ABSTRACT

A new experimental procedure for determining the solids flux curve is detailed. The procedure is based on measuring the concentration at various heights of a bed of settled solids formed during a semi-continuous sedimentation test. Unlike conventional thickener area calculations, the test procedure demonstrates the dependence of the flux curve on the system feed flux. At a given feed flux, there was good agreement between the solids flux curve determined by the method and continuous sedimentation results. The widely used method of Kynch was found to be restricted to relatively high effective feed fluxes and consequently there was poor agreement with continuous tests at low feed fluxes. The flux curve determined by the method of Coe and Clevenger was invalid for all feed fluxes. A case study shows how the required thickener area would vary for a number of fine coal processing options.

INTRODUCTION

Continuous sedimentation has an important role in coal preparation plants for clarifying and recycling the process water and reducing the volume of fine refuse for subsequent mechanical dewatering or disposal. The solids flux curve is used to model the process of continuous sedimentation. It is employed mainly in design calculations to determine the required thickener area, the most important design parameter, and ultimately the maximum steady state underflow concentration that is possible for a given solids throughput. In addition, the capital cost of a thickener will be dependent on the calculated required settling area. Therefore, it is important to provide adequate area but avoid excessive overdesign.

The principal assumption of the solids flux curve model is that the settling velocity of the solids in a slurry is a function of the local solids concentration only ($V_s = V_s(c)$). A number of researchers dispute this assumption (see discussion in Fitch, 1979) because in many cases the model has failed to accurately predict the required vessel area. The other limitation of the principal assumption is that the effect of bed height on underflow concentration cannot be assessed. Consequently, more complex models which take into account the forces involved in the system, such as inertial, buoyancy, compressive and drag forces have been developed. However, these theories require a great deal of experimental data and are much less practical than the solids flux curve model when it comes to design. In theory, the solids flux curve model can be used to predict the underflow solids concentration for any underflow rate, as well as the rise velocity of the bed for non-steady state conditions. The experimental methods of Coe and Clevenger (1916) or Kynch (1952) are often used to obtain the necessary data to determine the solids flux curve. In this study an experimental method for obtaining the data is outlined and flux curves determined by the three methods are compared with results from continuous sedimentation experiments. A case study shows the effect of various fine coal dealliming options on a tailings thickener underflow density and pumping requirements.

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THEORY

Basic Assumptions

Throughout the following discussion all vector quantities such as velocity and flux are taken as positive and, unless stated otherwise, they are given relative to the vessel. It is also assumed that:

i) the sedimentation vessel has a constant cross-sectional area

ii) the process is one-dimensional and so there is no radial variation in concentration for any horizontal layer

iii) the slurry can be treated as a continuum with a continuous liquid and solid phase interacting with each other.

In the sedimentation of a slurry there are three important velocities. Firstly, there is the settling velocity of the solids, $V_s$, which is a function of concentration. The settling flux $G_s$ is calculated from the product of $V_s$ and the local solids concentration, $C$. That is,

$$G_s = V_s C$$

The quantity $G_s$, when plotted against solids concentration $C$, results in the solids flux curve. A typical flux curve is shown in Figure 1.

The second important velocity is the upward velocity of propagation of each concentration in the sediment. This leads to a propagation flux component $G_p$, which is defined as,
The third velocity is the underflow velocity \( V_u \), which is defined as the volumetric underflow rate divided by the settling area of the vessel. This velocity is superimposed on the settling velocity of the solids, resulting in a total velocity inside the vessel, and relative to the vessel, of \((V_s + V_u)\). The underflow flux \( G_u \) is related to \( V_u \) by way of

\[
G_u = V_u C_u \quad (2)
\]

Therefore the settling velocity of the solids can be calculated if the velocity of propagation of each concentration in the range from \( C \) to \( C_{\text{max}} \) is known. These data can be obtained from the concentration profile of the bed. A semi-continuous test is performed by feeding a vessel at a constant solids feed flux without underflow removal. After a time \( T \) the concentration profile of the bed is determined experimentally. A typical profile is shown in Figure 2. The propagation velocity of each concentration is taken to be the height of the concentration divided by \( T \). That is,

\[
V_p = \frac{H}{T} \quad (9)
\]

**Solids Flux Curve Model Predictions and Validation**

The solids flux curve in Figure 1 is shown with an underflow operating line. The slope of this line is equal in value to the underflow velocity and the line is a tangent to the solids flux curve. A thickener operating with such an underflow velocity should have an underflow concentration \( C_u \) as shown. By material balance, the underflow solids flux should be \( G_u \) as given by equation 3. The underflow operating line will be tangential to the flux curve if the feed flux \( G_P \) is greater than \( G_u \) as is the case in Figure 1. By performing a continuous thickening experiment under non-steady state conditions \((G_P > G_u)\) at a particular underflow velocity, and determining the final underflow concentration \( C_u \), it is possible to fix the underflow operating line on the solids flux-concentration plane. Repeating this procedure at different underflow velocities, while ensuring that \( G_P > G_u \), a comparison can be made between results from continuous sedimentation and the solids flux curve determined by the method of Coe and Cleverger (1916), or Kynch (1952) or some other method.

**Determination of the Solids Flux Curve from a Concentration Profile**

The mathematical basis of the solids flux curve model is Kynch's (1952) continuity equation

\[
V_p = \frac{-dG_s}{dC} \quad (4)
\]

which depends on the \( V_s = V_s(C) \) assumption.

It follows from this equation that the propagation velocity of a particular concentration is equal to the slope of a tangent to the solids flux curve at that concentration.

Rearranging equation 4 leads to,

\[
dG_s = -V_p dC \quad (5)
\]

and noting that \( G_s = 0 \) at \( C = C_{\text{max}} \) since the solids virtually cease to settle to a higher concentration,

\[
\int_{G_s}^{C_{\text{max}}} dG_s = -\int_{C}^{C_{\text{max}}} V_p dC \quad (6)
\]

Therefore the settling flux is,

\[
G_s = V_p dC \quad (7)
\]

and from equation (1) it follows that

\[
V_s = \frac{1}{C} V_p dC \quad (8)
\]

This is effectively the same approach that Kynch (1952) used and depends on the assumption that each concentration from \( C \) to \( C_{\text{max}} \) is present at the base at time \( T = 0 \). It is also assumed that \( V_p \) remains constant over time for each concentration which is equivalent to the \( V_s = V_s(C) \) assumption.

The settling velocity of the solids can be calculated by combining equations 8 and 9. That is,

\[
V_s = \frac{1}{C T} \frac{H dC}{dC} \quad (10)
\]

By determining the area under the curve in Figure 2 between \( C \) and \( C_{\text{max}} \) and dividing the result by \( C T \), the settling velocity of the solids, \( V_s \) at concentration \( C \) can be found. Repeating the integration for different \( C \) values and plotting the product of \( G_s \) against \( C \) results in the solids flux curve.

**Determination of the Solids Flux Curve from Batch Tests**

In studies by previous workers, experimental data on settling velocities have been obtained from batch sedimentation tests. A slurry with an initial solids concentration \( C_1 \) is allowed to settle to a compact sediment in a vessel. A clear interface should appear between the slurry and supernatant. The height of this interface is plotted as a function of time and a batch settling test curve similar...
EXPERIMENTAL PROCEDURE

Figure 4 shows a diagram of the equipment used in the batch and continuous experiments. For the batch tests, the sedimentation vessel used was 1.5m high and for the semi-continuous and continuous tests the vessel was extended to 3.0m so that feed could enter at a depth of 1.5m, allowing the formation of a clarification zone and a reasonable bed depth. The vessel had an internal diameter of 95mm. On the wall of the vessel were sampling locations at heights of 10, 50, 150, 225, 300, 600, 900 and 1200mm for obtaining samples to determine the concentration profile. Full width samples could be collected.

The effective feed flux can be calculated for a batch settling test if the initial settling velocity is constant which is usually the case for low initial concentrations and a uniformly flocculated slurry. At low initial concentrations, a discontinuity is observed after a time $T_d$. The effective feed flux is equal to the mass rate per unit area that solids settle through the discontinuity and into the sediment. The effective feed flux is therefore,

$$G_F = C_1 H_1 / T_d$$  (13)

The effective feed flux of a batch test is useful when comparing the resultant flux curve with flux curves determined from semi-continuous tests at different feed fluxes.

The feed material used in this study consisted of a bituminous pulverised coal, with an ash content of 14.2% and a sizing of 70% by mass less than 0.075mm. A high molecular weight anionic polyacrylamide flocculant was used to flocculate the feed in a mixing vessel prior to sedimentation on a solids basis of 50%.
Batch Tests

Batch settling tests at initial solids concentrations ranging from 20 to 212 kg/m$^2$ were carried out to determine the solids flux curve by the methods of Coe and Clevenger (1916) and Kynch (1952).

Semi-Continuous Tests

Semi-continuous tests were carried out by feeding the vessel continuously at a constant solids feed flux without underflow removal. Between experiments, the feed flux was increased by increasing the volumetric feed rate at the same solids concentration rather than by increasing the solids concentration. Sufficient time for flocculant mixing was allowed (Waters, 1985) and hence the floc structure was similar for all experiments. This may not have been the case if the initial concentration was varied. To ensure that disturbances from feed rate changes did not occur near the bed, the feed entry was located well above the bed, allowing a uniform free settling zone to be established. The product of solids concentration and settling velocity (relative to the bed) was equal to the system feed flux throughout the free settling zone. Therefore, the interface between the free settling zone and the bed was only affected by the system feed flux.

The bed would rise up through the vessel at a constant velocity. After a time $T$ the bed was sampled to determine the concentration profile and hence the solids flux curve by the new experimental method.

Continuous Tests

In the continuous tests, the bottom sampling tube was connected to a pump and underflow was removed at a constant rate from the start of the experiment. The underflow concentration was determined by collecting samples at regular time intervals. It should be noted that steady state was not achieved in these experiments. However, a constant underflow concentration was eventually obtained. This meant that the underflow concentration was dependent on the underflow velocity only, and was not the result of a steady state mass balance. In other words, the underflow operating line would remain a tangent to the flux curve.

RESULTS OF FLUX CURVE CALCULATIONS

Determination of the Solids Flux Curve

Figure 5 shows the solids flux curve determined by the methods of Coe and Clevenger (1916), and Kynch (1952) and the concentration profile method. The difference between the flux curves determined by the Coe and Clevenger, and Kynch methods is a similar result to that obtained by Talmage and Fitch (1955). In their paper they note that the Coe and Clevenger procedure entails an additional assumption to the Kynch method, that the floc structure within the pulp is independent of the initial concentration of the batch test. They suggest that this is not the case by showing that the final sediment concentration increases as the initial concentration is increased, and so conclude that the Kynch procedure should be more reliable.

Batch Tests

The flux curve determined by the method of Kynch was obtained from batch tests at initial solids concentrations of 20, 30, 40 and 50 kg/m$^2$. There was only a small variation in the flux values obtained from the four batch tests and the Kynch flux curve shown in Figure 5 is the curve of best fit through the four batch test results. The effective feed fluxes (see Equation 13) for the four batch tests were relatively high and ranged from 0.14 kg/m$^2$-s for an initial concentration of 20 kg/m$^2$ to 0.30 kg/m$^2$-s for an initial concentration of 50 kg/m$^2$. Batch tests at initial concentrations less than 20 kg/m$^2$ were not feasible as the interface was unclear and the final sediment volume was very small. Consequently the Kynch method was restricted to effective feed fluxes greater than 0.14 kg/m$^2$-s.

Semi-Continuous Tests

The remaining flux curves were determined from concentration profiles obtained from semi-continuous tests. In each case a dashed line is shown extending from the flux curve, as a tangent, to the feed flux used in the semi-continuous tests. It is evident that as the feed flux of the test was increased, the resultant flux curve deviated toward the Kynch flux curve. This is in agreement with the fact that the effective feed fluxes used in the Kynch batch tests were relatively high. These results are contrary to the fundamental assumption of the solids flux curve model that $V_s = V_s(C)$ because the flux curve should be unique for a given feed material. It must be concluded at this stage that the flux curve is a function of the feed flux used to determine it.

Validation of the Solids Flux Curves Using Continuous Tests

This series of experiments was performed with the feed flux close to 0.083 kg/m$^2$-s. This was so that results could be compared with the semi-continuous flux curve shown in Figure 5, determined at the same feed flux. Figure 6 shows the bed height and underflow concentration as a function of time and the final concentration profile for one of the continuous sedimentation experiments. It is evident that the system was not at steady state (see the section 2.2 on validation of the flux curve) because the bed height was increasing, and the underflow concentration approached a constant value which was independent of bed height. This suggests that compressive forces were not significant.

Four additional experiments were performed, each at a different underflow velocity. The underflow solids flux was calculated using equation 3 for each experiment. The operating lines, drawn from the underflow solids fluxes on the flux axis, to the corresponding final underflow solids concentrations on the concentration axis, are shown in Figure 7. If the solids flux curve is accurate, then the underflow operating lines should be tangent to the flux curve.
To improve the flow properties of a particular coal, an investigation was made into the possibilities for modifying the circuit of a coal preparation plant to reduce the proportion of a higher ash ultrafines fraction in the filter cake by desliming the flotation feed and sending the slimes.

Figure 8 shows the flux curves determined by the Coe and Clevenger and Kynch methods, and the flux curve determined from the concentration profile of a semi-continuous test at the same feed flux (0.083 kg/m²-s) used in the continuous tests. There is excellent agreement between the continuous results and the flux curve determined by the concentration profile method. It is therefore concluded that the solids flux curve model is valid if the solids flux curve is determined at the same feed flux used in the continuous sedimentation test. This is a significant limitation of the model. Because the Kynch flux curve corresponded to a high feed flux, poor agreement was obtained. The Coe and Clevenger method resulted in a totally invalid flux curve.

An important implication of these results relates to thickener design and the determination of settling area. If the design underflow concentration is 300 kg/m³ for a thickener which is to be fed at a volumetric rate of 4 m³/s and at a feed concentration of 5 kg/m³, a significant difference in the calculated vessel diameters using the three flux curve methods is obtained as shown in Table 1. The Coe and Clevenger, and Kynch methods would result in settling areas that are too small and the steady state underflow concentrations would only be 150 and 250 kg/m³ respectively, according to the flux curves in Figure 5. A discussion of reasons for this can be found elsewhere (Galvin and Waters, 1987).

CASE STUDY

To improve the flow properties of a particular coal, an investigation was made into the possibilities for modifying the circuit of a coal preparation plant to reduce the proportion of a higher ash ultrafines fraction in the filter cake by desliming the flotation feed and sending the slimes.
Design Conditions: $Q_F = 4m^3/s$; $C_F = 5kg/m^3$; $C_U = 300kg/m^3$

<table>
<thead>
<tr>
<th>Method</th>
<th>Calculated Diameter (m)</th>
</tr>
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<tbody>
<tr>
<td>Coe and Clevenger</td>
<td>12</td>
</tr>
<tr>
<td>Kynch</td>
<td>18</td>
</tr>
<tr>
<td>Concentration profile (this study)</td>
<td>26</td>
</tr>
</tbody>
</table>

TABLE 1. Vessel diameters calculated using the three flux curve methods

...to the thickener. Because the thickener was already heavily loaded, the effect of additional feed on the thickener performance was a major concern. The aim of the study was therefore to indicate what level of additional solids loading the thickener could tolerate.

In order to assess the impact of the additional solids loading created by desliming on the thickener operation, sedimentation tests were carried out on samples of tailings and simulated deslimed flotation feed mixed with tailings. Three cases were simulated, based on desliming at 0.063mm.

**Proposed flowsheets**

(i) **Current Operation**

Figure 9 shows the existing process flowsheet for fine coal treatment at the washery. The solids loading to the flotation cells was 160 tph. The thickener solids feed loading was estimated to be 48 tph. It should be noted that these values were already higher than design values.
(ii) Desliming at 0.063mm

The proposed modification to include desliming cyclones is shown in Figure 10. Sizing analysis showed that 35% of the flotation feed was less than 0.063mm. All of this material would be sent to the thickener. As a result, a higher flotation yield of about 85% was expected. The resultant thickener solids feed loading would then be 72 tph. This thickener feed effectively consisted of 48 tph of existing thickener tailings from flotation cells and 24 tph of the fine sized existing flotation product.

(iii) Addition of degritting cyclones

The proposal to include degritting cyclones to alleviate the additional thickener feed loading in Case (ii) is shown in Figure 11. The cyclones would remove the coarse tailings from the thickener feed. This material would then be combined with the refuse from the coarse coal treatment section of the plant. It was also assumed that all of the coarse tailings would report to the cyclone underflow. Hence a best case scenario was assumed. Although no additional solids are assumed to report to the thickener, the overflow from the degritting cyclones would report to the thickener, contributing a substantial volumetric flow which could have an effect on flocculant requirements to ensure adequate clarity of the plant process water (thickener overflow). The thickener loading would then be 56 tph, effectively consisting of 32 tph of fine sized tailings and 24 tph of existing fine sized flotation product.

Experimental

Sedimentation tests were performed to assess the effect of load changes on the thickener using the techniques previously described. The proposed flowsheet modifications outlined in section 5.1 not only lead to an increase in thickener feed loading but also to a much finer thickener feed size distribution.

An initial experiment was carried out on a sample of existing thickener feed. In another two tests, thickener feed was combined with mass proportions of 23% and 53% of minus 0.063mm flotation feed material. The minus 0.063mm material was obtained in the laboratory by wet screening a sample of the flotation feed, which was sampled from the flotation feed sump after switching off the reagent addition.

Results and Discussion of case study analysis

The effect of each of the loading changes on the performance of the thickener was calculated. For the existing condition (Case i), it was found that a solids loading of 48 tph should result in a steady state underflow concentration of 300 kg/m$^3$ (approximately 26% solids). This value is reasonable and could be slightly increased if a higher flocculant dosage is used (Galvin and Waters, 1985). It also agreed well with measured values of thickener underflow obtained during the period when the sample was taken.

Table 2 shows the thickener diameters required to achieve a steady state underflow of 26% solids for the three cases. The diameter of the existing thickener was 45m. It is evident that Cases (ii) and (iii) require a vessel diameter significantly greater than the existing thickener, with calculated diameters of 68m and 65m respectively. Consequently, the use of degritting cyclones would not satisfactorily help to alleviate the increased loading in Case (ii). The similarity between the results for Cases (ii) and (iii) is due to the dependence of the settling velocity on the proportion

<table>
<thead>
<tr>
<th>Case</th>
<th>Vessel Diameter (m)</th>
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<tbody>
<tr>
<td>(i)</td>
<td>45</td>
</tr>
<tr>
<td>(ii)</td>
<td>68</td>
</tr>
<tr>
<td>(iii)</td>
<td>65</td>
</tr>
</tbody>
</table>

Table 2. Required vessel diameter to obtain an underflow of 26% solids.
of minus 0.063mm material. It was therefore concluded that it would not be possible for the thickener to operate at its present efficiency (underflow concentration) if either proposal was incorporated.

The effect of proposed modifications to the plant on the steady state underflow percent solids of the existing thickener is shown in Table 3. Again, a higher flocculant dosage could lead to a slightly higher predicted underflow solids content. However, it is evident that a 25% reduction in underflow solids is likely for Cases (ii) and (iii) with approximately 19% and 20% solids respectively.

The predicted volumetric underflow rates at steady state are probably the best guide to the detrimental effect of the proposed changes, and these are also shown in Table 3 for the three cases. Case (ii) would result in the volumetric underflow rate increasing by about 115%. The Case (iii) proposal would help to alleviate the problems of Case (ii) to a large extent. However, even this proposal results in a 60% increase in the volumetric rate of tailings disposal. Desliming at a lower size fraction, e.g. 0.038mm would not significantly alter the situation, because of the small amount of floatation feed in this size fraction (typically 5%), and any change in feed flux would be counteracted by the lower settling rates of this material. Hence the net effect is that desliming at 0.038mm would have a similar effect on thickener operation as desliming at 0.063mm.

<table>
<thead>
<tr>
<th>Case</th>
<th>Solids Content (%)</th>
<th>Volumetric Rate (m³/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>26</td>
<td>160</td>
</tr>
<tr>
<td>(ii)</td>
<td>19</td>
<td>343</td>
</tr>
<tr>
<td>(iii)</td>
<td>20</td>
<td>254</td>
</tr>
</tbody>
</table>

Table 3. Predicted steady state underflow percent solids and underflow rates for the thickener.

CONCLUSIONS

An experimental procedure for determining the solids flux curve from the concentration profile of the bed during semi-continuous sedimentation tests was discussed. The advantage of this method was that the solids flux curve could be determined at low and high feed fluxes. The solids flux curve was found to be dependent on the solids feed flux to the system. At very high feed fluxes, higher than used in practice, the resultant flux curve approached that obtained by the Kynch method. This was in agreement with the observation that the effective feed fluxes of the batch tests used in the Kynch method were always relatively high. The flux curve determined by the Kynch method was therefore invalid for systems at the much lower feed fluxes typical of industrial practice. The Coe and Cleveenger method was found to be invalid for all feed fluxes. The solids flux curve model was shown to be accurate when the flux curve was determined at the same feed flux used in continuous sedimentation. The Kynch method and in particular the Coe and Cleveenger method underestimated thickener area.

Results of a washery modification study showed that the proposed desliming changes would adversely affect the performance of the thickener and would result in a significant increase in the volume of tailings for disposal. The use of degritting cyclones would be of little value in overcoming these problems.

REFERENCES


NOMENCLATURE

C solids concentration (kg/m³)
Cf solids concentration, feed (kg/m³)
Cmax maximum solids concentration (kg/m³)
Cu underflow solids concentration (kg/m³)
GF effective feed flux (kg/m²-s)
Gp propagation solids flux component (kg/m²-s)
Gs settling solids flux component (kg/m²-s)
Gu underflow solids flux (kg/m²-s)
H height of sediment (m)
QF volumetric flow rate (m³/s)
T sedimentation time (s)
Vp propagation velocity (m/s)
Vs settling velocity (m/s)
Vu underflow velocity (m/s)

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