conventional grinding.

Grinding Circuits

There is no such thing as a "best" configuration of equipment for grinding. The circuit which works well on one ore may be totally unsatisfactory for another ore. The general trend in industry today is flowsheet simplification, using larger parallel circuits instead of multiple-line circuits with smaller mills. This change has been brought about by the savings of both capital and operating costs that are associated with larger equipment.

In any grinding installation a choice must be made between wet or dry grinding. Most mineral processing facilities utilize wet grinding, which has the following advantages over dry grinding:

1) lower power consumption per ton of product
2) higher capacity per unit of mill volume
3) makes possible the use of wet screening or classification for close product control
4) elimination of dust problems
5) allows the use of simple handling and transport methods (i.e., pumps, pipes, launders, etc.)

A choice must also be made between open and closed circuit operation. While both have their own advantages, the final decision is usually based on factors such as mill selection, product requirements, tonnage requirements and ore characteristics.

Open Circuit Grinding

Open circuit grinding consists of one or more grinding mills, either parallel or in series, that discharges a final ground product without classification equipment and no return of coarse discharge back to the mill. Some very simplistic examples of open circuit grinding are (Fig. 8):

1) Rod mill
2) Ball Mill
3) Rod mill, ball mill combination

Not all ores can be ground in an open circuit type of arrangement. Some conditions which do favor open circuit grinding are:

1) small reduction ratios
2) reduction of particles to a coarse, natural grain size
3) recirculation of cleaner flotation middlings for regrinding
4) a non-critical size distribution of the final ground product

Some advantages of operating in the open circuit mode vs closed circuit are:

1) minimum equipment requirements
2) high pulp density discharge
3) simplicity of operation

Closed Circuit Grinding

Closed circuit grinding consists of one or more mills discharging ground product to classifiers which in turn return the coarse product from the size separation back to the mill for further grinding. In this circuit, grinding efficiency is very dependent upon the size separation effected so care should be exercised in selecting the type and size of classifier used to close the system.

This type of grinding is the most common circuit found in mineral processing facilities, mainly because a lot of ores and product requirements are not suitable for open circuit grinding. Some advantages presented by grinding in closed circuit are:

1) This arrangement usually results in higher mill capacity and lower power consumption per ton of product.

2) It eliminates overgrinding by removing fines early.

3) It avoids coarse material in the final ground product by returning this material to the mill.

Although closed circuit grinding offers many choices for arrangement of the equipment as well as combinations of equipment, some of the more common circuits are (Fig. 9):

1) Rod mill/Classifier

2) Ball mill/Classifier

3) Rod mill/Ball mill/Classifier

4) Rod mill/Classifier/Ball mill/Classifier

CONTROL OF CIRCUITS

The importance of the grinding circuit to overall production in any facility should be obvious by now. Because of the responsibilities assigned to grinding it becomes essential that a grinding mill accepts a certain required tonnage of ore per day while yielding a product that is of a known and controllable particle size. This leads to the conclusion that close control over the grinding circuit is extremely important.

Variables

There are many factors which can contribute to fluctuations in performance of a mill, but some of the most common found in industrial practice are:

1) changes in ore taken from different parts of the mine
2) changes in crusher settings
3) wear in the crushers
4) screen damage in the crusher circuit

These are a few things that operators should look for when changes in mill performance are noticed. Stockpiling of ore ahead of the mill can aid in smoothing out some of the fluctuations although it must be stored in such a manner that no segregation occurs.

In operating a grinding circuit, like any other unit process, variables key to the performance must be dealt with. Some of the principle variables affecting control of grinding mills are (Fig. 10):

1) changes in feed rate

2) changes in circulating loads

3) size distribution of the ore

4) hardness of the ore

5) rate of water addition

6) interruptions in operation (i.e., stoppage for replacement of grinding medium, clearing of choked classifiers, etc.)

Of these variables, feed size distribution and ore hardness are the two most significant because they can affect the actual grinding mechanics.

The only two variables that can be independently controlled by the operator are feed rate and water addition. All other variables depend on and respond to changes in these two items, thus these are used to control the grind. Too dilute a pulp, which can arise from a decrease in feed rate or an increase in water addition, will decrease retention time in the mill, resulting in a coarser product and increased wear. Under opposite circumstances, (high feed rate, decrease in water) pulp densities can become too high. This can lead to an increase in retention time resulting in a finer product. However, in most situations concerning too thick of a pulp the pulp viscosity will become so great that the grinding medium may begin to float and actual grinding will cease, and if not corrected, the mill will eventually choke.

Automatic Control

Since grinding is extremely energy intensive and the product from grinding affects subsequent processes, close control is essential. It is now generally accepted that some form of automatic control is required.

In implementing instrumentation for process control the objective must first be clearly defined. Some examples of these objectives relating to grinding circuits may be:

1) to maintain a constant product size at maximum thruput

2) to maintain a constant feed rate within a limited range of product size

3) To maximize production per unit time in conjunction with downstream circuit performance.

Instrumentation to control process variables is receiving more and more attention lately. Some general methodology used to control some of the more critical variables are as follows:

1) Feed Rate - Variable speed belt feeders, audio controllers, which are sound sensitive devices that will measure feedrates according to noise levels in the mill.

2) Medium Charge - Can be controlled thru continuous monitoring of mill power consumption.

3) Flowrates and Densities - Can be measured with magnetic flowmeters and nuclear density gauges.

4) Sump Levels - Controlled by bubble tubes, capacitance type devices or other electronic devices.

5) Product Particle Size - Can be measured directly by the

use of on-line monitors or inferred thru mathematical modelling.

This is a very general run down of types or methods of control although specific devices are readily available. The best choice of controllers will depend on the characteristics of individual circuits.

CIRCUIT DESIGN

There are many methods available to size grinding mills and to design grinding circuits. The majority of these methods have been around for some time and are largely a matter of using data from laboratory tests, applying empirical equations to this data, and fine tuning with correction factors that are often based on accumulated experience. Different manufacturers of grinding mills use different methods and it is at times difficult to check on the validity of the sizing estimates when estimates from different sources are widely divergent.

This part of the discussion will focus on two methods of sizing mills. The first will be the Bond method, which has been considered by most as the standard for a long time. The second will be a computer simulation developed at Penn State University which utilizes the relatively new concept (as applied to grinding) of the size-mass balance theory. These two methods are radically different in their approach and exemplify the differences between the older (empirical relationships) and more recent theories.

Bond Method

The Bond method of sizing grinding mills has enjoyed wide acceptance in the mineral industry. It has two major engineering advantages, which are its simplicity and the fact that experience has shown that it does work for many (but not all) circumstances.

The Bond method can be broken down into 5 major components:

1) Standardized grindability tests on the material

2) An empirical equation which is designed to convert the test results to observed results in an 8 ft. diameter wet overflow mill operating in closed circuit with a circulating load of 250%.

3) An empirical equation to allow for overall reduction ratios in closed circuit operation

4) Scale-up relationships to predict results for larger mills

5) A series of empirical correction factors, based on experience, to allow for milling conditions

By looking at each of these 5 steps in detail, a good idea of the methodology of the Bond method can be obtained (Fig. 11).

Step 1: Grindability Test

The Bond method does utilize a standard laboratory test procedure to supply the basic data. The feed for this test is crushed to 100% -6 mesh and approximately 80% finer than 2000 micron. The ore charge to be used is specified at 700 cm^3, the weight of which is determined by packing the material in a graduated cylinder. This charge is then placed dry in a 12 X 12 in. mill rotating at 70 rpm (85% of critical) with a specified ball charge of:

43 balls - 1.75 in.

67 balls - 1.17 in.

10 balls - 1.0 in.

71 balls - 0.75 in.

95 balls - 0.61 in.

This ball charge should equal a total of 20.1 kilograms.

This charge, with the above conditions, is ground for a set number of revolutions, removed from the mill and sieved at the desired screen size. The undersize is removed and weighed while the oversize is returned to the mill with fresh feed added to reestablish the initial material charge weight. The new charge is reground and the process is continued until a constant 350% circulating load is achieved. At this point the net grams of screen undersize produced per mill revolution is determined (G_{bp}).

Step 2: Conversion to an 8 ft. Mill

From the results of the grindability test (Step 1) a work index (Wi) that is unique to the ore being tested can be determined using the following equation:

$$\text{eq.1} \qquad Wi = (1.10)(4.45) \Big| \; (P_1{}^{0.23})(G_{bp})^{0.82} \left[\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right] \text{kwh/t}$$

Where P_i = opening of test sieve in microns

P_{80} = 80% passing size of the test product (microns)

F_{80} = 80% passing size of the test feed (microns)

Step 3: Allowance for Size Reduction Ratio

Equation 1 calculates the work index which is used to calculate the specific grinding energy from the following equation:

eq.2 $$E = Wi\left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right), \text{ kwh/t}$$

Where P_{80} = 80% passing size of circuit product

F_{80} = 80% passing size of circuit feed

Step 4: Scale-up to Larger Mills

To convert to a larger mill than one having an 8 ft. diameter, the value of the work index (Wi) must be scaled accordingly using the following equation:

eq. 3 $$Wi = (W_i)_{test}\,(2.44/D)^{0.2}, \quad D<3.81\text{m (12.5 ft.)}$$

$$Wi = (W_i)_{test}\,(0.914), \quad D>3.81\text{m (12.5 ft.)}$$

This allows a new specific energy to be calculated for larger mills from eq. 2.

Using the correct specific energy value (E) for the desired mill diameter, the required shaft power for a desired feed rate can be determined from:

eq. 4 $$Mp = QE, \text{ kw}$$

Where MP = shaft power

Q = feed rate

The shaft power, Mp, can be related to mill power as a function of mill dimension using:

eq. 5 $$Mp = 7.33J\,\phi_c\,(1-0.937J)\left(1 - \frac{0.1}{2^{9-10\phi_c}}\right)\left(P_b\,LD^{2.3}\right)$$

Where: P_b = density of grinding medium (t/M^3)

ϕ_c = fraction of critical speed

J = formal ball loading

L = mill length

D = mill diameter

All values in eq. 5 have either been determined or can be specified, therefore the necessary values of mill length necessary to give this power can be determined.

Step 5: Corrections for Milling Conditions

Correction factors for specific mill conditions can be calculated or taken from tables to adjust the work index, Wi. This may not be necessary, although equations and/or values are available for conditions such as allowances for oversized feed, fineness of grind, low reduction ratios and converting from closed circuit to open circuit.

PSU GRINDING CIRCUIT SIMULATOR

A simulation of any physical process is a mathematical model which behaves on computation in a manner identical to that of the real process. Generally, a simulation is only an approximation to the real behavior, especially for a process as complicated as milling and the mathematical models can be more or less complex depending on how closely one wishes to simulate the real situation.

The Penn State University Grinding Circuit Simulator utilizes the size-mass rate balance concept in simulating a grinding circuit. This method uses two breakage concepts as its basis (Fig. 12).

1) Specific Rate of Breakage - fractional rate of breakage per unit of time per mass of the size particles are broken from. This is expressed mathematically by eq. 1 shown in Fig. 12.

2) Primary Breakage Distribution Function - Description of Function - Material breaks and the fragments produced are mixed back in with the general mass of powder in the mill. If this distribution of fragments can be measured before any of the fragments are reselected for further breakage, the result is the Primary Breakage Distribution. The mathematical expression for this function is also shown in Fig. 12.

These two principles sum up to define the overall basis of the program as the rate of some size i particles leaving the mill is equal to the rate of size i particles entering the mill plus the rate of production of size i particles from breakage of larger particles within the mill minus the rate of disappearance by breakage of size i particles within the mill.

The details of the program itself (i.e., flowsheet, computations, etc.) and the laboratory procedures required to generate much of the input data are much too complex to be included within this discussion. However, once the necessary data has been compiled, the program becomes very "user friendly," allowing easy input of the data as well as changing of conditions.

The program allows the selection of 5 basic mill circuits, which is an indication of the flexibility of the system. These circuits are (Fig. 14):

1) Open Circuit
2) Normal Closed Circuit
3) Reverse Closed Circuit
4) Open Circuit/Scalped Feed
5) Combined Closed Circuit

This is the first choice offered by the program. Once the proper circuit has been selected, it is shown on the monitor for verification. Following this selection, the program asks for the input data.

The required input data for simulation of the desired circuit includes feed size distribution, classifier size selectivity values, mill selection values, breakage values and time of grind. To be more specific, examples of required input are:

- No. of size intervals desired in evaluation
- No. of different ball diameters in both simulated and test mill
- Diameter of both simulated and test mill
- Maximum feed size
- Scale-up constants which correspond to effects of mill and

ball diameter on specific breakage rates and large particle size corrections

- Operating conditions such as media charges, powder charges, and critical speeds utilized in both the simulated and the test mills
- Whether grinding is wet or dry
- Parameters for specific breakage rate calculations (determined from laboratory testing)
- Parameters for primary breakage distribution calculations (determined from laboratory testing) Four values are required for each ball diameter
- Feed size distribution
- Classifier data such as d_{50} values, sharpness index and apparent by-pass
- Time of grind

After all the data has been entered, the computer will calculate the specific rate of breakage, a residence time distribution for the simulated mill and the primary breakage function. The output generated will use these values to predict the performance of the simulated mill under the desired conditions. The predicted performance includes flowrates of each stream in the simulated circuit, predicted size distributions for each stream, classifier performance based on predicated mill discharge and % circulating loads, if applicable.

This simulation can be extremely helpful to operators of existing mills because it will automatically predict the variation in mill output for different feed makeups, changed mill conditions and changed classifier parameters. Alternatively, from a design viewpoint, the effects of these same parameters on mill size, for a given feed rate, can also be predicted.

In contrast to other methods of design, this technique sizes the mill based on the rates of breakage occurring in the mill. The energy input necessary to run the mill then follows from the size and mechanics necessary to turn the device. This does seem to be a far more logical approach rather than estimating the energy first followed by making the mill big enough to consume this amount of energy.

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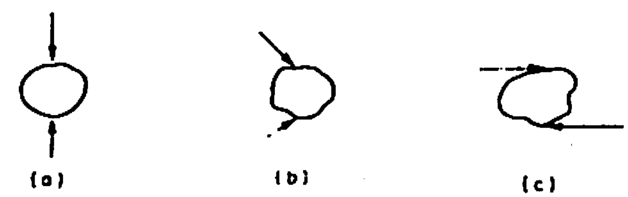
Fig. 1. Breakage Mechanisms



Mechanisms of breakage: (a) impact or compression, (b) chipping, (c) abrasion.

Fig. 2. Grinding Laws

Rittinger's Law

$$E_R = C_R \, (1/d_2 - 1/d_1)$$

Where:
E_R = Specific Energy
C_R = Constant
d_2 = Product Size
d_1 = Feed Size

Kirk's Law

$$E_K = C_K \log \, (d_2/d_1)$$

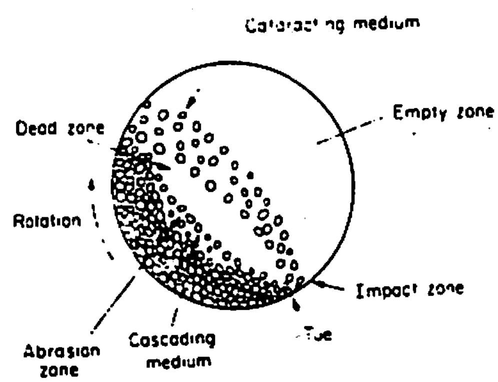
Where:
E_K = Specific Energy
C_K = Constant
d_2 = Product Size
d_1 = Feed Size

Bond's Law

$$W = \frac{10 \, Wi}{\sqrt{P}} - \frac{10 \, Wi}{\sqrt{F}}$$

Where:
W = Work Input
Wi = Work Index
P = 80% Passing Size of Product
F = 80% Passing Size of Feed

Fig. 3. Patterns of Tumbling

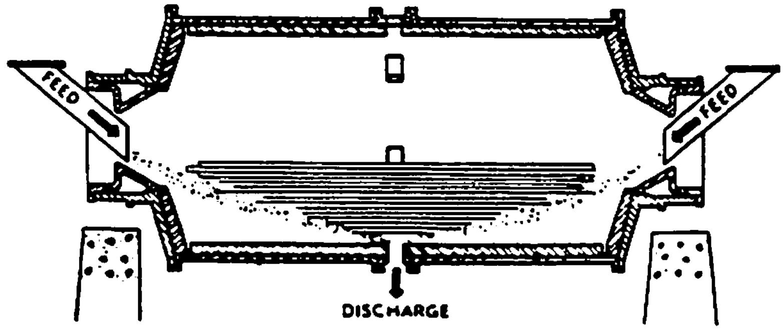


Motion of charge in a tumbling mill.

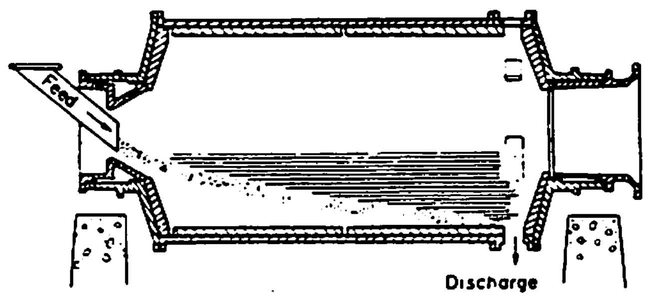
$$Nc = \frac{42.3}{\sqrt{D-d}}$$

Where Nc = Critical Speed
D = Mill Diameter
d = Charge Diameter

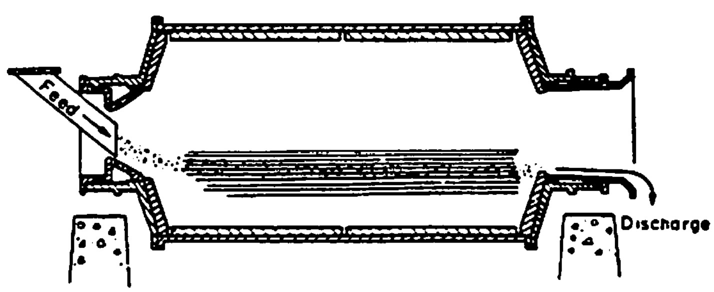
Fig. 4. Classification of Rod Mills



Central peripheral discharge mill.



End peripheral discharge mill.



Overflow mill.

Fig. 5. Determination of Media Charge

$$M = \sqrt{\frac{FWi}{KC_s}\sqrt{\frac{S}{\sqrt{D}}}}$$

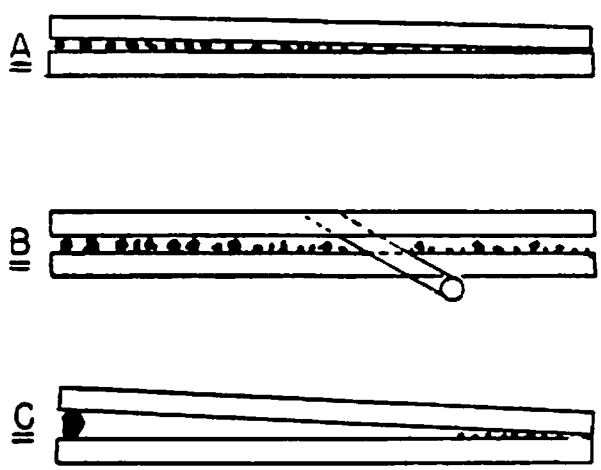
M = diameter of topsize media in inches
F = size in microns of the screen opening which 80% of the feed will pass
Wi = work index
C_s = percent of critical speed
S = specific gravity of feed
D = diameter of mill inside liners in feet
K = constant, the value of which is 200 for ball mills and 300 for rod mills

TABLE OF INITIAL CHARGES

Rod Diameter (Inches)	Rod Size Distribution For Startup Charge (% Weight)				
	5	4½	4	3½	3
5	19				
4½	17	21			
4	16	19	24		
3½	15	18	23	28	
3	13	17	20	22	30
2½	10	15	18	20	28
2	10	10	15	17	23
1½				15	21

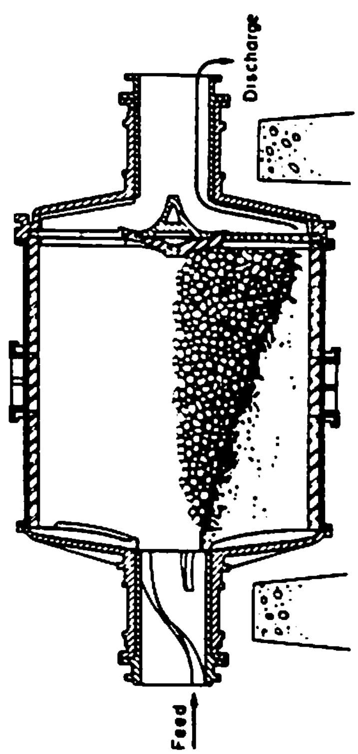
Ball Diameter (Inches)	Ball Size Distribution For Startup Charge (% Weight)											
	5	4½	4	3½	3	2½	2	1½	1¼	1	¾	½
5	17.0											
4½	25.0	16.0										
4	20.0	30.0	20.0									
3½	15.0	21.5	32.0	22.0								
3	10.0	14.0	21.0	35.0	26.0							
2½	6.4	9.1	12.5	19.0	36.0	[illegible]						
2	3.8	5.4	8.6	14.6	22.0	39.0	38.0					
1½	2.8	2.4	3.4	5.3	9.2	16.5	35.0	28.0				
1¼		1.6	1.2	2.0	[illegible]	6.1	13.0	36.0	30.0			
1			1.3	1.0	1.7	2.9	6.4	16.0	32.0	22.0		
¾				1.1	1.3	1.4	3.1	8.0	14.5	52.0	24.0	
½						2.1	4.5	12.0	23.5	26.0	76.0	100

Fig. 6. Grinding Action in Rod Mills



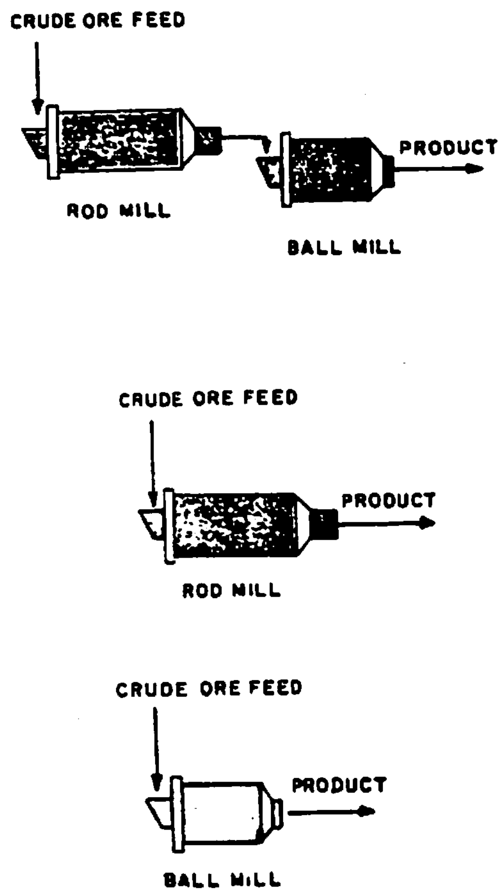
Particle Distribution in the Rod Mill

Fig. 7. Classification of Ball Mills



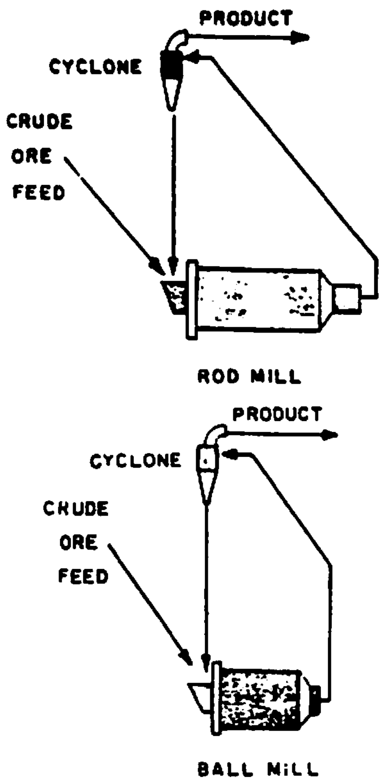
Grate discharge mill.

Fig. 8. Open Circuit Grinding

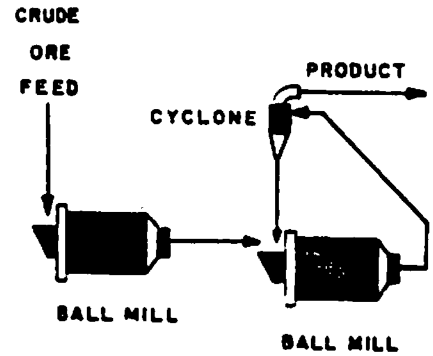


Open-Circuit Grinding

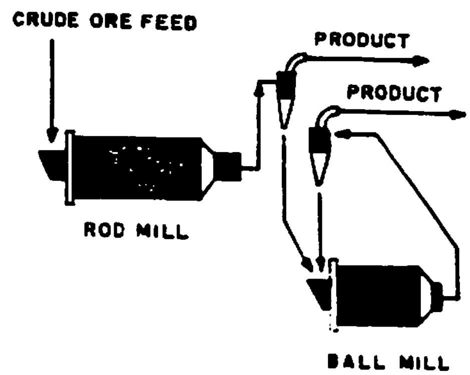
Fig. 9. Closed Circuit Grinding



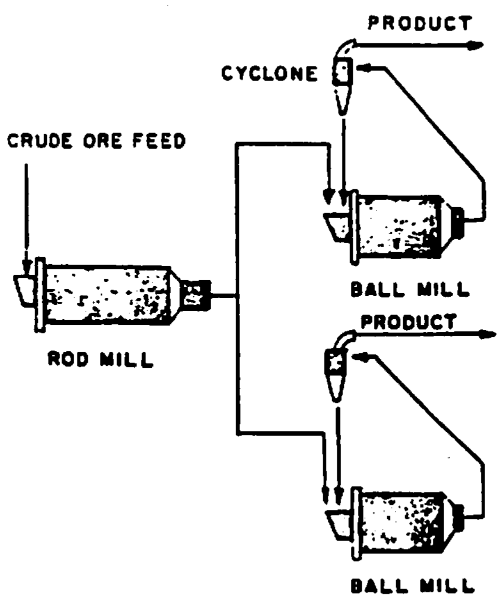
Single-Stage
Closed-Circuit Grinding



Ball Mill-Ball Mill,
Closed-Circuit Grinding

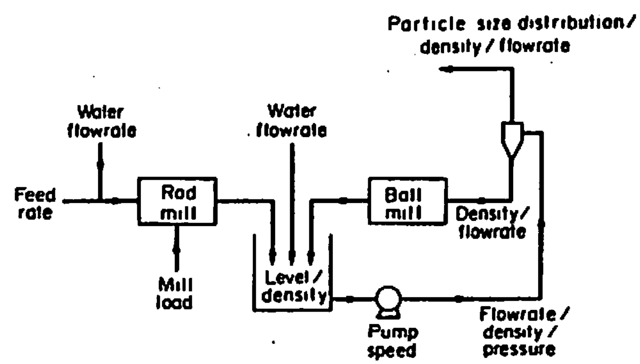


Two Stages Grinding
and Classification



Rod Mill-Ball Mill,
Closed-Circuit Grinding

Fig. 10. Grinding Circuit Variables



Grinding circuit control variables.

Fig. 11. Bond's Equations

eq.1 $$Wi = (1.10)(4.45)\Bigg|\left((P_1{}^{0.23})(G_{bp})^{0.82}\left[\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}}\right]\right)$$

Where: Wi = Work Index
G_{bp} = Net gms per revolution
P = 80% Passing Size of test product
F = 80% passing size of test feed

eq.2 $$E = Wi\left(\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}}\right)$$

Where P = 80% passing size of circuit product
F = 80% passing size of circuit feed

eq. 3 $$Wi = (W_i)\ \text{test}\ (2.44/D)^{0.2},\quad D<3.81\text{m (12.5 ft.)}$$
$$Wi = (W_i)\ \text{test}\ (0.914),\quad D>3.81\text{m (12.5 ft.)}$$

eq. 4 $$Mp = QE,$$ Where: Mp = shaft power
Q = feed rate

eq. 5 $$Mp = 7.33J\ \phi_c\ (1-0.937J)\left(1 - \frac{0.1}{2^{9}-10\phi_c}\right)\left(P_b\ LD^{2.3}\right)$$

Where: P_b = density of grinding medium (t/M^3)
ϕ_c = fraction of critical speed
J = formal ball loading
L = mill length
D = mill diameter

Fig. 12. Equations for Size Mass Balance Method

SPECIFIC RATE OF BREAKAGE FUNCTION - SCALE UP FORMULAS

$$S_i(d) = a_T (x_i/x_o)^{\alpha} \left(\frac{1}{1+(x_i/C_1 \mu_T)^{\Lambda}}\right) C_2 C_3 C_4 C_5$$

where

$$C_1 = (D/D_T)^{N_2} (d/d_T)^{N_3}$$

$$C_2 = (d_T/d)^{N_0} \left[\frac{1+(d^*/d_T)^{\lambda^*}}{1+(d^*/d)^{\lambda^*}}\right]$$

$$C_3 = \begin{cases} (D/D_T)^{N_1} & D \le 3.8\text{m} \\ \left(\frac{3.8}{D_T}\right)^{N_1} \left(\frac{D}{3.8}\right)^{N_1-\Delta} & D \ge 3.8\text{m} \end{cases}$$

$$C_4 = \left(\frac{1+6.6J_T^{2.3}}{1+6.6J^{2.3}}\right) \exp[-c(U-U_T)]$$

$$C_5 = \left(\frac{\phi_c - 0.1}{\phi_{cT} - 0.1}\right) \left(\frac{1+\exp[15.7(\phi_{cT}-0.94)]}{1+\exp[15.7(\phi_c-0.94)]}\right)$$

BREAKAGE DISTRIBUTION FUNCTION

$$B_{i,1} = \Phi_1 (x_{i-1})^{\gamma} + (1-\Phi_1)(x_{i-1}/x_1)^{\beta} \qquad n \ge i \ge 1$$

$$\Phi_j = \Phi_1 (x_j/x_1)^{-\delta} \qquad \delta > 0, \Phi_j \le 1$$

FIGURE 13 - NOMENCLATURE UTILIZED IN PSU SIMULATOR

NOMENCLATURE

A	$(x_i/u_T)/[(D/D_T)^{N2}(d_{max}/d_T)^2]$ (-)
b	d_{min}/d_{max} (-)
c	constant in Eq. 1 which defines the effect of hold-up on breakage rates (-)
d	ball diameter (L)
d_T	ball diameter used in laboratory test mill (L)
d_{max}	make-up ball diameter (L)
d_{min}	minimum diameter of ball in equilibrium ball charge (L)
D	mean internal mill diameter (L)
D_T	D for the laboratory test mill (L)
J	volume fraction of mill filled by ball bed (-)
J_T	J for the laboratory test (-)
m_k	weight fraction of ball charge of ball size class k (-)
N_o	exponent in Eq. 1, exponent of effect of ball diameter on specific breakage rates (-)
N_1	exponent in Eq. 1, exponent of effect of mill diameter on specific breakage rates (-)
N_2	exponent in Eq. 1, exponent of effect of mill diameter on large particle size correction (-)
S_i	specific rate of breakage of particle size interval i, fraction per unit time (1/T)
U	fractional interstitial filling of voids of ball bed by particles (-)
U_T	U for the laboratory test mill (-)

x_i upper size of particle size interval i (L)

x_o standard size (L)

X d/d_{max} (L)

a_T pre-exponential factor in Eq. 1, proportionality factor for specific breakage rates (1/T)

α exponential factor in Eq. 1, defines variation of specific breakage rates with particle size (-)

Λ factor in Eq. 1, defines the decrease in specific breakage rates of large particles (-)

μ_T the particle size for which the specific breakage rate is one-half that expected from $S_i = a(x_i/x_o)^\alpha$, in the laboratory test mill with ball diameter d_T (L)

ϕ_c rotational speed of mill as a fraction of critical speed (-)

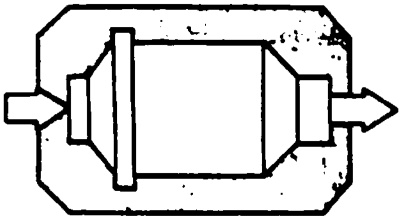
ϕ_{cT} ϕ_c for the laboratory test mill (-)

Φ_j is the intercept of the breakage distribution function - eqn (2)

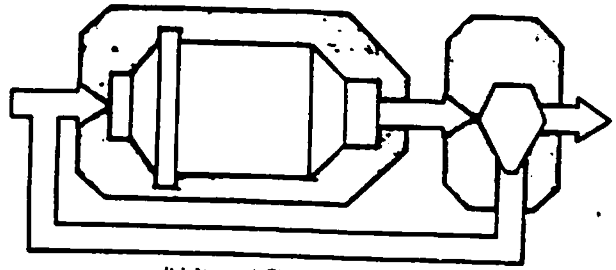
γ is the slope of the lower side of the breakage distributions function - eqn (2)

β is the slope of the upper side of the breakage distribution function

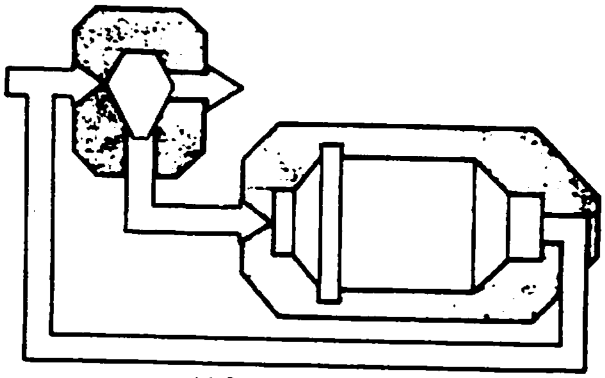
Fig. 14. Grinding Circuits in PSU Grinding Circuit Simulator



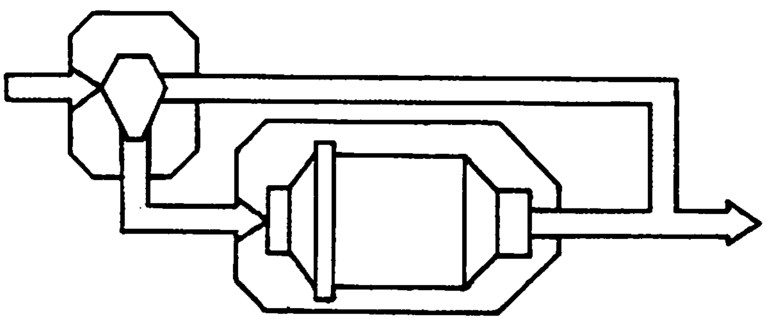
(a) Open Circuit



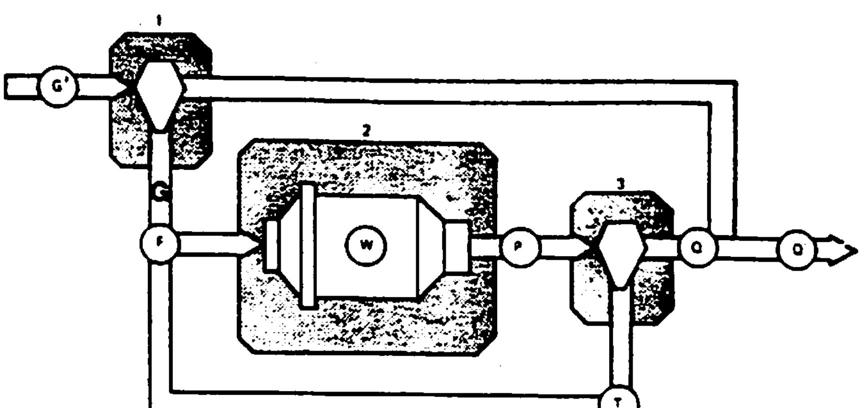
(b) Normal Closed Circuit



(c) Reverse Closed Circuit



(d) Open Circuit With Scalped Feed



General single-stage mill circuit with pre- and post-classifier: the combined closed circuit.