients and the associated stoppages and failures.

Blockages in the pump suction can be caused by rocks, roots, and clay balls. The severity of the problem depends on the size of these solids, how frequently they occur, and the size of the pumps. Some sort of screen or 'grizzly' may be useful, as noted previously, but it is often necessary to compromise the hydraulic performance of the pump and the location of its design point in order to ensure that it can pass very large solids. Increasing the size of solids that can be passed by a pump usually entails reducing the
number of vanes in the impeller. If it is necessary to increase the inside impeller shroud width, the effect will be to shift the best efficiency point of the pump to flows higher than the duty flow. Although it is sometimes possible to counteract this effect with a specially-designed shroud, it is not uncommon to find large pumps operating extremely inefficiently on the underdischarge, and wearing excessively, because they have been oversized to eliminate the chance of blockage.

Some operators use a so-called 'root cutter,' which consists of a bar welded to the suction pipe parallel to the axis and extending into the eye of the impeller. This is a crude device for keeping the impeller free. It is bound to have an adverse effect on pump performance, but the extent of this effect has not been characterized.

Jet pumps may be used at the entrance to a suction piping system, or even at the suction of the pump, to improve solids feeding. A jet pump adds to the TDH, but such devices are usually less than 50 percent efficient, and the water they add reduces the solids concentration and thus has an adverse effect on specific energy consumption. Nevertheless, jet pumps do eliminate some wearing parts, and have a role to play in handling difficult slurries. They are most likely to be useful where dilution can be tolerated, operating efficiency is unimportant, and the system head is low.

Pump Layout and Spacing

Considerations of wear and ease of maintenance usually dictate that a centrifugal pump must be a single-stage machine. Furthermore, because of the limitations on pressure rating, a single centrifugal pump cannot be used to transport a slurry over a distance of more than a kilometer or so. Several pumps in series may, therefore, be needed for a medium-length pipeline, although centrifugal pumps as prime movers are generally limited to pipelines not much longer than about 10 kilometers or, say, six miles. For this configuration, the total head generated by pumps in series is simply the sum of the heads developed by each pump at the common flowrate. Several alternative selections are often considered, based on different numbers of pumps in series. The most cost-effective selection usually involves fewest pumps, and this should be the initial assumption. When a selection has been made on this basis, the operating and capital costs of alternative multiple-pump systems can be compared before a final decision is made. Ideally, the pumps should be the same size.

To limit the pressure each pump must withstand, it is desirable to space the pumps at roughly equal distances along the pipeline. However, this configuration increases the difficulties associated with startup and control, and adds to the cost of power supply and the cost of providing pump houses, if these are required. Conversely, if each pump house contains a group of pumps, all but the first pump will experience elevated operating pressures, which can affect the design of casing, shaft, and seals. With pumps spaced along the line, simple flexible pipes or expansion joints are sufficient to prevent piping loads from damaging the pumps.

In locating the pumps along the line, it is desirable to have about one atmosphere of positive pressure at the suction of each pump. In lines with three or more pumps, it is not uncommon to break the line with an open sump to prevent hydraulic transients being transmitted down the line.

Because of the higher pressures involved, multiple-unit pumping stations must use bases bolted to concrete foundations. The pressure loads from internal and external piping must be taken into account, along with the loads on the pumps, the method by which the pipes are fixed and the magnitude of any thermal-expansion loads which they carry. In such cases, a full stiffness analysis of the pumps and associated piping may be required, involving the three loads and three torque's at each of the two flanges of every pump.

The layout of pumps within a series station depends on the number of units and requirements of maintenance, access and safety. A photograph of a six-pump station is shown in Figure 16, with two lines of three pumps in series. In this case, access to all pumps is excellent, and reasonable work areas are provided adjacent to each pump. Note that the use of different levels makes it possible to employ simple single-bend pipe sections between pumps. Though not readily visible on the photograph, large launderings are provided to each pump, connecting down the center to a single wet sump. More compact arrangements can be achieved by rotating the pump branches and using straight pipes between adjacent discharge and suction.

Operation of pumps in parallel is not common for pipelines transporting settling slurries, but is sometimes used to deliver a large flow against a relatively low system resistance or where the slurry to be pumped is non-settling. Parallel pumps may also be installed where backup spares are needed, where a very wide range of flows is to be handled and each individual pump is to be kept down to a manageable size, or where wear limitations and suction requirements make it preferable for each pump to handle less than the total flow. In parallel pumping, the composite 'pump characteristic' is obtained by adding the individual pump flows at each value of head. The overall system operation is defined by the intersection of this combined characteristic with the system characteristic. Each individual pump operates at the flow, efficiency and NPSH corresponding to this head. Even more than for series pumping, it is usually important to ensure that all the pumps are identical.

Figure 16. Interior of Pumping Station; Showing Three Pumps in Series.

The profile of the pipeline can cause difficulties if there are any large changes in level. When the line is shut down, any local high spots are susceptible to subatmospheric pressures, which are sometimes severe enough to cause vaporization. On subsequent startup, the collapse of the vapor pocket could initiate severe hydraulic transients. To avoid these, vacuum relief valves should be fitted at all high points, or else the line must be completely drained after each shutdown. Vacuum relief valves can also be used on the suction of a pump that is prone to cavitation, to let air into the suction, thereby reducing the effects of cavitation. This same idea is applied to dredge suction pipes, where a hoover valve is used, letting in water rather than air.
Flow Control and Valving

In dredging and in some mining operations, the solids throughput and size may vary widely, sometimes in a matter of minutes or less. In dredging operations, the system resistance may halve or double over a period of days, as the dredge is moved about. Control of pump and system operation is required to maintain reliability and efficiency. For this reason variable-speed drive is usually provided. Longer-term adjustments can be made by changing the drive to a different fixed speed or range of variable. As a more extreme alteration, the impeller can be changed for one with a smaller diameter.

When a system is being painted, filled, and started, there is little or no system head. Particularly for a long system, operation at fixed speed can lead to excessive overdischarge, so that a variable-speed drive should again be used. In systems with several pumps in series, it is common practice to provide variable-speed drives on the first and last units. The speed of the first pump is varied during filling, with each subsequent fixed-speed pump brought into service as its suction pressure shows sufficient available NPSH for it to be operated without cavitation. The variable speed on the last pump is used to control flow during subsequent operation. This method is preferable to varying the speed of any of the upstream pumps, because reducing speed could lower the discharge pressure to the point where the NPSH available to the next pump falls below the NPSH required and it cavitates.

Dredges with diesel or electric motors normally have variable speed drive, for control purposes. However, full control extends beyond simple adjustments to speed and level, and requires more comprehensive instrumentation to measure flows and concentrations. Some available instruments are reviewed later. Many dredges now have full instrumentation with onboard computer systems for sensing, monitoring and controlling operation, and, in more sophisticated application, for producing a historical production record and for online maintenance planning. This kind of system is starting to penetrate into the mining industry.

It has already been noted that flows in implant mine and tailings systems are typically less variable. Variable-speed drives can sometimes be avoided in these applications by allowing the liquid level in the sump to vary, provided the range of variation in sump level is significant in relation to the total head charge through the system. The action of this type of ‘passive control’ is illustrated in Figure 18. If the flow into the sump reduces, the level falls. The static lift required increases correspondingly, so that the system curve moves up until operation stabilizes at point B1 with reduced throughput. Similarly, if flow into the sump increases, the level rises and the system requires less head, so that operation stabilizes with the increased flow at point B2. For a pump with a flat characteristic, as in Figure 18, relatively wide variations in flow can be accommodated in this way, provided that the pump is able to operate over a wide enough range.

Figure 18. Schematic Illustration of Stabilizing Influence of Sump on Intercept of Pump and System Characteristics.

Instrumentation

The abrasive and sometimes corrosive nature of most slurries is such that it is difficult to make instrumentation accurate and reliable. Great care must be taken with selection, installation, calibration and maintenance if instruments are to perform as required. Safe operation and troubleshooting dictate the bare minimum of instrumentation needed. More extensive instrumentation is necessary to maximize output and achieve cost-effective operation. As noted before, instrumentation can also play a vital role in the maintenance of system components and can be used to provide a record of operation.

To ensure stable pump operation, it is essential to monitor suction conditions, indicated by sump level or suction pressure. Pressure at the pump discharge can be used to give an immediate indication of changes in operating conditions, but otherwise has limited value as a diagnostic or control measurement because it depends on several variables. The pressure increase across a pump depends on delivered slurry density, but also depends weakly on discharge rate because most slurry pumps have gently drooping head-discharge characteristics. Taken together, the suction and discharge pressures and the power drawn by the pump can provide useful diagnostic information. For example, a rise in pump power and discharge pressure together usually indicates a rise in solids concentration. Systematic control requires measurement of other parameters. The most important are flowrate and solids concentration.

The abrasive properties of slurries exclude orifice plates for flow measurement. Venturi and nozzle meters can be used, but they are still subject to wear even when constructed from a wear-resistant material so that the calibration must be checked regularly. A simpler device, which lends itself readily to slurry use, is the bend meter shown schematically in Figure 19. The pressure difference is measured between the inside and outside of a bend. A particular advantage of this technique is that it can be applied to an existing bend, and requires no additional disturbance to the slurry. Precautions such as regular flushing must be used to ensure that the pressure readings are reliable. This type of device becomes less reliable if the flow is stratified, and it is, therefore, preferable to
apply the measurement at a bend where the entering flow is
to the top of a ‘riser’ or the bottom of a ‘downcomer.’
Experience at the author’s company has shown that, provided these
precautions are taken, the bend meter is reliable and gives repro-
ducible measurements. As for related devices (such as an orifice
plate or Venturi) that obtain flowrates by measuring the pressure
difference due to a change in the momentum of a flowing fluid, in
the bend meter the flowrate is proportional to the square root of the
pressure difference, corrected for any difference in elevation be-
tween the two measurement points. To a first approximation [19]
the slurry flowrate for a 90 degree bend, with pressures measured
at the 45 degree positions as shown in Figure 19, is

\[ Q_m = \pi D^2 \sqrt{\frac{R \Delta p}{D p_{in}}} \]  

(35)

Figure 19. Bend Meter (schematic).

Here D is the pipe diameter, R is the bend radius, and \( \Delta p \) is the
pressure difference from the outside to the inside of the bend.
While Equation (35) may be used to calculate the range of a bend
meter and the range of \( \Delta p \) to be determined, it is essential to
 calibrate any specific bend meter against some ‘absolute’ measure-
ment such as a magnetic flowmeter.

For precise measurement, the most widely used flow instru-
ments are magnetic flowmeters and Doppler meters. A magnetic
flow meter measures total volumetric flow and is nonintrusive, i.e.,
it does not disturb the flow. Readings from a magnetic flowmeter
can be accurate to within 0.5 to 1.0 percent, of full scale reading,
although if the solids are significantly ferromagnetic the device
will indicate an incorrectly high flow. Furthermore, magnetic
flowmeters of the AC type respond anomalously to large particles,
and are, therefore, affected by stratification. Entrained air also
causes a magnetic flowmeter to give an incorrect reading of the
slurry flowrate. For these reasons, it is desirable to locate a meter
of this sort in a straight vertical section of pipe.

Doppler meters are also popular, partly because of their rela-
tively low cost. A Doppler meter uses a sensor in the pipe wall,
transmitting ultrasonic waves into the flow and measuring the
frequency shift of waves reflected by the transported particles.
Thus, a Doppler meter is also noninvasive. Because the device
responds to the particle velocity, the reading can be distorted by
stratification, to the point where the velocity indicated depends on
the sensor location in a stratified flow [20]. Again, it is usually
preferable to locate the instrument in a straight vertical pipe.

To measure solids concentration, or the equivalent parameter of
mean mixture density, a particularly useful device is the ‘U-loop,’
shown schematically in Figure 20. The pressure gradient is
measured in two vertical pipes, a ‘riser’ and a ‘downcomer.’ The
delivered solids concentration for this are given by

\[ S_{md} = \frac{(p_1 - p_2) + (p_3 - p_4)}{2gZ\rho_w} \]  

(36)

\[ C_{vd} = \frac{1}{(S_s - 1)} \left[ \frac{(p_1 - p_2) + (p_3 - p_4)}{2gZ\rho_w} - 1 \right] \]  

(37)

where P1 to P4 are the static pressures in the slurry at the four mea-
surement locations shown in Figure 20. If the pressure gradients
are actually measured by connecting the tappings to a pair of trans-
ducers with the connecting lines filled with water, the recorded
pressure differences will be

\[ \Delta p'_A = (p_1 - p_2) - \rho_w gZ \]  

(38)

and

\[ \Delta p'_B = (p_1 - p_2) - \rho_w gZ \]  

(39)

In terms of these measured values

\[ S_{md} = \frac{\Delta p'_A + \Delta p'_B}{2gZ\rho_w} + 1 \]  

(40)

\[ C_{vd} = \frac{\Delta p'_A + \Delta p'_B}{2(S_s - 1)gZ\rho_w} \]  

(41)

Figure 20. Schematic of Inverted U Loop for Measurement of
Delivered Slurry Density and Solids Concentration.

The limits to the accuracy of these equations, examined in detail
by Clift and Clift [21]. The U-loop concept lends itself well to field
measurements, and the device is often used on dredges. An
example is shown in Figure 21 of such a device used in a field test
on a phosphate matrix pipeline.

As an alternative, slurry density may be measured using a meter
that relies on the attenuation of some form of radiation passed
through the slurry. Gamma-rays are most commonly used so that
this type of instrument is sometimes referred to as a ‘nuclear den-
sitometer.’ The parameter measured is the mean density along the
radiation path, which indicates the in-situ density rather than the
delivered value: this inconvenient feature is sometimes overlooked by operators who use the results from this type of instrument to calculate throughput. Furthermore, in a stratified flow the reading will depend on the location of the gamma-ray beam within the slurry. Therefore, as for most of the other instruments reviewed here, a radiation densitometer is best installed on a straight vertical section of pipe.

An interesting future development rests on the possibility of mapping the local density within the slurry by using gamma-ray attenuation measurements along a number of paths at different orientations through the pipe [22]. A time-averaged image of the density profile in the pipe is then reconstructed by computer-aided tomography (CAT), in exactly the same way as medical CAT-scanners are used to obtain noninvasive sections through a living body. Although tomographic imaging is not likely to become widespread, it promises to become a valuable research tool for slurries and other multiphase flows.

Selection of the system components is important, but careful overall design of the system, together with correct selection of the pumps, is absolutely necessary to ensure satisfactory operation.

**Pump Solids Effect**

Although the presence of solid particles introduces more complicated effects than those accompanying a viscosity increase in a simple fluid, where is some rough qualitative similarity between slurry flow and the flow of a fluid having values of both density and viscosity greater than those for water. The effects on pump characteristics are shown schematically on Figure 22, which is a definition sketch for illustrating the reduction in head and efficiency of a centrifugal pump operating at constant rotary speed and handling a solid-water mixture. In this sketch, and the discussion which follows, \( \eta_w \) represents the pump efficiency in slurry service, and \( \eta_m \) is the clean-water equivalent. Likewise, \( P_m \) and \( P_w \) are the power requirements for slurry service and clean water service, respectively. The head, \( H_w \), is developed in slurry service, measured in height of slurry, while \( H_m \) represents the head developed in water service, and not in water in height of water. The head ratio, \( R_H \), and the efficiency ratio, \( \eta_m \), are defined as \( H_m/H_w \) (or \( \eta_m/\eta_w \)), respectively. The fractional reduction in head (the head reduction factor) is denoted by \( R_H \) and defined as \( 1 - H_m \); for efficiency the fractional reduction (efficiency reduction factor) is \( R_{\eta} \), given by \( 1 - \eta_m \).

Obviously, it is important to relate quantities such as \( R_H \) and \( R_{\eta} \) to the slurry properties. These include the relative density of solids \( S_s \), the delivered volumetric concentration \( C_{vd} \), and the relative density of the slurry 5th, which equals \( 1 + (S_s - 1)C_{vd} \).

The main work done in this area has been by Sellgren [23, 24, 25, 26], who has put together a generalized solids effect diagram for pumps shown in Figure 23.

**Figure 22. Effect of Slurry on Pump Characteristics (schematic).**

This diagram from Wilson, et al., [27] gives \( R_H \) in terms of pump impeller diameter \( D \) and solids size \( d_{50} \), with corrections for concentration \( C_{vd} \), relative density of solids \( S_s \), and content of fine particles \( X_f \), where \( X_f \) is the mass fraction of particles smaller than 75 micron. This correction applies for \( X_f \) in the range 0.05 to 0.50.

**Figure 23. Generalized Solids-Effect Diagram**

**Wear in Pump and Pipelines**

The useful life of most slurry transport equipment is limited by erosive wear of the wetted passages. As a result, wear performance must often be evaluated in connection with the design or operation of slurry systems. Wear is a common industrial problem, leading to frequent maintenance and replacement of components, and possibly also to reduced operating efficiencies. In simplest terms, erosive wear amounts to the progressive removal of material from a solid surface. In practice, the mechanisms by which this erosion occurs are diverse.
As the factors affecting wear performance are manifold, and the gamut of slurry applications broad, a good deal of wear-performance evaluation has occurred post-facto, when the system is already in operation. A body of experience and insight gathered by this method has accumulated over time, and much of the current design for wear performance of slurry systems is based on this experience.

Recent years have also seen the introduction of more rigorous approaches to wear-performance evaluation. These include standardized laboratory tests for ranking slurry abrasivities and material wear resistance's, and electron microscopy for providing close examination of the micro-mechanisms of wear for both laboratory and field-collected samples. The approach of numerical modeling of slurry flow is also gaining popularity as more powerful computers become widely available, and as numerical techniques become more refined.

In slurry pipeline systems the wear of pump components is of major practical importance. As might be expected it is found that wear varies considerably from pump to pump, and from application to application. Nevertheless, there are several typical or recurrent patterns of wear which are often observed in slurry pump casings and impellers, and thus merit particular attention. Specifically, slurry-pump casings often experience maximum wear in the outer radius or belly (Figure 24), a zone where sliding beds of solids form as a result of centrifugal forces, as noted previously. Some specific area on the circumference of the casing generally experiences the maximum wear rate, but the location of this area can shift with the operating conditions and the geometry of the casing. For increasing flowrates, and casings of more annular geometry, it is found that the location of maximum wear tends to shift toward the pump discharge and away from the tongue.

Another common wear pattern in slurry-pump casings is gouging or extreme localized wear in the side wall of the casing just downstream of the tongue (Figure 25). This gouging is initiated by the three-dimensional eddy (Figure 14), generated where the tongue part the fluid. The severity of the gouging depends on the design but is typically dominated by the ratio between the operating flowrate and best-efficiency flowrate. Specifically, pumps operating well below the best-efficiency flowrate recirculate large volumes of flow past the tongue, increasing the velocity and size of the gouging eddy.

Gouging eddies may also occur at other locations in the slurry-pump casing, or the impeller. In some instances, such eddies can be identified as having their origin in geometric discontinuities, and can be eliminated by judicious design modifications. In other instances, they may be traced to operational considerations, such as operation well away from BEP flow, as noted above. In all cases, it is vital to have a sound understanding of the large-scale flow patterns within the pump. This understanding depends on careful observation and also on analysis, using both basic fluid mechanics and numerical modeling. A healthy sense of three-dimensional visualization is extremely useful in suggesting design or operational modifications that can improve overall wear performance.

Gouging eddies in the casing may also affect flow patterns in the clearance between the impeller and the suction-side liner, producing secondary gouging here. The localized wear can be severe (Figure 26), and in this instance a practical method of extending the life of the liner is to rotate it 180 degrees when one-half to three-quarters of its estimated life has elapsed, thus moving the wear zone to a new portion of the surface.

As the slurry pump casing is a consumable part, its wear performance can often be improved by noting the wear patterns experienced, and applying the known operational and geometric considerations which affect those patterns. Then, through experimentation, the casing size, geometry and operating conditions can be modified in a direction which indicates increased wear life.

Wear of the impeller of a slurry pump is often closely related to the hydraulic efficiency that is exhibited. This seems reasonable, since improved hydraulic efficiency generally coincides with reduced velocities in the recirculating eddies which cause localized wear. However, a well-designed slurry-pump impeller can often sustain considerable wear before its pumping capacity is reduced to an unacceptable level, and may appear to be worn out long before it needs to be replaced from an operational standpoint (Figure 27). On an impeller the two areas of worst wear are usually
near the inlet and the outlet. At the inlet, eddies can be induced by changes in curvature of the flow, or by the intrusion of the recirculating flow from the clearance between the impeller and the suction liner. The high velocities at the impeller outlet are the major cause of problems there.

![Figure 27: Typical Worn Impeller.](image)

The key to improving the overall wear performance of any slurry-pump component is often found in identifying the causes of localized wear and either eliminating them or spreading them over a wider area. Consideration of design geometry and operating conditions play a role in this process. The materials of construction may also be varied in order to improve wear performance.

Typically, some balance between strength, toughness, wear resistance, serviceability and cost must be struck, with no one material holding the advantage in all respects.

The wear of other components of a slurry-pipeline transport system is also important. The pipe itself can wear, of course, both by the particle impact associated with fluid turbulence and by sliding abrasion. The latter mechanism is usually dominant in pipe flow, with wear concentrated in the lower portion of the pipe for both fully-stratified and heterogeneous flow regimes. Sometimes the resulting wear can limit pipe life, and in this case it may be worthwhile to rotate the pipe through 120 degrees at the estimated one-third and two-thirds points of its life-span; thus limiting the wear to any one part of the pipe wall. In more serious cases, a lined pipe may be considered, but this step is seldom required except when the slurry flow combines with chemically corrosive conditions. A stationary deposit of solids in the pipe produces a characteristic wear pattern, with the wear concentrated in bands at each side of the lower portion of the pipe interior, say at the 4:00 and 8:00 positions. Sometimes, the occurrence of this wear pattern provides the first indication that a system has been operated at too low a mixture velocity, i.e., in the inefficient stationary-deposit flow regime.

In bends and other fittings wear is usually much more severe than in straight lengths of pipe, with wear rates easily increasing by an order of magnitude or more. The basic reason for this enhanced erosion is the curvature of the streamlines, which produces centrifugal forces. This causes the particles to impact on the boundary, a point noted earlier in this chapter. In some cases, it is desirable to line the bends in order to avoid frequent replacement, in other instances the replacement cost is tolerated, or else the bends are fabricated of metal that is thicker, and possibly harder, than that used in the straight lengths of pipe.

For systems pumping simple fluids in small-diameter pipes it is quite common to control the flowrate by a throttling valve in the pump discharge pipe. This configuration is highly wasteful of energy, as the pump produces a higher head than is required, and the excess head is dissipated at the valve. The resulting energy loss may sometimes be tolerable in small fluid systems, but designers accustomed to this approach sometimes use the same configuration in slurry applications, generally with unfortunate results. Slurry going through a partly-closed valve can produce extremely rapid wear, and at the same time the pump will be forced to operate well into the underdischarge range, inviting the gouging type of wear described earlier. The appropriate solution is usually to reduce the diameter of the pump impeller, eliminate the throttling valve and use other methods of flow control such as the variable-speed drives discussed earlier.

**NOMENCLATURE**

- $C_D$: Particle drag coefficient
- $C_T$: Relative volumetric concentration of solids, $C_{vd}/C_{vb}$
- $C_{sm}$: Value of $C_T$ at which deposition velocity is a maximum
- $C_v$: Volumetric concentration-volume fraction of solids
- $C_{vb}$: Volumetric solids concentration in loose-poured bed
- $C_{vd}$: Deliver volumetric solids concentration
- $C_{vi}$: In situ volumetric solids concentration
- $C_w$: Solids concentration by weight (or mass)
- $d$: Particle diameter [L]
- $d_{50}$: Mass-median particle diameter [L]
- $D$: Diameter (internal diameter of a pipe, diameter of a pump impeller) [L]
- $f$: Moody friction factor for the pipe $f = 8 \tau_f / \rho V^2$
- $f_f$: Value of $f$ for equal volumetric flow of fluid
- $f_w$: Value of $f$ at equal volumetric flow of water
- $g$: Gravitational acceleration [L T$^{-2}$]
- $h$: Height [L]
- $h_v$: Vapour-pressure head [L]
- $H$: Head [L]
- $i$-HD: Hydraulic gradient (head lost due to friction/length of pipe)
- $i_f$: Value of $f$ for equal volumetric flow of fluid (height of water/length of pipe)
- $i_m$: Value if $f$ for flow of mixture (height of water/length of pipe)
- $i_{mh}$: Value of $f$ for flow of homogeneous mixture (height of water/length of pipe)
- $i_g$: Hydraulic gradient for plug flow, (height of water/length of pipe)
- $i_w$: Value of $f$ for equal volumetric flow of water (height of water/length of pipe)
- $i_{im}$: Friction gradient in height of mixture/length of pipe
- $k$: Coefficient in power-law rheologic model or roughness (L)
- $l$: Length of particle (L)
- $L$: Length, measured along pipe (L)
- $M$: Exponent in stratification-ratio equation
- $n$: Flow behavior index
- $p$: Pressure (M L$^{-1}$ T$^{-2}$)
- $\Delta p$: Pressure difference
- $Q$: Volumetric flowrate (L$^3$ T$^{-1}$)
- $Q_{bep}$: Flowrate at best efficiency point (L$^3$ T$^{-1}$)
Radius [L]

Head reduction factor [___]

Efficiency reduction factor [___]

Pipe Reynolds number

Relative density of fluid [___]

Relative density of mixture [___]

Delivered relative density of mixture [___]

In situ relative density of mixture [___]

Relative density of solids [___]

Specific energy consumption [___]

Mean transport velocity of mixture, i.e., ratio of volume of mixture delivered per unit time to the cross-sectional area of the pipe; \( V_m = \frac{Q}{A} \)

Terminal settling velocity of a single particle \( \left[ \text{L T}^{-1} \right] \)

Hindered settling velocity \( \left[ \text{L T}^{-1} \right] \)

Mean velocity, \( 4Q/(\pi D^2) \left[ \text{L T}^{-1} \right] \)

Mean mixture of velocity \( \left[ \text{L T}^{-1} \right] \)

Relative velocity \( \nu_m \nu_m [___] \)

Value of \( \nu_m \) at limit of deposition \( \left[ \text{L T}^{-1} \right] \)

Maximum value of \( \nu_s \left[ \text{L T}^{-1} \right] \)

Distance, usually measured along pipe \( [\text{L}] \)

Angle defining interface in stratified flow [___]

or exponent in eqn. (4.13) [___]

or vane angle of pump [___]

Roughness height \( [\text{L}] \)

Relative excess pressure gradient [___]

Value of \( \zeta \) at very large, \( \nu_r [___] \)

Efficiency of machine [___]

viscosity

Density of fluid \( [\text{M L}^{-3}] \)

Density of mixture \( [\text{M L}^{-3}] \)

Density of mixture delivered \( [\text{M L}^{-3}] \)

Density of mixture in situ \( [\text{M L}^{-3}] \)

Density of water \( [\text{M L}^{-3}] \)

Shear stress \( [\text{M L}^{-1} \text{ T}^{-2}] \)

Shear stress at flow boundary \( [\text{M L}^{-1} \text{ T}^{-2}] \)

SUBSCRIPTS

d = Discharge (or throughput)
f = conveying fluid and friction
m = Mixture of solid and conveying fluid
o = At pipe wall
s = solids
v = Volume Cranfield
w = Weight

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