Tests on the Hardinge Conical Mill

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Introduction

THE major portion of the work described in this paper was performed by R. W. Young,† a graduate student in the department of Mining and Metallurgy, Sheffield Scientific School, Yale University, working under a coöperative agreement between the Hardinge Conical Mill Co., the Sheffield Scientific School, and himself.

Since this coöperative scheme is at present in effect in the case of a considerable number of other students in the department and since it is the hope of the school that the privilege thus extended may be utilized even more freely in the future by mining and manufacturing companies, it may not be amiss at this point to give a summary of the general plan. Briefly it is as follows:

A graduate student, whose undergraduate work in this or other universities shows promise of ability to handle research work, is chosen by conference between the company and instructor involved. The aim of the company in the agreement then entered into is to obtain the solution of one or more of the technical problems with which it may be confronted, or, at the end of 1 or 2 years, to obtain as an employee a man especially trained in its work. As a means to accomplish one or both of these ends, the company furnishes the machine, apparatus, or material to be tested and pays the student during his graduate work a small salary, usually just sufficient to cover his living expenses, tuition and fees. The aim of the student is special training along a line in which he is particularly interested, the attainment of his advanced degree, and the chance to show to his future employer ability to handle such problems as may be presented to him. In return for the financial aid which he receives he agrees to devote at least half of his working time to the special problem submitted by his company. The other half is devoted to study of the collateral subjects required by the department for the granting of the degree which the student seeks. The student further agrees to enter the employ of the company in question at a wage not greater than that paid in like positions to recent graduates not specially trained and to remain with his employer at such wage for at east 1 year. If the student is to obtain a degree, the special work forming the basis of his investigation must be such as will involve real research and not mere routine manipulation. The subject is chosen by conference between the three parties to the agreement. The work is carried on under the direct supervision of the instructor involved. The school furnishes the general laboratory and library equipment essential to the pursuit of any extended investigation. In

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return it is expected that the results of the investigation shall be, in part at least, available for publication, if they are deemed of interest to the profession.

Mill Used

In the laboratory work described in the following pages, a 4½-ft. Hardinge mill with three removable cylindrical sections, 16 in. each in length, was used. Fig. 1 shows the mill with three cylindrical rings in place. This combination allows a mill 4½ ft. by 0 in., 4½ ft. by 16 in., 4½ ft. by 32 in., or 4½ ft. by 48 in., as desired. The conical and cylindrical sections were built of cast iron, 1¼ in. thick and were lined with chrome-

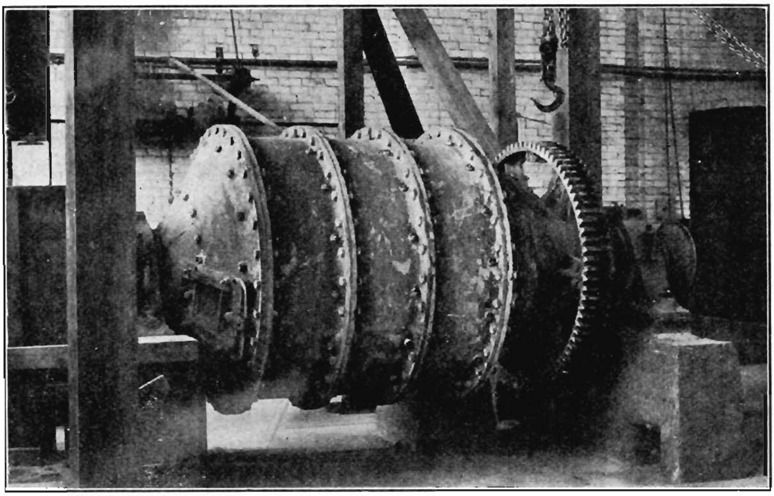


Fig. 1.—Hardinge mill used in grinding tests.

steel lifting bars 2½ in. high, 3 in. wide and 16 in. long, set on 11-in. centers. The head bearing was adjustable in height, thus allowing the mill to be tilted any desired amount.

Material

A majority of the tests were made on quartzite and trap. The quartzite contained an appreciable amount of white mica in flakes 1 to 2 mm. (0.04 to 0.08 in.) diameter, which made it rather easy to crush in the coarser sizes but difficult to grind when the finer sizes were reached. The trap was a variety of diabase quarried locally for road metal. The other materials tested (see tests 230 to 236) were of a special nature and will be described more particularly in connection with the record of the work done upon them.

Method of Feeding

The material to be ground in any given test was weighed up and divided into lots, each lot being sufficient to furnish the feed for the mill for 5 min. In general, this lot was divided by eye into five portions, so that a portion could be dumped into the feed box every minute. This procedure, aided by the low capacity of the scoop feeder on coarse material, assured a practically uniform feed rate. When, as was the case with finer materials, the scoop tended to take up the material meant for a 1 min. portion in two or three revolutions, the method of feeding was so changed as to take away from the scoop the burden of regulating the feed for even such a short interval as 1 min. The importance of this insistence on regular feed will be seen in Fig. 10, which presents a comparison of the feed and discharge rates of the mill, dry crushing. In wet feeding the same methods of introducing the rock were followed. The water was introduced into the feed box from a calibrated orifice at the proper rate to give the desired moisture percentage and the result was checked by moisture samples of the discharge.

Sampling

Feed samples were taken, in every case, by the method of alternate shovels. Large samples were cut to insure accuracy. Samples of the product consisted of the whole discharge stream caught for varying intervals according to the feed rate. The interval for wet samples was rarely less than 1 min. For dry samples the interval was never less than 1 min. and in all cases where the feed rate was less than 1 ton per hour the sample of the product consisted of the whole discharge for an interval of 5 min.

Screen Testing

Screen tests on feed samples were made in duplicate. The accuracy of the sampling was accepted as sufficient when cumulative graphs of the tests were closely coincident. Product samples were passed over the 6.680-mm. (0.26-in.) sieve. The total oversize on this sieve was then run through the coarser series. The undersize of the 6.680-mm. sieve was riffled down to not less than 200 gm. and then run through the remainder of the Tyler Standard Sieve Scale series of screens (1.414 ratio). The amounts of the aliquot parts of the whole sample remaining on these fine sieves and passing the last (0.074 mm.) were then calculated back into terms of the whole sample and the percentages given in Table 2 were calculated from the figures thus determined. Duplicate samples of the riffled undersize were run occasionally in order that frequent screen tests might not breed carelessness. In no case was the difference between duplicates greater than that to be expected in grading analyses.

Moisture Determination

Moisture determinations were made on products only, as the feed was, in every case, so dry as to be dusty. In all cases, the weight of the solid plus water in the sample and the weight of the dry solid were determined by direct weighing and the percentage of moisture calculated from these figures.

Power Measurements

Belt drive was used for the testing work. The power transmission is not so efficient, of course, as direct drive through herring-bone gears and the latter installations will give higher relative mechanical efficiencies than those recorded in this paper. The watt-hour meter and the voltmeter and ammeter were read at 5-min. intervals. The watt-hour meter readings are the basis for the figures of power consumption used, the readings of the indicating instruments being used for purposes of check only.

Outline of Testing Work

Objects

The specific object of the work described in the following pages was the determination of a set of constants and characteristic curves for the conical mill which could be applied to any installation. The 4½-ft. mill is large enough to do any class of work for which the conical mill is suited, its only limitation being a question of capacity. It was hoped to cover the question of variation in capacity due to variation in diameter by a few tests on mills of other sizes. It has, however, been impossible to do this, and the writer can offer but a tentative rule based on figures collected by correspondence.

Plan of Work

The plan of the work was to start with some given set of conditions, for instance, a 16-in. cylindrical section 4006-lb. load of mixed balls, a trap rock feed of a given size, no moisture, mill level (Test 202); and, keeping these conditions constant, vary one other condition (Tests 203 and 204), in this case the feed rate, and determine the effect of this variation on the character of the product, the horsepower, and the relative mechanical efficiency. By varying in similar manner the size of the feed, the kind of rock fed, the percentage of moisture, length of cylindrical section, slope of the mill, and the character and weight of the crushing charge, the effects of such variations on the performance of the 4½-ft. mill were determined.

In the collection of the aforementioned data various attendant phenomena of considerable interest were observed. Thus the distribution in the mill of the various sizes of balls composing a mixed charge was accurately determined, the effects of slope and moisture percentage on the possible maximum crushing charge were observed, and the lack of agreement between the feed and discharge rates over any short interval (5 min. for example), resulting in practice in a pulsation in the flow to subsequent machines in a mill flow sheet, was studied. These results are presented in their proper places later. A complete series of power tests, totaling more than 100, was made to afford a basis for a formula giving the horsepower required by a conical mill of any size.

Power Tests

Description of Tests

Six series of tests were made to determine variations in power consumption with varying conditions of loading, one series for each of the following conditions:

Test Series No.	Length of Cylindrical Section, Inches	Condition of Pulp
1	16	Dry
2	16	Wet
3	32	Dry
4	32	Wet
5	48	Dry
6	48	Wet

In each series the first set of power readings was made with the mill empty. Successive sets of readings were then taken with ball loads starting at 500 lb. (226.8 kg.) and increasing by 500-lb. steps. With each 500 lb. of balls, 170 lb. of trap rock was charged in order to prevent excessive wear and hammer in the mill. In the dry tests loading was continued until the surface of the load was considerably above the axis of the mill, discharge being prevented by plugging the discharge end. In the wet tests enough water was fed to produce a slight discharge throughout the series, and the tests were discontinued when discharge of balls commenced. Rock was fed in these latter tests from time to time to balance the rock carried off in the discharge, but no exact balance was attempted and the degree of balance attained is not known. The duration of the tests for each condition of loading varied from 30 to 90 min. Power readings were taken every 5 min. and the run was continued until the power-time curve became a horizontal line. The early readings in any given test were considerably higher than the last, due probably to cold bearings, slipping belts, etc., and were disregarded in making up the average power consumption for the run.

Fig. 2 is a graph covering all the power tests. The greatest variations from smooth curves occur near the end of the graphs for the 4½-ft. by 48-in. mill. These variations are due, in part at least, to an overloaded motor. Fig. 3 was plotted in an attempt to draw the curves for the different tests closely enough together to give a reasonable basis for an average curve upon which it would be possible to base an empirical form-

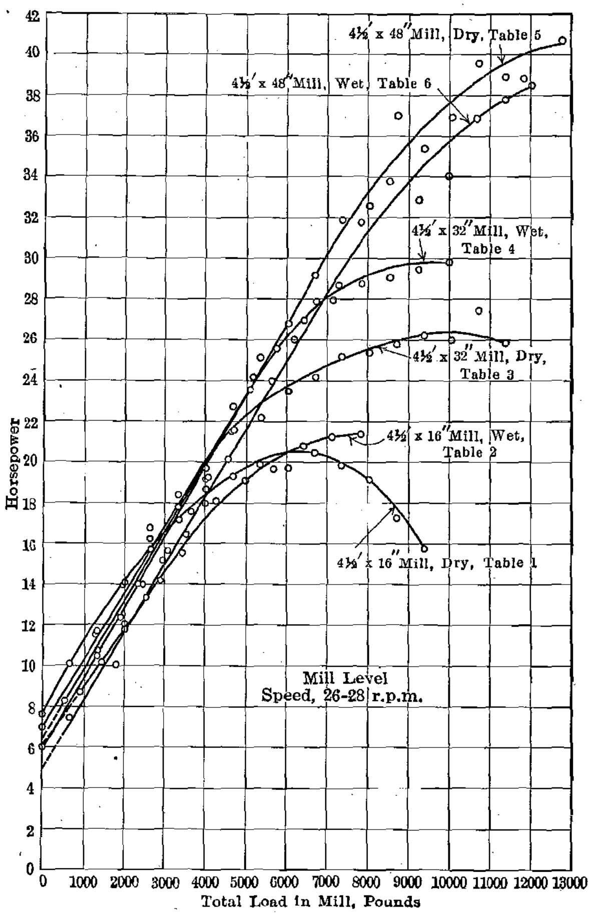


FIG. 2.—POWER CONSUMPTION OF 4½-FT. CONICAL BALL MILL AT HAMMOND LABORATORY.

ula for horsepower. As will be seen by reference to this figure, the curves are closely parallel throughout their respective lengths, the only graph departing seriously from parallelism with the others being that for the 4½-ft. by 16-in. mill, dry, series No. 1. It will be noted that the variation of this curve begins at the point where the charge in the mill rose above the horizontal axis. The mill was here working under unnatural conditions, so that this variation will not affect a formula designed to cover working ranges only.

From this point two methods of procedure were followed, resulting in the following formulæ for the horsepower of a conical ball mill within working ranges.

$$(1)\ Hp. = 0.002(100D^2L^{0.132} + L) \qquad (1)$$

$$(2)\ Hp. = \frac{D^{2.42}L^{0.086}}{6.53} + \frac{L}{1000}(0.025m + 1.4) \qquad (2)$$

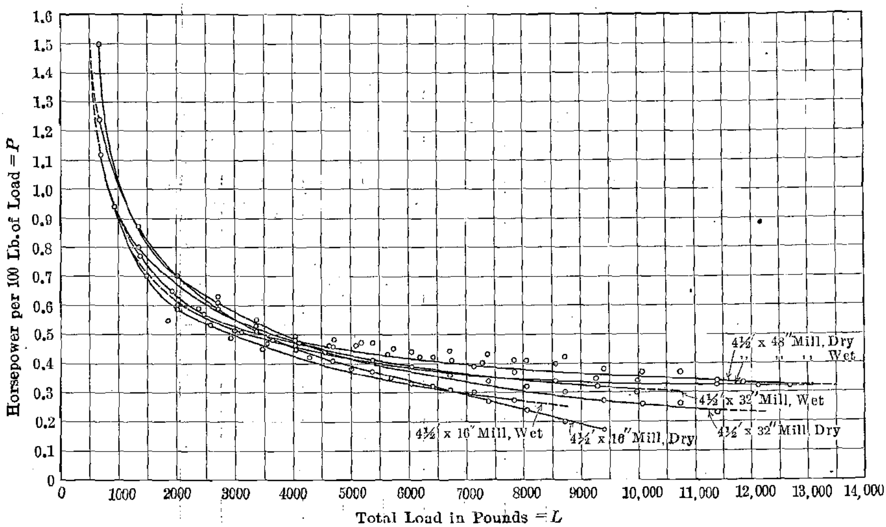


FIG. 3.—POWER CONSUMPTION PER 100 LB. OF BALL LOAD BY $4\frac{1}{2}$-FT. CONICAL BALL MILL WITH DIFFERENT CYLINDER LENGTHS.

where

D = the internal diameter of the mill in feet,
L = the total load in the mill in pounds,
m = the length of the cylindrical section in inches.

The first of these formulæ is of the nature of a preliminary trial and was developed from a free-hand average curve drawn through the curves on Fig. 3. It does not, therefore, give results which check throughout the range of operating conditions. Formula (2) was developed as outlined in the succeeding paragraph.

The first step in the determination of Formula (2) was to plot the average curve shown in Fig. 4 from the curves on Fig. 3. The points determining this curve were obtained by averaging the ordinates of the curves on Fig. 3 at the abscissæ 1000, 2000, etc. Arbitrary ordinates y and abscissæ x were then assigned to this curve and various functions such as x^2, y^2, $x^{1/2}$, $y^{1/2}$, $\log x$, $\log y$, $\frac{x}{y}$, $\frac{y}{x}$, $\frac{1}{x}$, $\frac{1}{y}$, etc., were plotted against each other in different combinations in an attempt to straighten out the curve. The best approximation to a straight line was obtained by plotting log $(y - 2)$ as ordinates and log (x) as abscissæ. The points thus obtained are shown on Fig. 5. The straight line drawn through these points was obtained by averaging ordinates and abscissæ. The equation for this line is:

$$\log (y - 2) = 1.213 - 0.914 \log (x) \tag{4}$$

From (4)

$$\log (y - 2)(x^{0.914}) = 1.213 \tag{5}$$

Taking antilogarithms of both sides

$$(y - 2)(x^{0.914}) = 16.33 \tag{6}$$

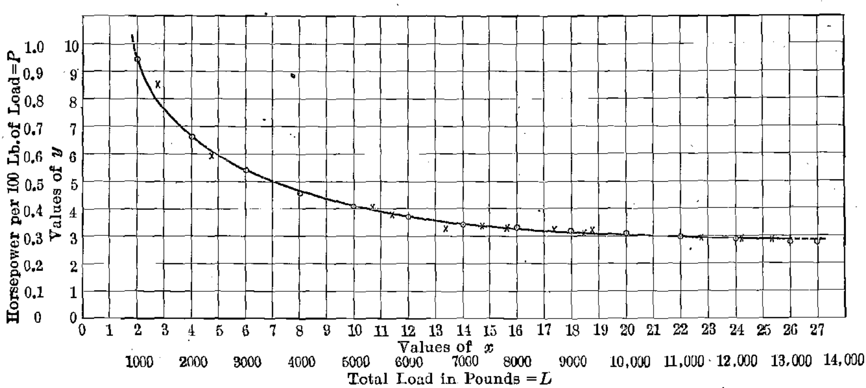


FIG. 4.—AVERAGE POWER CONSUMPTION OF $4\frac{1}{2}$-FT. CONICAL BALL MILL.

Clearing

$$y = \frac{16.33}{x^{0.914}} + 2 \tag{7}$$

But from Fig. 4

$$y = 10P \tag{8}$$

where P = hp. per 100 lb. of load

$$x = \frac{L}{500} \tag{9}$$

Substituting these values for x and y in equation (7) and clearing

$$P = \frac{478}{L^{0.914}} + 0.2 \tag{10}$$

But

$$Hp. = \frac{LP}{100} \tag{11}$$

Then

$$Hp. = 4.78L^{0.086} + 0.002L \tag{12}$$

Horsepowers solved for by this formula for different loads are indicated by crosses (x) on Fig. 4. These check solutions show a close agreement with the average curve, but show in some cases as much as 30 per cent. departure from the horsepowers determined experimentally. The variations are, as might be expected, greatest for the 16-in. and 48-in. cylindrical sections, as the average curve of Fig. 4 departs most greatly from the curves for these cylinder lengths. In order to eliminate this variation, Formula (12) was written as follows:

$$Hp. = 4.78L^{0.086} + CL \tag{13}$$

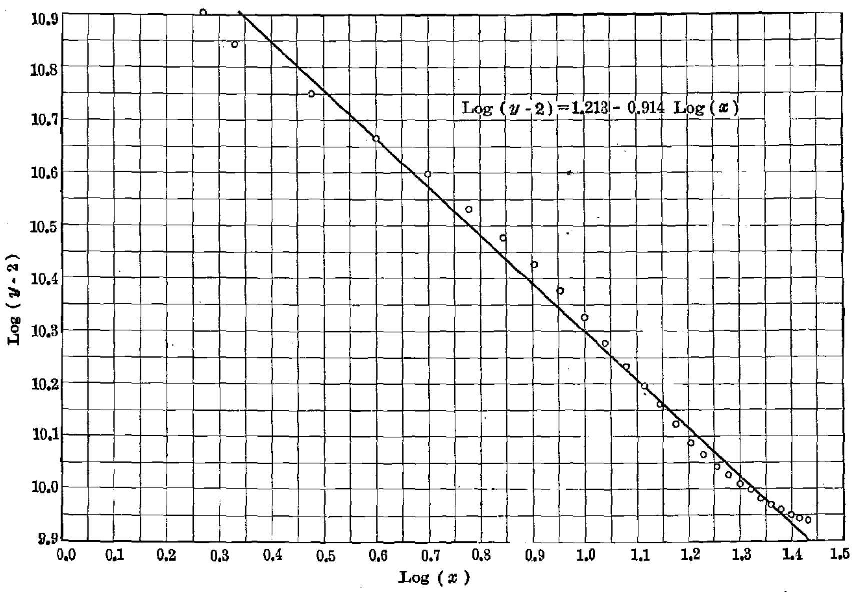


FIG. 5.

and average values of C were determined for different cylinder lengths by substituting known values of $Hp.$ and L corresponding to values of m from results of tests, series 1 to 6 inclusive. By this method the following corresponding average values of C and m were determined:

m	C
16	0.0018
32	0.0022
48	0.0027

The relation between these quantities can be expressed in the linear form

$$C = 0.000025m + 0.0014 \tag{14}$$

from which equation (13) may be rewritten as

$$Hp. = 4.78L^{0.086} + \frac{L}{1000}(0.025m + 1.4) \tag{15}$$

which gives values for the horsepower of the 4½-ft. ball mill that are accurate within a few per cent. The average curve for horsepower for mills of other diameters plotted with horsepower per 100 lb. (45.36 kg.) of load as ordinates and total load as abscissæ will be similar to the curve

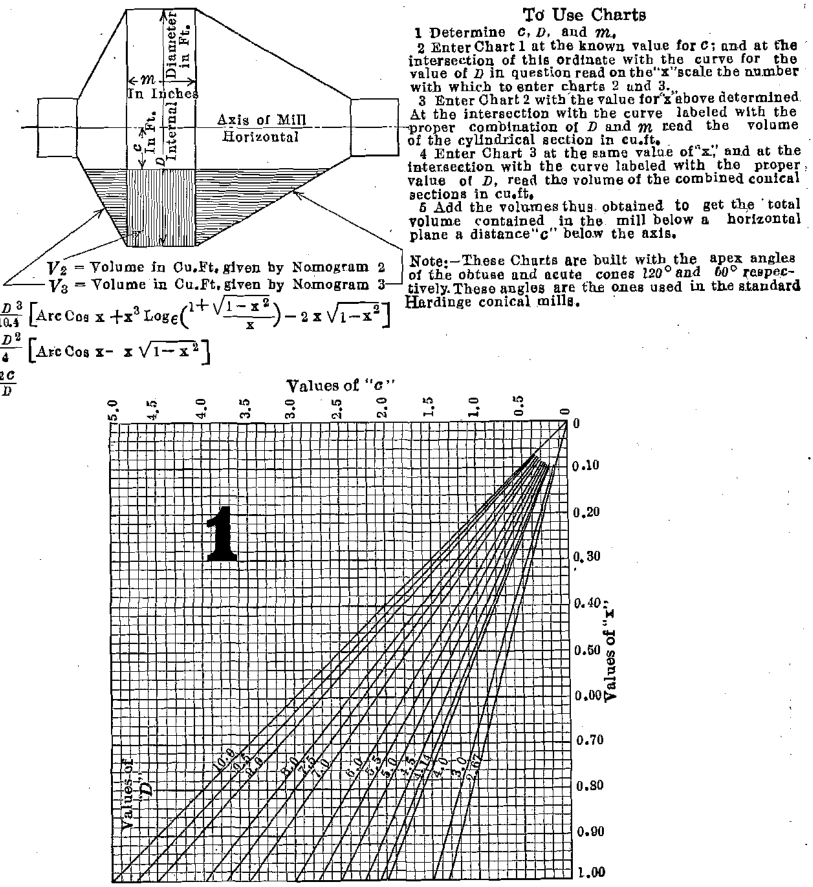
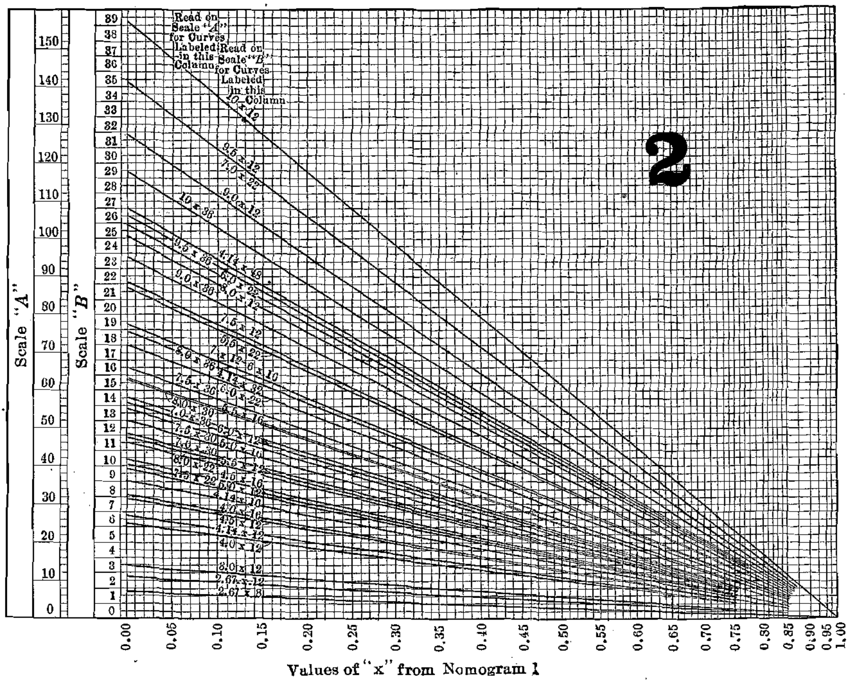


FIG. 6.

in Fig. 4 and, by changing the scale, can be made to coincide with this curve. If, then, corresponding values of x and L on this figure can be established for mills of several diameters and the proper substitutions made in equation (7) we will get a series of different numbers for the coefficient of the term $L^{0.086}$ in equation (15), corresponding to different internal diameters. The values of L corresponding to a given value of


2
Scale "A"
Scale "B"
Read on Scale "A" for Curves Labeled in this Column
Read on Scale "B" for Curves Labeled in this Column
10 x 12
9.5 x 12
7.0 x 22
9.0 x 12
10 x 36
9.5 x 36
4.14 x 48
6.0 x 22
8.0 x 12
9.0 x 30
7.5 x 12
5.5 x 22
4.14 x 22
6.0 x 22
7.5 x 36
8.0 x 30
4.0 x 36
7.5 x 30
7.0 x 30
8.0 x 22
7.5 x 22
8.0 x 12
2.67 x 12
2.67 x 8
Values of "x" from Nomogram 1

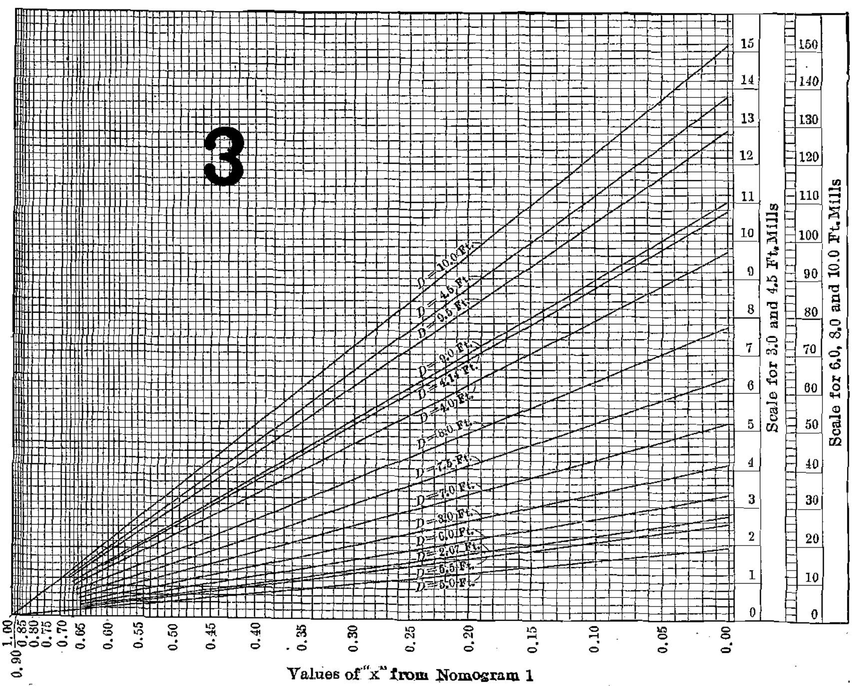

3
D = 10.0 Ft.
D = 4.5 Ft.
D = 9.5 Ft.
D = 9.0 Ft.
D = 4.14 Ft.
D = 4.0 Ft.
D = 8.0 Ft.
D = 7.5 Ft.
D = 7.0 Ft.
D = 3.0 Ft.
D = 6.0 Ft.
D = 2.67 Ft.
D = 6.5 Ft.
D = 5.0 Ft.
Scale for 3.0 and 4.5 Ft. Mills
Scale for 6.0, 8.0 and 10.0 Ft. Mills
Values of "x" from Nomogram 1

FIG. 6.

x, Fig. 4, will vary according to the volumes of the different mills. These values and the corresponding values of the coefficients of $L^{0.086}$ are as follows:

Internal Diameter of Mill in Feet	x	L	Coefficient of $L^{0.086}$
4.14	1	500	4.78
5.50	1	1,035	9.30
7.50	1	2,355	19.72
9.50	1	4,460	35.35

The logarithms of these coefficients of $L^{0.086}$ and the logarithms of the internal diameter of the mill, D, bear a linear relation to each other which is expressed in the equation

$$\log (D) = 0.413 \log (C) + 0.337 \tag{16}$$

From this equation

$$C = \frac{D^{2.42}}{6.53} = \text{coefficient of } L^{0.086} \tag{17}$$

Substituting this value for the coefficient of $L^{0.086}$ in equation (15) we have

$$Hp. = \frac{D^{2.42}L^{0.086}}{6.53} + \frac{L}{1000}(0.025m + 1.4) \tag{18}$$

This formula gives values accurate within a few per cent. for the horsepower of the conical ball mill throughout the range of operating conditions.

For pebble mills with smooth lining, results obtained by the above formula should be multiplied by the factor 0.65; with a semi-smooth lining, 0.8; with a rough lining, 0.95. It must be noted, however, in the use of the formula, that the load should be calculated on the assumption that the mill is horizontal, as the reduction in load due to tilting does not produce a corresponding decrease in power consumption.

The charts given in Fig. 6 and 7 will be found useful in determining the value of L in the horsepower formula. The use of these charts may be best explained by following through a calculation for the horsepower consumed by an 8-ft. by 30-in. ball mill crushing rock of a specific gravity of 2.6 with a moisture content of 50 per cent., using a 30,000-lb. ball load, composed of 5-in., 4-in., and 3-in. balls. For this condition

$D = 7.5$.

c (Fig. 6) $= 0$. (The volume contained in a mill in operation is more than that contained in the same mill at rest, and the assumption that $c = 0$ is legitimate.)

Then from nomogram 1,

$$x = 0.$$

Enter nomogram 3 with this value for x and read on line $D = 7.5$, $V = 63.7$.

Enter nomogram 2 with $x = 0$ and read on line 7.5×30, $V = 55.3$.

The working volume of the mill is, then, $63.7 + 55.3 = 119.0$ cu. ft.

The volume occupied by the balls is $30{,}000 \div 495 = 60.6$ cu. ft.

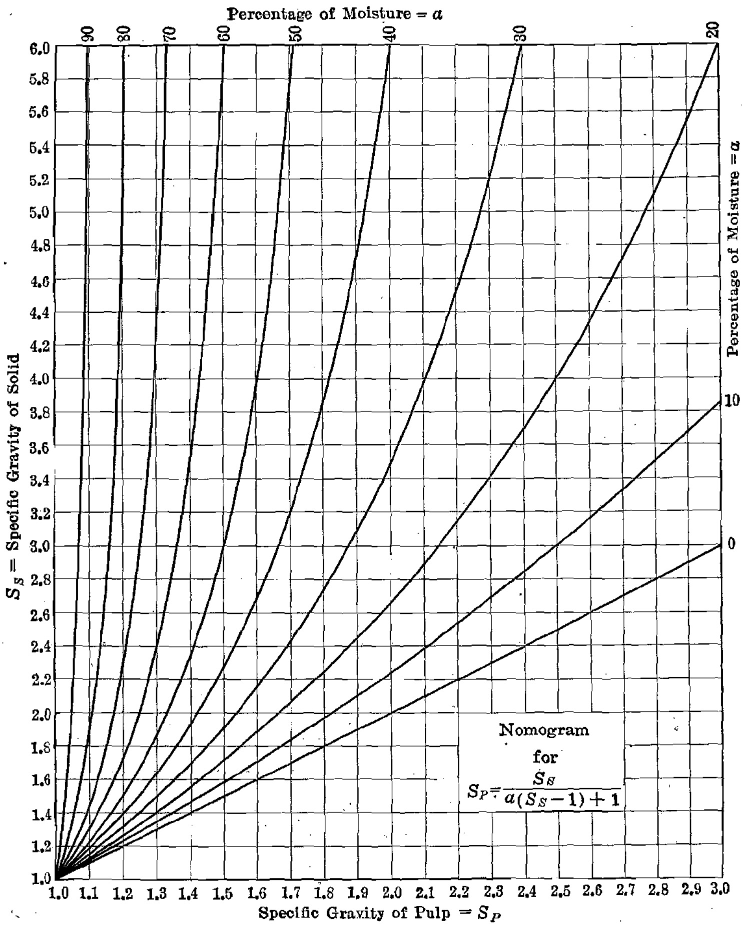


FIG. 7.

The volume occupied by the pulp is $119.0 - 60.6 = 58.4$ cu. ft.

The specific gravity of the pulp is determined from Fig. 7.

Enter at $S_S = 2.6$. At the intersection with the curve $a = 50$ per cent., read $S_P = 1.44$. The weight of the pulp in the mill is, then, $58.4(1.44)62.5 = 5250$ lb.

$$L = 30{,}000 + 5{,}250 = 35{,}250.$$

$$Hp. = \frac{(7.5^{2.42})(35,250)^{0.086}}{6.53} + \frac{35,250}{1,000}(0.025(30) + 1.4)$$
$$= 125.2.$$

The economy in calculation to be gained from the use of Fig. 6 is not so apparent in the foregoing instance, where c was taken equal to zero, as it will be if the information sought is the crushing load which fills a given mill to within a given distance of the center, or the depth to which a given load will fill a mill of a given size. In the course of 2 or 3 years' work with the mill, the writer has been confronted with a consider-

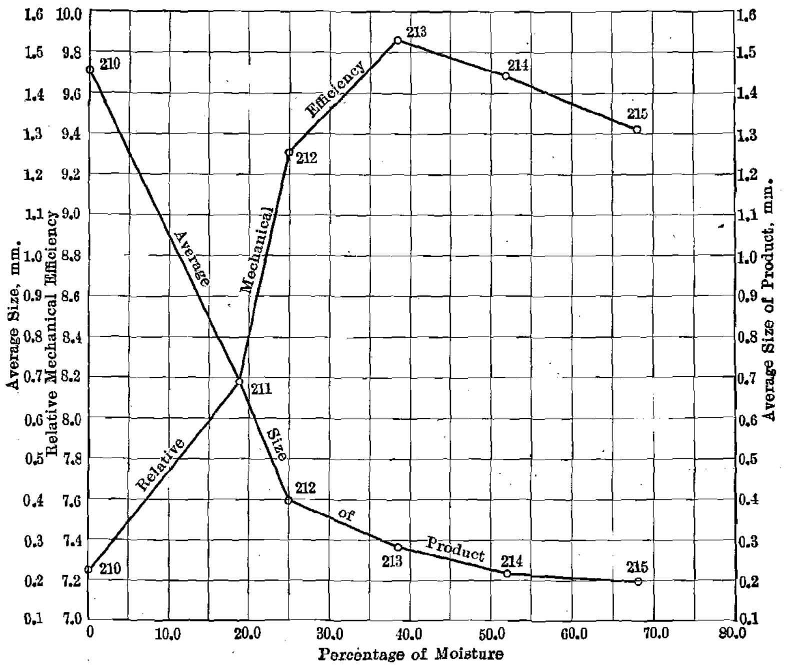


Fig. 8.—Effect of moisture on crushing efficiency and average size of product.

able number of such problems and it is because of the saving in time effected in their solution by the use of the chart, that it is inserted here. In such calculations the weight of a cubic foot of steel balls may be taken as 250 lb. and the weight of a cubic foot of pebbles as 100 lb.

Analysis of Operating Data

Thirty-five tests were run on the 4½-ft. mill to determine the effect of variations in operating conditions on the performance of the mill. As each test furnished some information that may be classified under several heads, it is not possible, without considerable repetition, to segregate them. They are, therefore, presented in Tables 1 and 2 in the order in which they were performed and will be referred to by number in the subsequent discussion.

The indicator commonly used in this paper for comparing the character of the work done by the mill under any given condition with that done under some other condition is the figure in the last column of Table 1 headed R.M.E. (Relative Mechanical Efficiency). A detailed explanation of the development of this conception is given in the article,

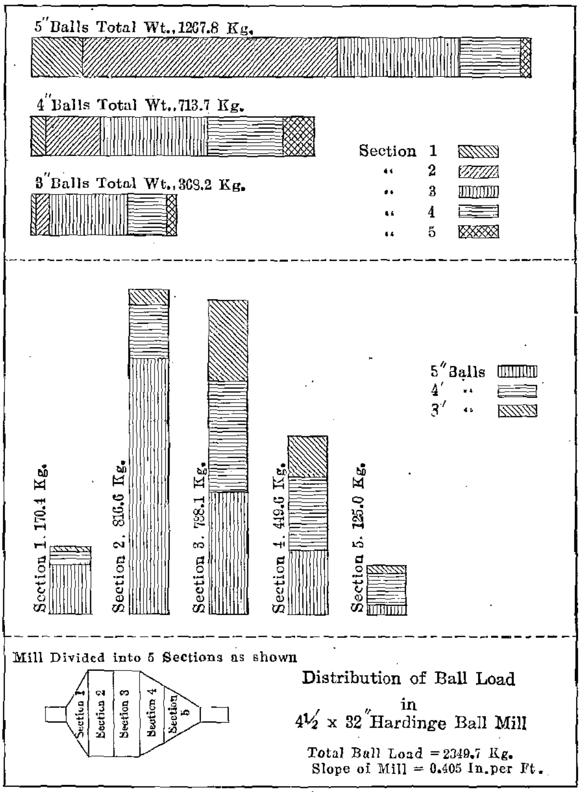


FIG. 9.

The Work of Crushing, *Trans.* (1914), **48,** 153. Briefly, it is expressed in the formula

$$\text{R.M.E} = \frac{\text{(Difference E.U. Feed and Product) (Tons per 24 hr.)}}{\text{Hp.}}$$

in which the term "Difference E.U. Feed and Product" is a measure of the useful work done per unit of weight in reducing the material in question from feed size to discharge size, and is determined by screen analysis.

Thus the total energy units, E.U., in the feed sample, screen test 22-A, Table 2, is 34.32 and is obtained by summing the products of the percentages on the different screens by their corresponding ordinal numbers. The same method applied to screen test 22–41 gives 1664.22 E.U. in the product of run No. 202. Then

$$\text{R.M.E.} = \frac{(1664.22 - 34.32)12}{21} = 930.$$

In order to make this figure accord with commonly accepted figures of efficiency, the R.M.E. thus obtained is divided by 100, giving for test 202 a value of 9.3.

Rate of Feed

The effect of rate of feed on the relative mechanical efficiency of the conical ball mill is given in the two groups of tests 202 to 204 and 213, 216 to 219. In the first group, trap rock of an average size of 24.58 mm. (0.96 in.) was fed dry at the rates of 1000 lb. (453.59 kg.), 1500 lb., and 2000 lb. per hour. The axis of the mill was horizontal and the ball load was a mixture of 5-in., 4-in., 3-in., and 1¾-in. balls in approximately the same proportions that would be found in commercial operation after the mill had settled down. The relative mechanical efficiencies, 9.3, 9.13 and 10.3 respectively indicate the result, confirmed in later tests, that the ratio of useful work done by the mill to power input increases with the feed rate. That there is, of course, a limit to this proportionate increase at the point of overload is shown in the second series of tests above mentioned. In this series the mill was tilted 2⅝ in., or 0.405 in. per foot, toward the discharge end. One result of this tilting was to decrease the ball capacity of the mill by about 1,200 lb. Quartzite of an average size of 9.90 mm. was fed with an average of about 38 per cent. moisture at rates of 1500 lb., 3000 lb., 6000 lb., 9000 lb. and 12,000 lb. per hour. The relative mechanical efficiencies corresponding to the above rates were 9.86, 17.30, 29.21, 43.50, and 41.10. In this series of tests the relative mechanical efficiency of the machine increases with the feed rate up to 4.5 tons per hour, beyond which we have an apparent condition of overloading. Table 3 gives the reduction in average size of particle in the different tests above discussed.

These figures present three different cases for consideration. Test 202 is in a class by itself, the machine is patently underfed for all purposes except that of producing a practically finished, fine, dry product at one passage through the machine. It is a surprising fact that in doing this kind of work the machine uses power so efficiently. Tests 203, 204, 218 and 219 compared with tests 213, 216 and 217 point the moral that for most efficient work it is not wise to attempt too great reduction at one passage through the mill. When the large amount of power consumed

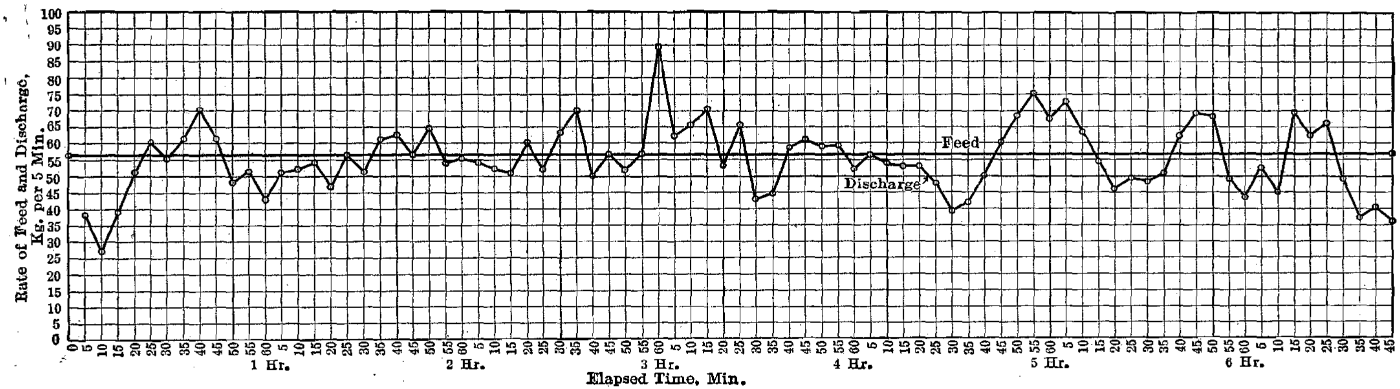


FIG. 10.—COMPARISON OF FEED AND DISCHARGE RATES OF CONICAL BALL MILL DRY CRUSHING.

TABLE 1

Test No.	Size of Mill	Crushing Charge: Character	Weight, Pounds: 5-in.	4-in.	3-in.	1¾-in.	Total	Kind of Rock Crushed	Screen Test No.: Feed	Product	Feed Rate, Pounds per Hour	Percentage of Moisture	Elevation of Feed End, Inches	R.p.m.	Hp.	R.m.e.
202	4½ by 16	Balls	1,960	953	712	381	4,006	Trap	22–A	22–41	1,000	0.0	0	28.0	21.0	9.30
203	4½ by 16	Balls	1,960	953	712	381	4,006	Trap	22–A	22–69	2,000	0.0	0	28.0	21.0	10.30
204	4½ by 16	Balls	1,960	953	712	381	4,006	Trap	22–A	22–77	1,500	0.0	0	28.0	21.3	9.13
205	4½ by 16	Balls	2,455	1,110	703	235	4,503	Trap	22–A	22–79	1,500	0.0	0	28.0	22.1	11.03
206	4½ by 16	Balls	2,400	522	.....	.....	2,922	Trap	22–A	22–88	1,500	0.0	4¼	26.5	23.0	8.66
207	4½ by 16	Balls	2,400	522	.....	.....	2,922	Trap	22–A	22–91	1,500	0.0	2⅝	28.0	22.2	9.33
208	4½ by 16	Balls	2,400	522	.....	.....	2,922	Trap	22–A	38–4	1,500	19.5	2⅝	26.5	19.9	11.46
209	4½ by 16	Balls	2,400	522	.....	.....	2,922	Quartzite	39–A	39–3	1,500	0.0	2⅝	25.5	18.4	8.09
210	4½ by 16	Balls	1,460	885	385	.....	2,930	Quartzite	39–A	40–1	1,500	0.0	2⅝	26.0	19.2	7.25
211	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	41A–1	1,500	18.8	2⅝	27.0	17.7	8.18
212	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	41B–1	1,500	25.0	2⅝	27.0	17.7	9.31
213	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	41C–1	1,500	38.5	2⅝	27.0	17.7	9.86
214	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	41D–1	1,500	51.8	2⅝	26.0	19.3	9.67
215	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	41E–1	1,500	68.2	2⅝	26.0	20.1	9.42
216	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	42–1	3,000	37.4	2⅝	26.0	18.5	17.30
217	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	42–2	6,000	37.3	2⅝	26.0	17.7	29.21
218	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	42–3	12,000	38.3	2⅝	26.0	18.5	41.10
219	4½ by 16	Balls	1,460	878	481	.....	2,819	Quartzite	39–A	42–4	9,000	38.9	2⅝	26.0	16.9	43.50
220	4½ by 16	Balls	1,772	1,067	711	.....	3,550	Quartzite	39–A	44–1	1,500	31.7	1¼	26.5	20.1	8.45
221	4½ by 16	Balls	2,125	1,284	855	.....	4,264	Quartzite	39–A	44–2	1,500	36.5	0	26.0	20.1	9.06
222	4½ by 16	Balls	2,125	1,284	855	.....	4,264	Trap	45–A	45–1	1,500	35.4	0	25.0	20.9	10.20
223	4½ by 16	Balls	1,457	874	480	.....	2,811	Trap	45–A	46–1	1,500	41.7	2⅝	26.5	20.1	10.71
224	4½ by 16	Balls	696	417	293	.....	1,406	Quartzite	39–A	47–1	1,500	47.0	2⅝	26.5	15.3	9.83
225	4½ by 16	Balls	1,412	....	.....	.....	1,412	Quartzite	39–A	47–2	1,500	40.0	2⅝	27.0	13.7	10.72
226	4½ by 32	Balls	2,785	1,572	810	.....	5,167	Quartzite	39–A	49–1	1,500	39.4	3 3/16	25.0	26.6	7.15
227	4½ by 48	Balls	3,765	1,822	1,067	.....	6,654	Quartzite	39–A	50–1	1.500	40.4	0.405 in./ft.	24.5	33.5	5.88
228	4½ by 48	Pebbles	.....	....	3,125	.....	3,125	Quartzite	51–A	51–1	1,500	42.2	0	27.5	16.9	5.37
229	4½ by 48	Pebbles	.....	....	3,125	.....	3,125	Trap	52–A	52–1	1,500	37.6	0	26.5	16.4	1.89
230	4½ by 48	Balls	1,604	1,479	2,160	70	5,313	Cocoanut shell	32–1	32–2	1,000	0.0	2⅝	25.0	22.1	1.67
231	4½ by 48	Balls	343	1,480	2,720	758	5,301	Cocoanut shell	32–2a	32–3	550	0.0	2⅝	28.0	23.6	0.50
232	4½ by 48	Balls	343	1,480	2,720	500	5,043	Cocoanut shell	32–4	32–7	550	84.5	0	28.0	31.4	1.56
233	4½ by 48	Balls	343	1,480	2,720	500	5,043	Cocoanut shell	32–6	32–8	2,200	54.8	0	27.0	32.8	0.94
234	4½ by 16	Balls	.....	1,465	2,535	.....	4,000	Sawdust	5,321	5,331	300	0.0	0	27.0	15.5	0.48
235	4½ by 16	Balls	1,927	951	815	720	4,413	Brass ashes	1	2	4,480	42.9	2½	28.0	20.0	15.60
236	4½ by 16	Balls	1,927	951	815	720	4,413	Brass ashes	3	4	1,820	39.7	2½	28.0	21.0	10.30

TABLE 2.—*Screen Tests*

Ordinal Number	Screen Aperture, Millimeters	22–A	22–41	22–69	22–77	22–79	22–88	22–91
−1.0	38.100	2.15						
0	26.670	70.90	....	11.72	7.41	6.67	5.05	1.13
1	18.850	24.42	....	16.84	14.78	5.78	6.10	3.87
2	13.330	1.63	....	11.62	9.27	2.28	5.95	5.16
3	9.423	0.23	....	4.60	3.84	0.83	4.98	5.44
4	6.680	0.10	....	2.46	1.42	0.46	3.60	4.99
5	4.699	0.05	....	1.21	0.49	0.32	3.28	5.44
6	3.327	0.04	....	0.50	0.38	0.50	2.60	4.48
7	2.362	0.02	0.17	0.36	0.29	0.32	2.62	3.00
8	1.651	0.02	0.85	0.28	0.37	0.50	2.54	3.09
9	1.168	0.02	0.48	0.37	0.40	0.86	2.72	2.71
10	0.833	0.02	0.92	0.46	0.52	1.47	2.58	2.35
11	0.589	0.02	2.49	0.79	1.10	2.92	3.02	2.50
12	0.417	0.02	3.57	1.21	1.39	3.71	2.82	2.34
13	0.295	0.03	6.10	2.20	2.74	5.15	3.24	2.95
14	0.208	0.04	5.98	2.97	3.63	5.88	3.50	3.17
15	0.147	0.04	9.05	4.78	5.50	6.94	5.04	5.52
16	0.104	0.05	11.75	6.56	7.55	9.39	6.41	6.49
17	0.074	0.04	8.34	4.92	6.12	7.47	5.60	5.75
19	Through 0.074	0.16	50.30	26.15	32.80	38.55	28.35	29.62
	Aver. size of particle, mm.......	24.58	0.142	8.590	6.573	3.449	4.472	3.219
	Total energy units	34.32	1,664.22	936.00	1,114.80	1,387.64	1,138.47	1,183.64

TABLE 2.—*Screen Tests.*—(*Continued*)

Ordinal Number	Screen Aperture, Millimeters	38–4	39–A	39–3	40–1	41A–1	41B–1	41C–1
−1.0	38.100	....	0.66					
0	26.670	4.34	5.51	0.22	0.45			
1	18.850	3.22	16.36	0.76	1.44	0.03		
2	13.330	2.98	22.00	1.18	2.02	0.25		
3	9.423	2.32	11.18	1.00	1.63	0.45	....	0.01
4	6.680	2.30	7.46	1.16	1.37	0.72	0.11	0.02
5	4.699	1.82	5.49	1.38	1.83	0.70	0.28	
6	3.327	1.72	3.62	1.54	2.44	1.42	0.58	0.14
7	2.362	1.41	2.97	2.63	2.20	2.56	0.88	0.26
8	1.651	1.90	3.30	4.45	3.78	5.01	2.38	1.07
9	1.168	1.94	2.94	5.39	4.96	6.94	3.99	2.14
10	0.833	2.89	3.18	6.04	5.92	8.64	6.73	4.33
11	0.589	3.93	3.55	8.47	9.38	11.94	11.88	9.13
12	0.417	4.66	2.37	7.46	7.01	9.14	9.14	10.05
13	0.295	5.49	2.48	8.31	9.58	10.22	11.27	13.10
14	0.208	6.29	1.85	7.46	8.36	7.61	8.91	10.87
15	0.147	8.91	1.72	10.13	9.62	8.99	9.93	11.59
16	0.104	9.15	1.25	8.35	7.67	6.62	8.23	9.76
17	0.074	6.43	0.70	5.74	4.89	4.21	5.30	5.89
19	Through 0.074	28.30	1.41	18.33	15.45	14.55	20.39	21.64
	Aver. size of particle, mm.	2.896	9.900	1.052	1.456	0.688	0.397	0.288
	Total energy units	1,301.83	487.82	1,316.38	1,261.70	1,292.85	1,403.48	1,458.70

TABLE 2.—*Screen Tests.*—(*Continued*)

Ordinal Number	Screen Aperture, Millimeters	41D-1	41E-1	42-1	42-2	42-3	42-4	44-1
-1.0	38.100							
0	26.670	......	......	......	......	0.08		
1	18.850	......	......	......	......	0.76	0.58	
2	13.330	......	......	......	......	2.99	1.44	
3	9.423	......	......	0.02	0.04	3.38	2.55	
4	6.680	......	......	0.02	0.29	4.08	3.59	0.02
5	4.699	......	......	......	1.08	4.03	2.64	
6	3.327	0.11	......	0.29	2.47	5.40	3.67	0.16
7	2.362	0.05	0.06	0.83	4.56	6.96	5.96	0.41
8	1.651	0.33	0.15	2.45	8.22	8.70	8.02	1.27
9	1.168	0.75	0.48	4.34	9.30	8.06	8.10	2.76
10	0.833	1.93	1.58	6.92	9.86	7.79	8.80	5.41
11	0.589	6.22	4.68	12.37	12.35	9.88	11.37	10.50
12	0.417	8.91	7.98	10.15	9.18	6.99	8.40	10.96
13	0.295	12.20	12.57	12.40	9.72	7.73	8.42	12.45
14	0.208	11.27	12.48	10.55	6.91	5.91	6.57	9.35
15	0.147	13.21	14.73	10.88	8.10	5.93	6.74	11.73
16	0.104	11.50	11.87	8.48	5.72	4.24	4.84	9.37
17	0.074	7.29	7.35	5.12	3.78	2.44	2.84	6.14
19	Through 0.074	26.23	26.07	15.18	8.42	4.65	5.47	19.47
	Aver. size of particle, mm.......	0.215	0.196	0.388	0.768	2.129	1.628	0.316
	Total energy units	1,525.87	1,538.26	1,376.66	1,207.65	1,016.92	1,169.53	1,432.96

TABLE 2.—*Screen Tests.*—(*Continued*)

Ordinal Number	Screen Aperture Millimeters,	44-2	45-A	45-1	46-1	47-1	47-2	49-1
-1.0	38.100							
0	26.670	....	5.62					
1	18.850	....	14.72					
2	13.330	....	24.42					
3	9.423	....	15.92					
4	6.680	0.02	10.18	0.04	0.01	0.02	0.04	0.01
5	4.699	0.07	7.65	0.38	....	....	0.53	
6	3.327	0.08	4.92	0.08	....	0.78	1.21	
7	2.362	0.27	3.03	0.36	0.52	1.79	2.24	0.04
8	1.651	0.56	2.50	1.27	1.12	5.44	5.30	0.41
9	1.168	1.29	1.71	2.05	2.24	7.98	8.20	0.89
10	0.833	3.12	1.25	4.49	3.15	9.07	9.04	1.93
11	0.589	7.10	1.19	7.58	6.26	11.77	11.53	5.07
12	0.417	8.71	0.79	7.82	6.84	8.30	8.65	7.20
13	0.295	12.29	0.83	8.88	7.66	9.87	9.79	11.80
14	0.208	11.33	0.57	7.29	6.62	7.91	8.10	11.41
15	0.147	13.17	0.84	9.97	8.58	9.16	9.04	13.75
16	0.104	10.59	0.83	9.84	8.81	7.99	7.98	11.29
17	0.074	6.98	0.68	7.03	6.32	5.44	5.23	8.72
19	Through 0.074	24.42	2.35	32.92	41.87	14.54	13.12	27.48
	Aver. size of particle, mm.......	0.246	10.394	0.275	0.228	0.509	0.537	0.202
	Total energy units	1,500.73	384.29	1,525.27	1,580.21	1,324.53	1,305.35	1,543.11

TABLE 2.—*Screen Tests.*—(*Continued*)

Ordinal Number	Screen Aperture, Millimeters	50-1	51-A	51-1	52-A	52-1	32-1	32-2
−1.0	38.100	......	......	......	......	......	11.82	
0	26.670	......	......	......	......	......	45.47	2.88
1	18.850	......	0.14	......	......	......	20.97	12.58
2	13.330	......	0.39	......	0.02	......	12.67	25.14
3	9.423	......	0.76	......	0.06	......	3.96	21.01
4	6.680	0.02	1.19	0.02	0.18	......	1.88	15.26
5	4.699	......	3.60	......	1.78	......	0.65	6.46
6	3.327	......	5.15	0.36	1.33	......	0.41	3.34
7	2.362	0.04	5.44	......	1.50	0.06	0.48	2.02
8	1.651	0.43	8.47	0.25	2.31	0.12	0.22	1.33
9	1.168	0.60	8.47	0.32	2.20	0.23	0.22	0.94
10	0.833	1.34	8.79	0.38	2.43	0.52	0.22	0.75
11	0.589	3.89	11.01	1.08	3.60	0.67	0.25	1.37
12	0.417	5.89	8.66	2.12	3.79	0.58	0.19	1.15
13	0.295	10.76	9.06	5.29	5.79	1.42	0.15	1.14
14	0.208	11.65	6.82	9.78	5.71	3.48	0.09	0.88
15	0.147	14.90	7.49	17.18	9.69	11.44	0.09	0.76
16	0.104	11.58	5.10	17.65	9.63	15.41	0.11	0.92
17	0.074	9.69	2.93	12.26	8.37	13.60	0.05	0.62
19	Through 0.074	29.21	6.53	33.31	41.61	52.47	0.10	1.45
	Aver. size of particle, mm......	0.184	1.173	0.140	0.380	0.091	22.820	10.010
	Total energy units	1,583.89	1,130.23	1,635.04	1,563.10	1,735.92	83.00	390.15

TABLE 2.—*Screen Tests.*—(*Continued*)

Ordinal Number	Screen Aperture, Millimeters	32-2a	32-3	32-4	32-7	32-6	32-8	5321
−1.0	38.100							
0	26.670	4.67	0.09	....				
1	18.850	20.41	6.22					
2	13.330	40.80	27.25	....	......	6.00	0.80	
3	9.423	34.12	34.03	15.41	......	15.89	3.85	
4	6.680	....	18.23	27.77	0.20	23.27	13.32	0.13
5	4.699	....	5.35	20.86	0.81	12.88	9.79	0.60
6	3.327	....	2.15	16.75	1.18	14.72	13.83	2.10
7	2.362	....	0.69	8.23	2.31	8.91	7.99	5.81
8	1.651	....	0.17	4.40	3.85	4.19	8.01	19.42
9	1.168	....	0.07	2.39	6.11	2.13	6.03	23.27
10	0.833	....	0.07	1.34	9.38	1.53	5.70	25.11
11	0.589	....	0.09	1.15	14.94	1.39	7.46	17.89
12	0.417	....	0.11	0.33	14.78	0.86	4.71	4.33
13	0.295	....	0.18	0.43	12.63	0.84	4.63	0.80
14	0.208	....	0.18	0.24	8.01	0.57	2.79	0.18
15	0.147	....	0.36	0.14	6.36	0.73	2.49	0.12
16	0.104	....	0.53	0.19	5.10	1.02	2.47	0.09
17	0.074	....	1.34	0.13	3.27	1.73	1.42	0.09
19	Through 0.074	....	2.89	0.24	11.07	3.36	4.71	0.06
	Aver. size of particle, mm.......	13.750	9.600	5.170	0.583	5.280	2.810	11.720
	Total energy units	204.37	382.60	527.30	1,269.69	600.74	853.40	940.75

TABLE 2.—*Screen Tests.*—(*Continued*)

Ordinal Number	Screen Aperture, Millimeters	5331	1	2	3	4
−1.0	38.100					
0	26.670	......	....	......	7.81	
1	18.850	......	1.14	......	8.78	
2	13.330	......	7.87	0.24	25.76	
3	9.423	......	18.60	1.00	26.31	1.32
4	6.680	......	19.05	3.36	17.42	3.88
5	4.699	0.07	13.22	5.18	9.19	6.23
6	3.327	0.38	13.92	6.37	2.21	5.14
7	2.362	1.44	9.21	6.91	0.68	5.48
8	1.651	5.28	4.81	6.75	0.48	4.13
9	1.168	8.12	2.75	7.00	0.24	3.06
10	0.833	19.48	2.21	7.90	0.15	2.93
11	0.589	20.70	2.00	8.73	0.14	4.07
12	0.417	15.98	1.41	8.94	0.11	5.13
13	0.295	12.25	1.16	7.64	0.12	7.14
14	0.208	6.89	0.78	6.10	0.11	7.21
15	0.147	5.02	0.59	5.75	0.11	7.68
16	0.104	2.12	0.52	5.21	0.12	8.45
17	0.074	0.96	0.23	1.56	0.07	4.99
19	Through 0.074	1.31	0.53	11.36	0.19	23.18
	Aver. size of particle, mm.	0.644	5.746	1.350	11.350	1.219
	Total energy units........	1,148.81	534.39	1,125.61	294.75	1,284.98

TABLE 3

Test No.	Feed Rate, Pounds per Hour	Aver. Size of Feed, Millimeters	Aver. Size of Product, Millimeters	Ratio of Reduction	R.M.E.
202	1,000	24.58	0.142	173:1	9.30
203	2,000	24.58	8.590	2.88:1	10.30
204	1,500	24.58	6.573	3.74:1	9.13
213	1,500	9.90	0.288	34.2:1	9.86
216	3,000	9.90	0.388	25.5 :1	17.30
217	6,000	9.90	0.768	12.9 :1	29.21
218	12,000	9.90	2.129	4.65:1	41.10
219	9,000	9.90	1.628	6.08:1	43.50

by one of these mills is considered, together with the fact that the power consumption is practically the same whether the mill is loaded lightly or heavily, it should be apparent that it will pay well to expend the small amount of power necessary for handling the pulp in a closed circuit and thereby gain increased efficiency of the mill.

This conclusion is practically in accord with that reached by the usual method of analysis. Taking 0.295 mm. (48-mesh) as a limiting size sought, Table 4 shows the relative number of mills and, therefore, the relative amounts of power necessary to crush quartzite, the screen test of which is shown in S. T. 39-A, to pass a 0.295-mm. screen at the rate of 12,000 lb. per hour. The same relative figures will, of course, hold for any multiple of this desired capacity.

TABLE 4

Test No.	Feed Rate, Pounds per Hour	Total Feed Including Returns from 12,000 Lb. per Hour Original Feed, Pounds per Hour	Minus 0.295-Mm. Material in Feed, Pounds per Hour	Minus 0.295-Mm. Material in Product, Pounds per Hour	Minus 0.295-Mm. Material Produced, Pounds per Hour	Number of Mills Needed
213	1,500	20,000	104	896	792	13.0
216	3,000	23,920	208	1,507	1,299	8.0
217	6,000	36,400	416	1,978	1,562	6.0
219	9,000	45,000	624	2,382	1,758	5.0
218	12,000	51,600	832	2,778	1,946	4.3

This table is based on the assumption that the efficiency of reduction is the same on the smaller material returned to the mill as it is on the larger original feed. This is not quite true, but it is nearly enough true for the purposes of this argument.

Fig. 10 presents a fact which goes far toward explaining the irregular performance often met with in machines following a crusher of the ball- or tube-mill type. It will be noted that at the end of the 6¾ hr. operation the divergence between feed and discharge rate at any given minute is as great as at the beginning of the run, despite a careful, regular feed. The rising portions of the curve are accompanied by a progressively coarser product. At the peaks the screen tests show but little crushing. This irregularity in discharge rate and character of product is greater in dry crushing than in wet crushing, but it is also distinctly apparent in wet crushing. In most mill practice the irregularity is smoothed out by crushing in closed circuit, the circuit acting as a balance. Where no such balance occurs through other features of mill design, it will be wise to make special provision if the machines treating the discharge require a close adjustment.

Effect of Moisture Content

Fig. 8, summarizing tests 210 to 215 inclusive, Table 1, shows distinctly the effect of moisture on the crushing efficiency and average size of product of the conical ball mill. The true maximum of the efficiency curve probably lies somewhere between 40 and 50 per cent. moisture. The decidedly higher efficiency of wet crushing over dry crushing is confirmed in tests 207 and 208 where the relative mechanical efficiency rises from 9.33 to 11.46 due to the addition of 19 per cent. water, which is decidedly less than the most efficient water quantity. It will be noted, however, on referring to screen tests 40-1, 41A-1, 41B-1, 41C-1, 41D-1, and 41E-1, Table 2, that a progressively finer product is obtained by increasing the amount of water in the feed and that the decreased relative mechanical efficiency in tests 214 and 215 is due to increased power consumption.

Moisture content has an effect on the weight of crushing charge that can be held in a mill. If the mill is charged to the limit when pulp of a given moisture content is being fed, a slight decrease in the moisture content will cause the discharge of a considerable quantity of balls or pebbles, as the case may be. The converse of this statement is, of course, also true.

Effect of Slope

The principal factors in mill operation affected by changes in slope are the ball load and the character of the product.

The effect on the ball load is best shown in tests 205 and 206. At the end of test 205 the mill, then level, contained a charge of 4503 lb. consisting of 2455 lb. of 5-in., 1110 lb. of 4-in., 703 lb. of 3-in. and 235 lb. of 1¾-in. balls. At the end of test 206, which started with this load and was continued for several hours, the mill being set at a slope of 0.64-in. per foot, there had been forced out of the mill 55 lb. of 5-in., 588 lb. of 4-in., and all the 3-in. and 1¾-in. balls, leaving a total charge of 5-in. and 4-in. balls weighing but 2922 lb. The ratio of weight of rock in the mill to weight of balls was also reduced. This latter fact considerably lessens cushioning and increases the amount of crushing done by impact as compared to that done by abrasion. The result is reflected in the increased efficiency and more granular product obtained, as noted later.

The change in power required to operate at higher slopes is in no way commensurate with what would be expected from the decrease in ball load (see tests 205 and 206). This fact should be borne in mind in using the formula given for horsepower.

The effect of changes in slope on the relative mechanical efficiency is so small that contradictory results due, no doubt, to unavoidable experimental inaccuracies, are shown. Thus tests 212, 213, 220 and 221 show a point of least efficiency at 1¼-in. slope with higher efficiencies at 2⅝-in. slope and no slope. It is the writer's opinion that the relative mechanical efficiency increases with increase in slope within operating limits, but that the change will in all cases be small. Tests 222 and 223 compared show higher efficiency and finer grinding at the greater slope. The finer grinding in test 223 is due to the higher moisture content of the pulp, rather than to the increase in slope. The progressively coarser grinding with increasing slope in the quartzite series, tests 212, 213, 220 and 221, is typical of the results to be expected in this direction. In dry grinding (see tests 205, 206 and 207), a decided change in the character of the product takes place with change of slope. The material discharged from the mill when grinding with the axis horizontal, contains a large percentage of – 200-mesh material and a considerable percentage of the coarsest sizes with a decided minimum in the amount of the intermediate sizes. The product of the tilted mill, on the other hand, is more uniform. There is decidedly less coarse material and decidedly less dust, the bulk of the product lying in the intermediate sizes. This difference is undoubtedly due to the difference in the character of the crushing done in the two cases. With the mill horizontal a considerable proportion of the load is rock. This rock acts as a cushion to the falling balls in the mill so that crushing by impact is greatly lessened and crushing by abrasion forms an important part of the work done. In such crushing many of the large particles in the feed are reduced in size but slightly and pass out practically untouched, while such work as is effective produces very fine material. Thus we have the large percentages of very coarse and very fine ingredients in the product. On the other hand, when the mill is tilted the amount of rock that it contains at any time is small in proportion to the crushing load, there is little or no cushioning and the amount of crushing done by impact is large in comparison with that done by abrasion. Under such circumstances a granular product is to be expected.

Ball Load.—Varying the weight of the ball load affects the power consumption, fineness of grinding and relative mechanical efficiency. Power consumption increases with increase in the ball load, but the rate of increase in power consumption is not so rapid as the rate of increase of the ball load. Thus in tests 204 and 205 the ball load is increased 12.5 per cent. while the corresponding increase in power is but 3.8 per cent. In tests 222 and 223 an increase in ball load of 51.8 per cent. produces an increase in power consumption of but 4 per cent. It must be noted, however, that in the latter instance the increase in ball load is accompanied by a change in slope and that the mechanical efficiency of the power chain is unquestionably less when the mill is tilted than when it is horizontal. The increase in ball load in test 205 as compared with 204 causes reduction in average size of product, from the same feed, from 6.573 mm. (0.26 in.) to 3.449 mm. (0.14 in.) or 47.5 per cent. This material increase in the fineness of the product, with its corresponding increase in the mechanical value of the pulp, is sufficient to cause an increase of 20.8 per cent. in the relative mechanical efficiency of the machine, notwithstanding the increased power. As noted previously, the writer believes that the apparently contradictory result presented in tests 222 and 223, where the product of the lightly loaded mill is the finer, is due to the increase in percentage of moisture in the latter product and that with the same moisture percentage in both cases a result in agreement with the first case cited would have been obtained. It is, however, the writer's opinion further that the more lightly loaded mill, tilted and with a carefully aligned power chain, should show a higher relative mechanical efficiency, due to a reduction in power consumed, which would more than compensate for any increase in average size of the product.

Effect of Difference in Size of Balls.—A comparison of test 209 with 210 and of test 224 with 225 shows that the larger the average size of ball in the crushing load (up to 5-in. diameter) the smaller the power consumption and the higher the relative mechanical efficiency. It is to be further noted, that a mixture of 5-in. and 4-in. balls crushes finer than a mixture of 5-in., 4-in., and 3-in. balls of equal weight, when the crushing is done dry and the average size of the feed particles is 9.900 mm. (0.39 in.). When the work is done in the presence of water, as in tests 224 and 225, the product when the ball charge is a mixture of 5-in., 4-in., and 3-in. balls is slightly finer (0.509 mm. as against 0.539 mm.) than when the charge consists wholly of 5-in. balls, but in test 224 the moisture percentage was 47.0 per cent. as compared with 40.0 per cent. in test 225. By reference to the section on Effect of Moisture, it will be seen that this result is probably due to the difference in moisture content and that at the same moisture content the 5-in. balls would crush finer than the mixed charge. In any case the difference in fineness in favor of the mixed load is so slight as to fail to justify charging a ball mill working on coarse feed with anything smaller than 5-in. balls. The writer inclines to the belief that the presence of small balls is a hindrance, and that periodical sorting of the charge accompanied by removal of the small balls (less than 3-in. diameter) will increase capacity, decrease power consumption, decrease the average size of the product and materially increase the relative mechanical efficiency.

Size of Feed.—Comparison between tests 208 and 223 apparently indicates that the ball mill works more efficiently on a coarse feed (24.58 mm. (0.96 in.) average size) than on a finer feed (10.394 mm. (0.41 in.) average size). In test 223 the percentage of moisture present, 41.7 per cent., is practically that determined most favorable, while in 208, but 19.5 per cent. of water was present in the feed. Notwithstanding this fact the relative mechanical efficiency in crushing the larger feed is 11.46 as against 10.71 in the case of the finer feed. There is very little difference in the power consumption. This conclusion must, however, be limited by a statement as to the rate of feed, viz., 1500 lb. per hour. The reduction ratio is but 7.02 in the case of the larger feed as against 45.6 for the finer feed. In neither case was the mill fed up to its most efficient capacity. Comparing the results obtained here with those obtained in the rate of feed tests (213 and 216 to 219 inclusive) we may expect that by pushing the capacity in the case of the smaller feed until the reduction ratio is in the neighborhood of 7.0 that the relative mechanical efficiency will rise to about 40, while from the same series of tests it is obvious that lessening the ratio beyond this point in the case of the coarser feed, by increasing the feed rate, would result in lowering the relative mechanical efficiency. When these facts are taken into consideration, the smaller feed gives most efficient operation.

This conclusion cannot, however, be extended to finer and finer feeds, as is apparent when the pebble mill runs on trap and quartzite, 228 and 229, are compared with the ball mill runs on the same rocks, tests 213 and 223. In the latter tests, with feeds of approximately the same average size, the efficiencies varied by but 7.9 per cent., the trap showing the higher result. In the pebble mill tests the average size of the quartzite feed was 1.173 mm. and of the trap feed 0.380 mm. The corresponding relative mechanical efficiencies were 5.37 and 1.89, a difference of 65 per cent., all of which must be ascribed to the fineness of the feed.

Length of Cylindrical Section.—The effect of increasing the length of the cylindrical section in a ball mill is to reduce the relative mechanical efficiency. This is due to the fact that the ball load and power consumption increase with increased length much more rapidly than the fineness of the product increases. Thus, by reference to Table 1 we find that, all conditions being constant other than those noted above, an increase in power consumption amounting to 89 per cent. occurs with an increase in length of cylindrical section from 16 in. to 48 in., the corresponding decrease in average size of product is but 36 per cent., and there is a resulting decrease in relative mechanical efficiency of 40 per cent., the 16-in. mill being taken as the standard of comparison. Therefore, if the desired capacity of a plant is sufficient to justify the installation of more than one mill, additional mills placed in series, each making a relatively small reduction, will be more efficient than an installation which attempts a large reduction ratio in one mill by increasing the length of the cylindrical section.

Pebbles vs. Balls

Test 228 presents the pebble mill working at a reduction ratio of 8.3, which is close to the most economical ratio. Under these conditions the relative mechanical efficiency is 5.37. Test 227 presents a ball mill of the same cylinder length working on a coarser feed but making a reduction of 53.6 to 1. Even under this unfavorable condition the relative mechanical efficiency is 5.88. If the rate of feed is raised and the reduction ratio correspondingly lowered to a point comparable with the pebble mill, we may expect a relative mechanical efficiency much higher. It is obvious, then, that the ball mill is a more efficient crushing machine than the pebble mill. It is also obvious that it will grind as fine as the pebble mill, when the products of the two tests above cited are compared. The ball-mill product is 0.184 mm. average size, produced from a 9.9-mm. feed, while the pebble-mill product is 0.140 mm. average size produced from a feed only 1.173 mm. average size. Given a feed of the same size, the ball-mill product would have been finer.

Records were kept throughout of ball and pebble consumption, but the results were so contradictory, due to the relatively short duration of the runs, that they are not worth presenting.

Character of Feed

When the feed to a ball mill is a rock similar to an average ore, no great difference in efficiency is noticeable as between different kinds. Tests 221 and 222 give a comparison of grinding efficiencies on quartzite and trap of approximately the same average size. The reduction ratios in the two cases are 40.2 and 37.8 respectively, giving products 0.246 and 0.275 mm. average size. The relative mechanical efficiencies are 9.06 in the case of the quartzite feed and 10.20 in the case of the trap-rock feed. When, however, tough, soft materials such as cocoanut shells or sawdust are tested (tests 230 to 234 inclusive) the efficiencies fall off rapidly to figures ranging from 0.48 to 1.67. This means, of course, that a crushing device employing impact chiefly is not suitable for reducing such material.

An interesting and unexpected result is to be noted in tests 235 and 236. The brass ashes treated in these tests consisted of a mixture of unburned coal, coal ash, and a brittle slag containing metal shot. The coal ash and slag ground up with surprising ease, the coal was easily broken to an intermediate size and then seemed to float through on the surface of the load in the mill, while the metal particles were discharged with very little flattening or abrasion. Thus, due to the heterogeneous character of the feed, a higher efficiency was obtained than would be expected where one of the ingredients was so tough. When, however, it was attempted to grind slowly and pulverize metal, the relative mechanical efficiency of the machine fell to figures ranging from 0.04 to 0.71, confirming the comparison made in the first part of this section between rock and such tough materials as sawdust and cocoanut shells.

Capacity

As stated in the introductory part of this paper, it has not been possible to extend the series of tests to gain capacity figures on mills of different diameters. The writer has, however, some figures on the capacity of 6-ft. and 8-ft. ball mills which indicate that with a feed of average ore of 10 mm. average diameter, grinding wet to pass a 20-mesh (0.833-mm.) screen, the capacity will vary as a function of the cube of the nominal diameter of the mill. Approximate capacities for this duty for mills 4.5 ft. diameter and larger may be derived from the formula

$$C = 0.95D^3 - 65$$

where C = the capacity in tons per 24 hr. and D = the nominal diameter of the mill in feet. This formula should, however, be used with caution.

Distribution of Crushing Charge

At the end of one of the runs on the 4½ by 32 mill, the balls were sorted as they were taken from the mill into heaps corresponding to the portion of the mill from which they were removed. For the purpose of this classification the charge was divided by theoretical vertical planes into five sections, as shown in the diagrammatic sketch, Fig. 9. The heaps taken from each of these sections were then sorted into sizes, with the result shown graphically in Fig. 9. It will be seen from this figure that there is a marked segregation of large balls in sections 1 and 2 at the head end of the mill. The segregation is, however, by no means complete, as is shown by the fact that the average size of ball in the mixture in sections 3, 4, and 5 is greater than 4 in.

Conclusions

1. In crushing average ores the character of the gangue has but little effect on the relative mechanical efficiency of the conical mill.

2. The mill is not suitable for grinding soft, tough materials.

3. The ball mill works more efficiently on material of intermediate (0.5 in. to 0.75 in. average) size than on either a coarser or a finer feed.

4. A greater ratio of reduction in average size of material can be expected with feed of an intermediate size than with a coarse feed.

5. Steel balls are much more efficient crushing media than pebbles.

6. Steel balls will grind as fine or finer than pebbles when working on the same feed.

7. Increase in the weight of the ball load, other conditions remaining constant, increases the ratio of reduction and the relative mechanical efficiency of the mill.

8. The power consumption increases with increase in the weight of the ball load, but this increase in power consumption is not in direct proportion to the increase in load.

9. Power consumption decreases with increase in the average size of the balls composing the crushing load up to an average size of 5 in.

10. A ball charge composed of 5-in. balls makes a greater reduction in size of particle at one passage through the mill than a mixed charge composed of 5-in., 4-in., and 3-in. balls.

11. The relative mechanical efficiency of the ball mill increases with the average size of ball in the crushing charge up to 5 in. average diameter.

12. The relative mechanical efficiency of the mill increases with the rate of feed to the point of overload.

13. Increase in the length of cylindrical section in the conical ball mill increases the reduction ratio at the expense of a marked decrease in the relative mechanical efficiency.

14. Increase in the slope of the mill axis decreases the ball load materially, but the corresponding decrease in power consumption is in no way commensurate.

15. In general, increase in slope tends to produce a more granular product with less very fine and less coarse ingredients than are present in the product of the mill set with the axis horizontal.

16. Increase in slope has but little effect on the relative mechanical efficiency.

17. Other conditions being constant, the relative mechanical efficiency of the mill is a maximum at between 40 and 50 per cent. moisture content in the feed.

18. The relative mechanical efficiency in wet crushing is decidedly greater than in dry crushing.

19. The increase in the percentage of moisture in the feed causes an increase in the reduction ratio.

20. Power consumption increases slightly with increase in the moisture content of the feed.

21. The rate of discharge and the character of the product of the mill fluctuate continually through rather wide limits. This fluctuation is greatest in dry crushing.

22. The conical mill should be operated in closed circuit with a sizing device which will return to it the oversize from its product. In this installation the rate of feed should be raised until the relative mechanical efficiency shows a maximum. When operating as a ball mill, the ratio of length of cylindrical section to diameter should not exceed 0.3. This will be a much more economical installation than one which seeks, by slow feeding or long cylindrical section, to obtain a finished product at one passage through the mill. In wet grinding the moisture content of the feed should be kept about 40 per cent. The slope should be adjusted to mill requirements, but for ordinary concentrating-mill practice should be about 0.4 in. per foot. The ball charge should be the maximum that the mill will hold and should be kept as large in average size as is possible without too great sacrifice of small balls.

Acknowledgments

The writer is indebted to Percey F. Smith, Professor of Mathematics in the Sheffield Scientific School, for valuable assistance in the development of the formulæ, and to Professor Herbert L. Seward of the department of Mechanical Engineering for help with the nomograms.

Discussion

John W. Bell,* Montreal, Quebec, Canada (written discussion†).—The test results in Mr. Taggart's paper will, I am sure, be recognized as a notable contribution, and of great assistance in the study of the performance of the Hardinge mill.

I regret, however, being obliged to note that Mr. Taggart still retains such confidence in the Kick-Stadler method of computing the "relative mechanical efficiency" of crushing machines. The Rittinger-Kick graph submitted by Mr. Gates[1] and the tests made at McGill University disclosed precisely the same fundamental defect in the Kick-Stadler theory.

Consequently, I have been obliged to recalculate in terms of Rittinger surface units the results obtained in the 28 rock-crushing tests cited by Mr. Taggart, in order to find out what the relative efficiencies really were. In order to show the large discrepancies between the Stadler and Rittinger R. M. E.'s, the most efficient result disclosed by each method is represented by the number 100, and the R. M. E.'s for the other tests have been recalculated on this basis. The results will be found in Table 1. Personally, I look forward to the time when we shall cease to talk about "relative mechanical efficiency" and merely refer to the "efficiency" of a crusher. All that is required to accomplish this is to agree on a standard method for determining the "crushing constant" of a given rock and a standard method for calculating the efficiency. The figures in the fourth column of Table 1 have been derived by assuming a constant of 2000 for the quartzite crushed in the Yale tests.

The Stadler method sometimes indicates changes in efficiency produced by changes in operating conditions, as I pointed out in a paper describing rock-crushing tests made at McGill University. It is, however, not enough to determine that certain changes increase or decrease efficiency; surely it is equally important to determine the magnitude of these variations. By examination of Mr. Taggart's results, I have been obliged to conclude that the size of feed and amount of reduction greatly influence the Stadler R. M. E. figures, that they are positively misleading.

It should be mentioned that since the majority of the tests have been made at the least efficient feed rates, and since, moreover, it is one of the evidently very important factors affecting efficiency, it is possible that different results might be obtained by a high-tonnage feed series, and that some of the conclusions reached by Mr. Taggart or by me may require revision when this data has been obtained.

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† Received April 28, 1917.

[1] A. O. Gates, *Trans.* (1915), **52**, 898, Fig. 20.

TABLE 1

Test No.	Apparent R. M. E. Stadler	R. M. E. Rittinger	Efficiency Assuming Quartzite Crushing Constant = 2000 S. Units
202	21.4	56.6	
203	23.7	58.5	
204	21.0	54.0	
205	25.4	63.0	
206	19.9	44.1	
207	21.4	47.6	
208	26.3	55.1	
209	18.6	39.4	10.5
210	16.7	32.8	8.7
211	18.8	33.8	9.0
212	21.4	45.3	12.0
213	22.6	49.4	13.1
214	22.2	54.0	14.3
215	21.6	52.2	13.9
216	39.8	72.2	19.2
217	67.1	90.6	24.0
218	94.5	100.0	26.5
219	100.0	98.6	26.2
220	19.4	40.2	10.7
221	20.8	48.5	12.9
222	23.5	56.0	
223	24.6	69.2	
224	22.6	40.6	10.8
225	24.6	42.1	11.2
226	16.4	41.0	10.9
227	13.5	34.4	9.1
228	12.3	59.8	15.7
229	4.3	24.5	

But even more striking than the errors in magnitude of Stadler R. M. E.'s, are the errors they lead to in some of Mr. Taggart's principal conclusions. He says that (p. 141) the R. M .E. of the machine increases with the feed rate up to 108 tons per 24 hr. "beyond which we have an apparent condition of overloading." My conclusion is that the 144-ton feed rate (test No. 218) is the more efficient, and that there is consequently no indication of overloading. As an additional argument in favor of surface rather than energy units, I have plotted the results (see Fig. 1) of tests 213, 216, 217, 218, 219 given in Table 3 of Mr. Taggart's article, with the apparent Stadler R. M. E.'s and Rittinger R. M. E.'s figured on the same basis as in Table 1 of this discussion. That the Stadler R. M. E. should shoot up to a maximum value at 108 tons in a nearly straight line, and then down at 144, does not seem to me to be what one would reasonably expect. The Rittinger curve seems far more rational.

In regard to the result listed in Mr. Taggart's Table 4 (p. 148), I repeat a protest I have already made against the estimation of efficiency by considering the number of tons of −48-mesh material produced. The impression that is created by this table is, that a small feed will require 13 tube mills to produce 12,000 lb. per hour of −48-mesh material; whereas a high feed will require only four. On the assumptions made, numerically, this may be true, but a very practical consideration in this connection is that the 13-tube-mill plant will produce a product containing 21.6 per cent. of −200 grade and the 4.3-tube-mill plant product will only contain 4.7 per cent. of −200. Table 2 shows that by the addition of 1.7 tube mills, the amount of −200 could be nearly doubled with a

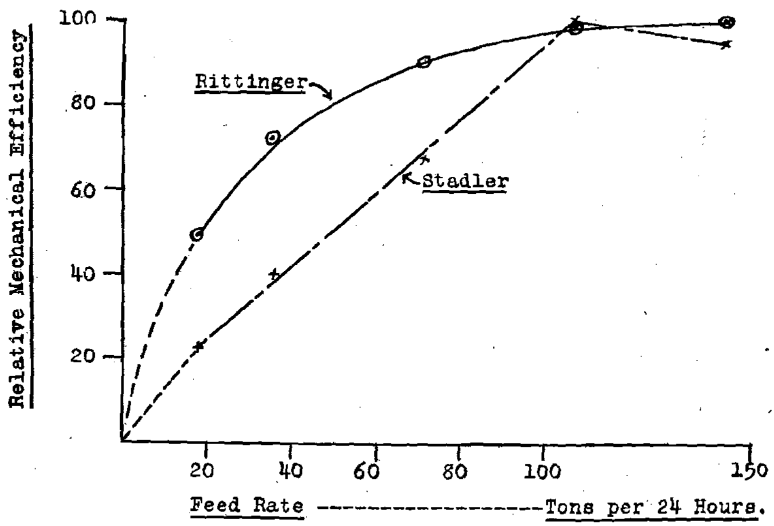

Fig. 1.

drop of only 2½ per cent. in the mechanical efficiency of the mills. The efficiencies are real (assuming the crushing constant to be 2000) in order to eliminate the exaggeration of the effect of tonnage feed created by calculating R. M. E.'s.

Table 2

Test No.	Tons Per 24 Hr.	Per Cent. −200 in Discharge	Efficiency
213	18	21.6	13.1
216	36	15.2	19.2
217	72	8.4	24.0
219	108	5.5	26.2
218	144	4.7	26.5

Speaking of his Table 4, Mr. Taggart says: "This table is based on the assumption that the efficiency of reduction is the same on the smaller material returned to the mill as it is on the larger original feed." When it is considered that the Stadler R. M. E. figures cited by Mr. Taggart for a coarse-feed test and the finest-feed test are as 100 to 4, approximately, it is clear that the assumption is dangerous, and is even dangerous by the Rittinger theory which gives a ratio of 100: 24½.

This naturally brings up the question of whether the oversize product from a ball mill should be returned to the ball mill or passed along to a second grinder (either ball or pebble mill) for final reduction.

On p. 152 Mr. Taggart compares tests 227 and 228 and says: "It is obvious, then, that the ball mill is a more efficient crushing machine than the pebble mill." My conclusion, arrived at by the Rittinger theory, is diametrically opposite, as will be noted by Table 3. That the pebble mill is much more efficient than the ball mill is well shown by the results in the last two columns.

TABLE 3

Test No.		Apparent R. M. E. Stadler	R. M. E. Rittinger	Tons —48-Mesh Material Per Horsepower
227	Balls	13.5	34.4	0.38
228	Pebbles	12.3	59.8	0.65

The fact that there was such a great difference in the feed diameter would render the comparison valueless or nearly so if the Rittinger R. M. E.'s had happened to be nearly equal. But since, in spite of this handicap, the pebble mill is able to demonstrate its great superiority, I am extremely doubtful of the advisability of returning anything but the very coarsest pieces in the oversize to the ball mill circuit, as it seems probable that the regrinding of the finer sizes could be done far more efficiently by a second pebble mill working in a closed circuit.

Mr. Taggart's conclusion "that the true maximum of the efficiency curve lies somewhere between 40 and 50 per cent. moisture" does not seem to be very well supported by his moisture-efficiency diagram, Fig. 8 (p. 139), since it would be, if anything, more reasonable to suppose that the maximum efficiency moisture was either 38½ per cent. or that it was somewhere between 25 and 40 per cent. The Rittinger results in Table 4 show that the maximum efficiency moisture will be found between 52 and 68 per cent. and is probably in the neighborhood of 55 per cent. It is worth noting, however, that the actual gain in efficiency realized by changing from a 25 to a 52 per cent. moisture would only amount to about 2½ per cent. of the power used (see 5th column, Table 1, tests 212–214). This of course applies to the tests cited. The increase might be appreciably greater for a large mill, fed at its maximum efficiency feed rate and feed size.

In regard to the efficiency effect of ball load, slope of mill, size of feed, dry versus wet crushing, etc., I do not think that very positive conclusions can be drawn because of the changes made in the mill adjustments before the required data was obtained. I am inclined to think that the trap crushes so much more easily than the quartzite, as to hardly warrant Mr. Taggart's first conclusion, which is based on tests 221 and 222. If the two rocks were similar, the R. M. E.'s would be the same. The Rittinger R. M. E.'s show an appreciably greater number of surface units produced per horsepower, and if we assume the crushing constant of the quartzite as 2000, the trap constant (assuming that the mechanical efficiency of the mill was the same in each test) would be about 2300. In test 223 an appreciably larger amount of work was done than in test 213 (see Table 4), and it is interesting to note that although the horsepower increased from 17.7 (in quartz test 213) to 20.1 (in the trap test) the R. M. E.'s are respectively 49.4 and 69.2.

If the mechanical efficiency of the mill was the same in these tests the trap constant would be raised to 2800. It should be noted, however, that in tests 221–222, the ball load is 4264 lb. (and the mill is level)

TABLE 4

Test No.	Rock Crushed	Ball Load, Pounds	Work Done per Unit, Surface Units.	R. M. E.	
				Stadler	Rittinger
213	Quartz	2,819	258	22.6	49.4
223	Trap	2,811	410	24.6	69.2
224	Quartz	1,406	183	22.6	40.6

whereas in tests 213–223 it is only about 2800 lb., with a mill slope of 2⅝ in. It is, of course, possible that the mill has a higher mechanical efficiency when grinding a softer rock, in which case the calculated constant 2800 would be reduced.

The most efficient feed size is a matter of great practical importance. In regard to this, Mr. Taggart says: "The ball mill works more efficiently on material of intermediate (0.5 to 0.75 in average) size than on a coarser or finer feed" (Conclusion No. 3). There can be no question about the inefficiency of a ball mill working on a very fine feed, but I have the liveliest suspicions of the correctness of this statement in regard to the feed coarser than the grade he fixes as most efficient. The coarse-feed tests (202 to 208) have noticeably high R. M. E.'s, but whether due to the coarse feed or the softer trap rock crushed it is difficult to say.

On p. 151, Mr. Taggart explains why he adopts conclusion No. 3. He points out that the Stadler R. M. E.'s. show the coarser-feed test (208) to be more efficient than the finer-feed test No. 223, but he explains that the probable reason for this is because the reduction ratio is only 7.0 in test 208 as against 45.6 in test 223. Consequently, he says, "we may expect, by pushing the capacity (feed rate) in the case of the smaller (size of) feed until the reduction ratio is in the neighborhood of 7.0, that the relative mechanical efficiency will rise to 40, while from the same series of tests (the feed-rate tests) it is obvious that lessening the ratio beyond this point in the case of the coarser feed, by increasing the feed rate, would result in lowering the relative mechanical efficiency."

This reasoning is not at all obvious to me, because in my conception the ratio of reduction in the coarse test is more nearly 108 than 7, and in the finer feed test I would fix the ratio of reduction to be 14 instead of 46, and since these figures are diametrically opposed to his in direction, the conclusion to be drawn from them, following his own argument, is also reversed, that is, by increasing the coarse-feed rate, until the reduction ratio was reduced from 108 to 14, the coarse-feed R. M. E.'s. would go up by leaps and bounds as shown by the "feed-rate" tests. (See Fig. 1 of this discussion.) It is quite probable that the high R. M. E. in test 223 can be partly accounted for in this way.

The foregoing will make clear my reasons for believing that some of the numbered conclusions in Mr. Taggart's paper should either be reversed, or commented on, as follows:

1. The indications are that the trap crushes more easily than the quartzite, and that the efficiencies are therefore appreciably affected. The effect of small differences in rock constants is lessened by the fact that crushing machines utilize usefully a comparatively small amount of the power they draw.

4. A greater ratio of reduction in average size of material can be expected with coarse feed than with feed of intermediate size.

5. Pebbles working on a fine feed are much more efficient than balls working on a relatively much coarser feed, on account of the large reduction in the power required to lift equal volumes of pebbles (100 lb. per cubic foot) compared with balls weighing 250 lb. per cubic foot. The powers are indicated to be roughly proportional to the weights per cubic foot given. It is to be expected that if the size of feed to the pebble mill was gradually increased, a feed size would ultimately be reached which could be crushed more efficiently by a ball than by a pebble mill. These conclusions are based on tests 227–228.

12. The relative mechanical efficiency of the mill increases to the point of overload, which, however, was not reached in the tests described.

13. I hardly think Mr. Taggart has sufficient data to draw the conclusion he gives.

14. .... "is in no way commensurate." NOTE.—Probably on account of the inefficiency of the chain drive.

17. The relative mechanical efficiency for the conditions prevailing

in the moisture series of tests, is at a maximum, when the moisture is at or slightly in excess of 52 per cent. of the weight of the pulp.

18. Probably dry crushing is less efficient than wet crushing, but the decrease does not appear to be very large.

22. It seems probable that the oversize from a ball mill could be more efficiently reduced in secondary mills using pebbles.

A. F. TAGGART (written discussion*).—The writer wishes to record his appreciation of the careful study bestowed by Mr. Bell on the paper on Tests on the Hardinge Conical Mill and of the labor expended in translating the data therein contained into such shape as to make them comprehensible to those who use the "surface-unit" method of analyzing crushing data.

He wishes further, however, to register emphatic disagreement with the conclusions drawn by Mr. Bell and summarized at the end of his discussion. Most of Mr. Bell's conclusions are so completely at variance with the experience of practical mill men as to make repudiation here superfluous were it not for the fact that they were arrived at by applying a method of calculation ably defended by many writers on crushing data and therefore not to be lightly ignored.

To refer in detail to a few of Mr. Bell's criticisms: (*a*) As to the insufficiency of the data: The writer realized throughout the course of the experiments that rigorous proof of the conclusions drawn demanded more work than it was possible to do under the conditions that obtained. For that reason the data upon which the conclusions were based were fully presented in order that each reader might himself judge of their sufficiency. The writer further corresponded with and talked with several operators of mills before submitting the paper for publication, in order to determine whether or not the conclusions reached differed radically from mill experience, and was pleased to find remarkable agreement.

(*b*) Mr. Bell apparently overlooks the fact that the point of overloading in the operation of any crushing machine marks a sudden change in the phenomena involved. In some machines, such as rolls, stalling occurs; in ball mills there is a practical cessation of grinding, the mill acting as a conveyor only. Bearing this point in mind, the Stadler curve in Mr. Bell's Fig. 1 is more rational than the Rittinger curve, which latter would indicate a broad maximum and a gradual diminution in efficiency as the point of overload is passed.

(*c*) In regard to Table 4, the writer is far from defending the mill method of using "per cent. − 48 mesh" or any other mesh as a measure of crushing efficiency. However, such a means of measurement is used as a guide for practical work by intelligent operators of wide

* Received June 27, 1917.

experience and carries weight for that reason. The near agreement reached by its use with the conclusions of the writer is not the least argument in favor of the Stadler method of measurement.

(*d*) Relation between relative mechanical efficiency and metallurgical treatment: No attempt was made in the original paper to analyze the suitability of any particular product to subsequent mill operations. The requirements of these operations differ with every ore and every process. The effects of changes in the operating conditions on the relative mechanical efficiency of crushing are in no way changed by these other matters. Such analysis can be left to the mill manager.

(*e*) Effect of size of feed on the efficiency of reduction: In discussing the writer's Table 4, Mr. Bell says:

> "When it is considered that the Stadler R. M. E. figures cited by Mr. Taggart for a coarse-feed test and the finest-feed test are as 100 is to 4 approximately, it is clear that the assumption is dangerous, and is even dangerous by the Rittinger theory which gives a ratio of 100 : 24½."

Mr. Bell has apparently compared tests 219 and 229, where practically the only similar conditions are that the tests were performed in the same laboratory with the same percentages of moisture, while for such comparison the only variable should be the size of feed.

(*f*) Pebbles *vs.* balls: It is practically the universal experience, where tests have been run in the mills, that the amount of grinding done in a ball mill per unit of power far exceeds that done in the pebble mill, and that it pays to install the additional power necessary and use balls instead of pebbles. Such a change has been made in many of the mills throughout the country. Mr. Bell notes that the pebble mill produces more "–48-mesh" material per horsepower expended than the ball mill. On the same basis of reasoning, the tube mill is a far more efficient crusher than the gyratory, yet such is not the usual conclusion of mill men. In test 227 the feed was 9.900 mm. average size, in test 228 it was 1.173 mm. Obviously –0.295-mm. material can be produced with a smaller expenditure of power in test 228 than in test 227.

(*g*) The effect of moisture, point 213, Fig. 8, is obviously an accidental maximum, the position of which was determined by the moisture content in that particular test. It will be apparent to anyone accustomed to reading curves that such an accidental maximum might occur at any moisture percentage between 35 and 50. But it will be obvious to the same reader that the maximum of a smooth curve averaging the experimental points will lie between 40 and 50 per cent. and, as a matter of fact, very near 40 per cent. The conclusion drawn by Mr. Bell, that the point of maximum efficiency lies between 52 and 68 per cent. moisture, is utterly at variance with all mill experience where the question of most efficient moisture content has been tried out.

(*h*) Size of feed: The writer is unable to follow Mr. Bell's argument under this heading, since the definition of ratio of reduction used by Mr. Bell is so widely divergent from the common definition, viz.:

$$\frac{\text{Average size of particle in feed}}{\text{Average size of particle in product}}.$$

The fallacy of his method is proved by mill experience, which has taught operators to feed ball mills with a product in which a large percentage will pass a 1-in. ring, whenever the plant is of sufficient capacity to justify the installation of heavy rolls or disk crushers between the breakers and the ball mill.

(*i*) Conclusions:

1. The writer can see no reason from his data or Mr. Bell's analysis of the same to change his conclusion No. 1. There are unquestionably ores so hard and ores so soft that a comparison of the relative mechanical efficiencies of the conical ball mill working on two ores at the extremes of the list would show a marked difference, but for average ores the writer still believes that the character of the gangue has little effect on the relative mechanical efficiency of the mill.

4. As previously mentioned, Mr. Bell's definition of reduction ratio precludes discussion on this point.

5. This conclusion in the original paper is almost unanimously supported by mill experience.

12. Mr. Bell is working under the disadvantage of not having seen the experiments and not visualizing accurately from the screen tests reported.

13. An operator of mills will have little trouble in agreeing with this conclusion.

14. Chain drive was not used on the mill in the Hammond Laboratory.

17, 18, and 22. The conclusions drawn here by Mr. Bell, using the "surface-unit" method of measurement, are the strongest arguments against the method that the writer has yet seen.

JOHN W. BELL (written discussion*).—Mr. Taggart's reply illuminates a puzzling element in his original paper. Unconsciously, he has allowed practical considerations to influence some of his conclusions. In certain cases, he accepts the direct conclusions indicated by his Stadler efficiencies, and these are proved wrong by *both* the Rittinger and practical method for estimating efficiency. In other cases, a direct conclusion based on his Stadler figures would be so unsatisfactory to him that he is compelled to argue how he could have obtained satisfactory results if he had done something that he did not do; and he bases his final conclusion on the hoped for result. Perhaps the best evidence of his perfect sincerity in arguing as he did is that in one

* Received Aug. 23, 1917.

instance I was just as completely mislead by the plausibility of the argument as he was himself. I failed to note that the very fact that he was compelled to argue a reason for non-acceptance of the Stadler facts, was proof in itself that Stadler's theory had tricked him. The only value my discussion of Mr. Taggart's paper has, will be the proof that Stadler's theory will mislead him, and others, in the future, if they continue to employ it, just as it has misled Mr. Taggart in the present instance. A part of the proof has been submitted and the rest follows.

Let us first consider the facts and the Taggart argument relating to the efficiency effect of "size of feed." The main facts are that Mr. Taggart and Mr. Young made two tests to find out whether a coarse feed (22A) was favorable to an increase or a decrease in efficiency in comparison with an intermediate feed (39A). The efficiency figures were calculated in Stadler energy units. Now if I ask Mr. Taggart—Did the ball mill work more efficiently on the *coarse* feed than on the intermediate feed *in the tests he made* to determine this point?—he is obliged to answer, Yes. My reply is that both the Rittinger and −48-mesh efficiency figures show that the ball mill worked more efficiently on the *intermediate* feed *in the tests he made* (which are the important ones in this discussion) and that consequently his conclusion 3 is supported by the Rittinger and −48-mesh efficiency figures and opposed by his own efficiency figures. Not appreciating that experimenting with a bomb and experimenting with the aid of Stadler's theory are almost equally safe occupations, Mr. Taggart was obliged to argue a reason for avoiding a direct conclusion based on his efficiency figures, and it is a very pretty argument until it is examined closely. The argument is this: that the reason for the lower efficiency indicated by his intermediate-feed test was that there was too much crushing done, or, as Mr. Taggart expresses it, too great a reduction ratio. In short, the very reason that correctly explains why the intermediate-feed test *was* more efficient than the coarse-feed test is the reason Mr. Taggart gives for its being indicated to be less efficient by the Stadler efficiency figures.

The screen analyses of the coarse and intermediate feed discharges afford indirect evidence of the correctness of the Rittinger and − 48-mesh findings, since the coarse-feed discharge contains 17 per cent. of material coarser than 6.7 mm. whereas the intermediate feed contains none. Evidently the coarse feed was too coarse for efficient reduction with 4 and 5-in. balls. On the other hand, if 29 per cent. of fine material was eliminated from the intermediate feed, it seems reasonable to suppose that a much higher efficiency would have been attained.

In my first analysis of Mr. Taggart's data, instinct told me correctly that Mr. Taggart's conclusion No. 3 was not justified by his facts. Misled just as completely as he by an argument with fatal defects, the contradiction between his figures and his conclusion seemed to be explained by the difference between the diameters he gave and the diameters I found by taking the reciprocals of the mechanical values of feeds and discharges. By completely disregarding my Rittinger data, especially the "work per ton," I unconsciously countered a fallacy with a fallacy and I would not be frank if I failed to acknowledge it. All that was needed to obtain the correct explanation, was to reason it out again and pay attention to my Rittinger facts. Mr. Taggart was quite right in claiming that his ratio of reduction conforms with the usual definition but after he notes how dependable Rittinger's theory is, I hope he will agree that the ratio of reduction is a quantity which is valueless in connection with crushing tests, for the reason that it takes 10 times as much power to reduce 1/10-in. particles to 1/100 in. as to reduce 1-in. pieces to 1/10 in., the ratio of reduction being the same in both cases.

Mr. Taggart has evidently failed to appreciate how seriously his own data indicts the Stadler theory, and this will account for statements he has made which I am confident he will withdraw after an unprejudiced examination of the facts in the case.

First, he says that the results indicated by Rittinger's theory are incompatible with the findings of practical men, and secondly, that the Stadler method agrees with the −48-mesh efficiency method. Evidently, Mr. Taggart neglected to work out the −48-mesh figures because all that is required completely to refute these statements is to submit the figures in the following tables.

The tests in Tables 5, 6 and 7 are arranged in ascending order of efficiency, and one has only to note the orderly ascension of both the Rittinger and −48-mesh efficiency figures in Table 5 and the total lack of relation between their findings and the Stadler findings to realize that the assertions quoted above have no foundation in fact. There is an inconsistency between the Rittinger and −48-mesh figures after passing Test No. 222 in the trap tests, which is worthy of note, but after examining this, my conclusion is that the evidence is in favor of the Rittinger figures, for any practical man will be willing to concede that his method does not give proper credit to a machine for its production of −200-mesh material. Taking this defect into account, the agreement between the Rittinger and −48-mesh figures is astonishing. It is so astonishing that I cannot resist the temptation to ask Mr. Taggart to note that both Rittinger and −48-mesh select pebble mill Test No. 228 to be *the test of highest efficiency* in the low tonnage series of quartz tests, while the Stadler method selects it *to be the test of lowest efficiency* in the same series. He might also note (see Table 6) that −48-mesh agrees with Rittinger that a feed rate of 144 tons is more efficient than a 108-ton feed rate. After considering his argument in this connection I have to admit that I cannot imagine how a ball mill could be fed, "so that there would be a practical *cessation* of crushing, the mill acting as a

TABLE 5.—*Low Tonnage Tests*

Quartz Test No.	Relative Mechanical Efficiency		
	Rittinger	−48-Mesh	Stadler
210	32.8	29.0	16.7
211	33.8	28.2	18.8
227	34.4	29.8	13.5
209	39.4	33.3	18.6
220	40.2	34.8	19.4
224	40.6	35.5	22.6
226	41.0	35.2	16.4
225	42.1	38.0	24.6
212	45.3	36.9	21.4
221	48.5	42.2	20.8
215	52.2	46.5	21.6
214	54.0	46.2	22.2
228	59.8	51.6	12.3
Trap Test No.			
229	24.5	18.6	4.3
206	44.1	30.1	19.9
207	47.6	32.2	21.4
204	54.0	37.0	21.0
208	55.1	42.1	26.3
222	56.0	42.1	23.5
202	56.6	37.6	21.4
203	58.5	40.8	23.7
205	63.0	43.7	25.4
223	69.4	47.4	24.6

TABLE 6.—*Feed Rate Series of Tests*

Test No.	Tons per 24 hr.	Relative Mechanical Efficiency		
		Rittinger	−48-Mesh	Stadler
213	18	49.4	42.5	22.6
216	36	72.2	66.7	39.8
217	72	90.6	83.8	67.1
219	108	98.6	98.9	100.0
218	144	100.0	100.0	94.5

conveyor only" unless the feed was inserted by something far more active and powerful than a scoop feeder. At any rate, the evidence in Table 6 satisfies me that instead of the cessation, there was a slight increase in efficiency when the feed rate was increased from 108 to 144 tons. In regard to the relative merits of the Rittinger and Stadler curves, even the Stadler curve fails to afford evidence of a practical cessation of crush-

TABLE 7.—*Moisture Series of Tests*

Test No.	Moisture, Per Cent.	Relative Mechanical Efficiency		
		Rittinger	−48-Mesh	Stadler
210	0.0	32.8	29.0	16.7
211	18.8	33.8	28.2	18.8
212	25.0	45.3	36.9	21.2
213	38.5	49.4	42.5	22.6
214	51.8	54.0	46.2	22.2
215	68.2	52.2	46.5	21.6

ing, because after it reaches the 108-ton point, the efficiency decreases gradually. One is compelled to conclude, therefore, that the Stadler critical point indicated is in the nature of a mirage. But perhaps an even more important conclusion to be drawn from Table 6 is the entire absence of merit there is in a method which declares that the R.M. efficiency of a crusher is 23 per cent. when it should be 49 per cent., 40 per cent. when it should be 72 per cent., and 67 per cent. when it should be 91 per cent. Mr. Gates and I both know the reason for this lamentable showing, and we have published it.

In regard to moisture, the results figured by the three methods are given in Table 7. In his reply to my discussion, Mr. Taggart calls for help from the practical men, to support his contention and his curve (suitably modified with a smooth curve) that 40 per cent. is *the* moisture. I am afraid his supplications will fall on deaf ears when practical men find that their own method of figuring efficiency supports the Rittinger conclusion that the high moistures result in highest efficiency. The only difference in the findings is that −48-mesh selects the highest moisture and Rittinger the next to the highest, but both show that there is very little change in efficiency after reaching 52 per cent. moisture. It does not seem to have occurred to Mr. Taggart that the findings of practice are based on tests carried out under conditions quite different from his and that consequently agreement with mill practice is not proof of the correctness of his conclusions. At any rate, Mr. Taggart will appreciate how dangerous it was for him to state a definite conclusion about moisture when he notes that the method he used indicated a change of only 1 per cent. in efficiency in going from 38 up to 68 per cent. moisture and that it indicated a decrease instead of the increase shown by the other two methods.

In his reply, Mr. Taggart claims that he did not analyze the suitability of any product to subsequent mill operations, but he forgot, for the moment, his conclusion No. 22, in which he tells all and sundry to return a ball-mill oversize to a ball mill. This conclusion conforms with his other conclusions that a ball mill is more efficient than a pebble mill, and that balls are more efficient than pebbles, but all of these are founded on two tests, Nos. 227–228. Here again, he could not reconcile himself to acceptance of a conclusion his efficiency figures pointed to, which was, that for all practical purposes there was but little difference (only 1.2 per cent. in favor of the ball mill) between the efficiency of the ball-mill test and the pebble-mill test, and he is compelled to argue a reason for the trifling difference in favor of the ball mill, and a remedy for rectifying it. Now I think Mr. Taggart will agree with me, that it would be impossible to establish the relative merits of pebble and ball mills unless the several machines were operated under the special operating conditions each required to attain maximum efficiency. Therefore the only question of any importance in connection with Tests 227–228 is whether the ball mill made the execrable showing indicated by Rittinger and −48-mesh efficiency, or was the opposite conclusion indicated by the Stadler method another one of its disconcerting tricks. In this one instance, I cannot give an absolute proof of its error because the operating conditions were so vastly different in the two tests, but a sufficiency of proof will be found in Tables 5, 6 and 7. The force of a conclusion, which Mr. Taggart says is obvious in paragraph (*f*), is lessened almost to the vanishing point when it is remembered that the amount of crushing done by the pebble mill was only 13 per cent. less than the amount done by the ball mill, and was accomplished by the pebble mill with an expenditure of *half the power* drawn by the ball mill. This suggests the possibility that *a too heavy ball load* had something to do with the poor performance of the ball mill although such an explanation is directly opposed by Mr. Taggart's conclusions about ball loads. However, one has only to examine the facts to see that Stadler's theory again failed to supply Mr. Taggart with the evidence required to form correct conclusions about ball loads. The more important facts in this connection will be found in the following table.

Table 8

Test No.	Ball Load	Relative Mechanical Efficiency		
		Rittinger	−48-Mesh	Stadler
204	4006	54.0	37.1	21.0
205	4503	63.0	43.9	25.4
223	2811	69.4	47.6	24.6
222	4264	56.0	42.1	23.5
224	1406	40.6	30.9	22.6
213	2819	49.4	42.1	22.6

Mr. Taggart's conclusions about ball loads were founded on Tests 204–205, Tests 223–222, and an argument to explain, as he says, "the apparently contradictory result presented in Tests 223–222." Now, in justice to Mr. Taggart, I must point out that in the sentence following the one quoted, he evidently suspected that the more lightly loaded mill should have been more efficient, although the two sentences seem contradictory. Nevertheless, he concluded that "increase in ball load increases efficiency" and that "the ball load should be the maximum the mill will hold." Here again Mr. Taggart was misled by the colorless indication of his Stadler efficiency figures in Tests 223–222. The Rittinger and −48-mesh figures show in an unmistakable way that the heavy ball load *decreased* the efficiency of the mill in Test 222 and the decrease would have been greater if the power chain (which I inadvertently called the chain drive) effect had been equal in both tests. Evidently there is a ball load of maximum efficiency, depending on the test conditions, above which and below which there is a decrease in efficiency. Probably, therefore, the 6654-lb. ball load in Test 227 was adverse to the ball-mill performance. The unreliability of the Stadler method is again illustrated by Tests 224–213 (Table 8) in which the test conditions permit drawing a conclusion about the effect of ball load. The strict neutrality of the energy unit efficiency figures is belied by the findings of both Rittinger and −48-mesh which declare that doubling the ball load in these tests resulted in a decided increase in efficiency.

In paragraph (*e*) Mr. Taggart has ground for objecting to a 108-ton feed rate test, but there are many others in the series to illustrate my argument, and it is well known that a coarse (but not too coarse) feed is much more favorable to mechanical efficiency than a fine feed.

In regard to the comparative merits of a gyratory and a ball mill, it would not surprise me if a ball mill *was* mechanically more efficient than a gyratory, but nobody could express an opinion about this that would be of value, without first comparing the relative powers with the total number of units of crushing produced by each machine.

Conclusion 4 is correct as originally stated by Mr. Taggart, although, for the reason given, the expression "work per ton" is preferable to the "ratio of reduction." It was conclusion 3 that I wished to enter a protest against, which I now do; not because it is wrong by the correct facts but because it is opposed to the Stadler facts. Just why anyone would continue to support and make use of Stadler's theory after considering the evidence against it, is not at all clear to me. Perhaps the new evidence will be more convincing than the first.

R. B. T. Kiliani, New York, N. Y.—I do not care to discuss Mr. Taggart's paper in the light of theory, as that has been very well done by Prof. Bell, but I should like to criticize some of his conclusions, in the light of actual operating practice at plants all over the country.

1. Mr. Taggart's first conclusion is that in crushing average ore, the character of the gangue has little effect on the efficiency of the mill. This, I believe, is not in accordance with the usual practice, since in crushing a hard ore the capacity will be much reduced below what it would be with a softer ore, while the power consumed will be practically independent of the character of the ore, being proportional only to the load of ore and balls in the mill.

2. Mr. Taggart's second criticism is that the Hardinge mill is not suitable for grinding soft, tough material. In answer to this I might mention that the mill is being used for grinding tough, ductile material, such as metallic aluminum, and also for grinding licorice root.

3. He says that the ball-mill works more efficiently on material of intermediate size, say, ½ to ¾ in., than on either coarser or finer feed. This is true as to coarser feed. For the most efficient work, I believe that a ball mill should be fed with material not coarser than 1½ in.; it will handle material up to 3 and even 4 in., but the reduction from 3 or 4 in. to 1½ in. can be done much more cheaply and efficiently by rolls or disk crushers than in the ball mill.

4. As to Mr. Taggart's fourth conclusion, that a greater ratio of reduction can be expected with feed of an intermediate size than with a coarse feed, I have not enough information to express an opinion.

5. His next conclusion is that steel balls are much more efficient crushing media than pebbles. Steel balls are undoubtedly more efficient for crushing coarse feed. On fine material they are also more efficient as to tons per horsepower crushed to 10-mesh, but on fine material I think it will be found that flint pebbles are cheaper than cast-iron balls, per ton of ore, although there will be a saving in power per ton by using cast iron instead of flint. The increased cost of crushing with cast iron will be due to the higher cost of iron at the present time.

6. Mr. Taggart's sixth conclusion coincides with present practice, that steel balls will grind as fine or finer than pebbles when working on the same feed.

7. It is also true that an increase in weight of the ball load, other conditions remaining constant, increases the ratio of reduction and the relative mechanical efficiency of the mill. However, I believe that there is a certain load which is most efficient, and that this is not the maximum load the mill will hold, filled to the center line, but when loaded up to the trunnion line or about 6 in. below the center line.

8. Mr. Taggart's eighth conclusion, that the power consumption increases with increased weight of ball load, but not in direct proportion, agrees with my observations.

9. He says that power consumption diminishes with increase in the average size of balls, up to an average size of 5 in. We have found, when using smaller balls, that the mill requires more power than with the

same load of large balls; this is probably due to the fact that with small balls it takes a large number to make up the same weight, and therefore more friction is produced when those balls roll over each other.

10. Mr. Taggart concludes that a ball load composed of 5-in. balls performs a greater reduction in size of ore at one passage through the mill than a mixed charge composed of 5-in., 4-in., and 3-in. balls. This does not agree with ordinary practice, because we have found that when we want to crush fine in a ball mill, using only 4- and 5-in. balls, we cannot obtain so great a capacity as when we use a certain number of balls of small diameters. The addition of small balls will usually increase efficiency too, if not too numerous.

11. Mr. Taggart's eleventh conclusion, that the mechanical efficiency of the ball mill increases with the average size of ball in the crushing charge up to 5-in. average diameter, has just been answered.

12. His twelfth conclusion, that the relative mechanical efficiency of the mill increases with the rate of feed, to the point of overload, I believe, is correct.

13. He says that increased length of cylindrical section in the conical ball mill increases the reduction ratio, but at the expense of a marked diminution in mechanical efficiency. That larger ratio of reduction is not very pronounced, although it is distinguishable, although in certain cases that increase in efficiency can be taken care of where large capacity per foot of floor space is desirable, and then it may be advantageous to use a mill of larger diameter. However, I do not believe it is good practice, especially when grinding in closed circuit with a classifier, to use a mill having too small a cylindrical section. Better results are obtained with larger diameter and shorter cylinder. That has been proved by some data I obtained recently. One pebble mill of 8-ft. diameter, operated by a 75-hp. motor, was lagged down to a diameter of 6 ft., while another was reduced to 7 ft. diameter; better results were obtained with the 7-ft. than with the 6-ft. mill.

Mr. Taggart's fourteenth, fifteenth and sixteenth conclusions, regarding the slope of the mill, seem to be borne out by present practice.

17. His next conclusion is that, other conditions being constant, the relative mechanical efficiency of the mill is a maximum when the feed contains between 40 and 50 per cent. of water. Professor Bell claims that 58 per cent. water gives better results. In actual mill practice, that will depend on the character of the ore, since a very dirty ore will require much more water than a granular ore not containing much natural colloidal slime. I know cases where it has been necessary to run the mill with 30 per cent. solids; if the pulp were thicker, no crushing would be accomplished. At another plant they are grinding with 75 per cent. solids and getting very satisfactory results, probably due entirely to the character of the ore. As a general rule, to obtain the best results,

I believe that the pulp should be as thick as possible, as is the usual practice with cylindrical tube-mills.

Mr. Taggart's eighteenth conclusion, that the relative mechanical efficiency in wet crushing is decidedly higher than in dry crushing, I believe has been thoroughly proved. His nineteenth and twentieth conclusions seem to be satisfactory. As to his twenty-first conclusion, I am inclined to doubt it, but I have not sufficient information on which to base definite opinion.

22. His last conclusion is that the conical mill should be operated in closed circuit with a sizing device which will return the oversize to the mill. Apparently better results are obtained by crushing in two stages than in one stage. It is perfectly possible to crush in one stage, and in a small plant this is the proper thing to do, owing to the higher initial cost of a two-stage plant; but in a large mill, where sufficient machinery can be installed, two-stage or even three-stage crushing is considerably more efficient than one-stage crushing.

Mr. Taggart says, "When operating as a ball mill, the ratio of length of cylindrical section to diameter should not exceed 0.3. This will be a much more economical installation than one which seeks, by slow feeding or long cylindrical section, to obtain a finished product at one passage through the mill." For fine crushing, the mill should always be operated in closed circuit, by returning the oversize to the mill itself, so long as the ratio of reduction is not too great; that is, not more than, say, from 8 mesh to 48 or 65 mesh, but not from 1 or 1½ in. to 65 mesh in one stage."

Lastly, Mr. Taggart says, "The slope should be adjusted to mill requirements, but for ordinary concentrating mill practice should be about 0.4 in. per foot." If the inclination of the mill axis is too great it will diminish the ball load unnecessarily. The inclination should not be over 0.2 in. per foot, and when the mill is operated in closed circuit with a classifier, I believe it should be set level. "The ball charge should be the maximum that the mill will hold and should be kept as large in average size as is possible without too great sacrifice of small balls." The ball charge should not be all that the mill will hold, but should be somewhat less than that; neither should it be kept at as large an average size as possible, because by so doing the mill will naturally get all the larger sizes. However, if too many small balls are present, they will probably interfere with the crushing.

Arthur F. Taggart (written discussion*).—The numbers in the following reply to Mr. Kiliani's discussion refer to correspondingly numbered conclusions in the original article.

(1) This conclusion is based on comparative tests with a hard, tough, homogeneous trap, and a rather soft, micaceous quartzite. These two

* Received Dec. 3, 1917.

rocks may be considered fairly representative of the two extremes of average ores. In crushing these rocks, under similar conditions, there was very little difference in the relative mechanical efficiencies of the mill. In coarse crushing, these rocks would have shown considerable difference because the quartzite would break easily along the planes where sericitization had taken place. But when crushing is carried to the point where the quartz or other component minerals of the rock have to be pulverized, and such is always the case in ball- or pebble-mill grinding, the difference disappears. Unless the ore is exceptionally hard and tough, or exceptionally soft and friable (in neither of which cases could it be called an average ore), the relative mechanical efficiency of a mill crushing different rocks under similar conditions will be approximately a constant.

(2) Mr. Kiliani does not state in his criticism of this conclusion what efficiencies are being obtained in the mills he mentioned. The conclusion stated in the original paper was based on performances of the 4½-ft. mill grinding sawdust, cocoanut shells, and metallic copper. Relative mechanical efficiencies ranged, as stated, from 0.04 to 1.67 with the mill pushed to maximum capacity. These figures are to be compared with the efficiency figure 43.50, test 219. The conclusion is, of course, obvious.

(3) The statement of size in conclusion (3) in the original paper is 0.5 in. to 0.75 in. *average*, which covers the range in average size of the product of any coarse breaker delivering material to pass a 1.5-in. ring. Mr. Kiliani apparently agrees with this conclusion. It should be borne in mind, however, that the work on which this conclusion is based was done in a 4½-ft. mill. This size is the lower limit, or, if anything, somewhat below the lower limit in size for ball mills in ore-milling plants. Mr. Kiliani speaks of a ball mill handling 3-in. or even 4-in. material. The 4½-ft. mill will not handle this size at all. The diameter of the mill is not great enough to give sufficient fall to the balls to break such large lumps. It should be expected, therefore, that the larger sizes in the tests presented in this paper would be more satisfactorily reduced in the 6 and 8-ft. mills. These remarks apply also to conclusion (4).

(5) This conclusion was based on crushing efficiencies without regard to the consumption or cost of crushing media. From such a standpoint, it is apparent that Mr. Kiliani's experience agrees. The economic efficiency would obviously vary as between metal balls and flint pebbles, both with locality and with metal and pebble prices. No general statement can, therefore, be made on this score.

(7) The data presented will not bear out Mr. Kiliani's contention that there is a point of maximum efficiency in the ball load below the point of maximum charge which the mill will hold. Nor does such a conclusion seem reasonable. If the assumption is made, and it seems

to the writer a proper one, that crushing is done by all the balls in the mill, then any increase in the ball load which is not accompanied by an increase in the power consumption should add to the efficiency of the operation. This is the case when loading is carried beyond the horizontal axis of the mill (See curves, p. 131).

(10, 11) Mr. Kiliani's facts agree with these conclusions, although he does not draw the same conclusions from them. Any attempt to crush fine in a ball mill (when the term "ball mill" is used to describe a mill taking feed of 1½-in. maximum, or greater), will result in reduction of capacity and in a corresponding reduction in relative mechanical efficiency.

(13) The writer has found that the increase in reduction ratio with a 4½-ft. by 48-in. mill as compared with a 4½-ft. by 16-in. mill is marked, as is also the accompanying decrease in relative mechanical efficiency. The stenographic transcript of this part of Mr. Kiliani's comment is confused, but he is apparently discussing increase in diameter rather than increase in length of cylindrical section.

(17) The only exception which Mr. Kiliani cites to this conclusion is that of a very dirty ore, by which statement it is presumed that he means a clayey ore. Such material is, of course, entirely different from any of that tested in the work leading up to the present paper. Subsequent experience confirms the original conclusion that on average ores the relative mechanical efficiency will be at a maximum with a feed containing 40 to 50 per cent. moisture, and in the usual case the maximum point lies nearer to 40 than to 50 per cent.

(21) This conclusion is confirmed by a considerable number of observations other than those cited in the paper. In fact, no one characteristic has been so invariable in the writer's experience with the conical ball and pebble mills as this one of variation in the tonnage and character of discharge under conditions of constant feed. No tonnage measurements or sizing tests made on a single sample of the discharge should be depended upon as being representative of the average performance of either mill.

(22) Mr. Kiliani questions the statement in regard to the best slope for a mill for ordinary concentrating-mill practice. The advantage of the larger slope is confirmed by the rather remarkable results obtained by cylindrical ball mills using some such means as a sand elevator in the discharge end to aid the egress of fine material. The result of such continuous removal of fine material is to cause the pulp load in the mill to be small, thus doing away with all cushioning and making every blow of the balls effective in crushing ore. The same result can be obtained in the Hardinge mill by giving the mill a very decided slope toward the discharge end. If it is aimed to make a large reduction in the ball mill at one passage, this can be done at small capacity by increasing the

length of the cylindrical section and setting the mill level, but where more than one-stage crushing is practised, and high relative mechanical efficiencies are desired, they can be obtained only by this method.

The criticism of the statement in this conclusion that "the ball charge should be the maximum that the mill will hold and should be kept as large in average size as is possible without too great sacrifice of small balls" is commented upon in paragraphs numbered 7, 10 and 11 above.

JOHN W. BELL (written discussion*).—In his discussion of Mr. Taggart's paper, Mr. Kiliani assigns to me the claim "that 58 per cent. moisture gives better results." I have not claimed, and have no intention of claiming, that 58 per cent. moisture will give better results in practice than the lower percentages of moisture which large-scale tests have shown to be preferable. I do claim, and have submitted the proof, that of the six tests made by Mr. Taggart to determine the effect of moisture on efficiency, 52 per cent. was decidedly more efficient than 38½ per cent. moisture.

I have never had the least doubt that the relative mechanical efficiency of a ball or pebble mill increases to the point of overload. Practical operators offered convincing evidence on this point long ago. But what I do claim is that Mr. Taggart erroneously concluded that he had reached and passed the point of overload, and that the error resulted by his adherence to a theory which falls down hopelessly when it is tested practically, that is, by experiment.

At McGill we have made a number of tests on a trap rock and on a quartz gangue. They are "average" rocks and yet 1 h.p. expended on the quartz will produce 1½ times as many units of crushing as are produced when the same power is expended on the trap. These tests were fine-crushing tests.

Mr. Kiliani says that he does not care to discuss Mr. Taggart's paper in the light of theory, because I have done that. I do not accept his kind impeachment because I think the only practical way to test Taggart's conclusions is to examine them carefully in the light of the facts and figures which have been offered in support of them and against them.

I am sure Mr. Kiliani will agree that it would be desirable to be sure that the Stadler-Kick method of estimating efficiency is so defective as to preclude drawing correct conclusions from its findings. Of a number of proofs of this, perhaps the one which will enable Mr. Kiliani and others to make a quick decision is the fact that in a series of 13 tests, the one test which the Stadler-Kick method indicates as giving the

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lowest efficiency is found, both by the Rittinger and the −48 mesh method, to give the highest efficiency in the whole series.

How to measure and how not to measure the efficiency of a crusher is a question which should be of interest to practical men. The facts and figures submitted by and in connection with Mr. Taggart's paper are very illuminating in their bearing on this question. It would seem to be in the interest of science and practice to discuss this aspect of Mr. Taggart's paper thoroughly.