APPENDIX A.

ON THE DRAWING OF CRYSTAL FIGURES

In the representation of crystals by figures it is customary to draw their edges as if they were projected upon some definite plane. Two sorts of projection are used; the *orthographic* in which the lines of projection fall at right angles and the *clinographic* where they fall at oblique angles upon the plane of projection. The second of these projections is the more important, and must be treated here in some detail. Two points are to be noted in regard to it. In the first place, in the drawings of crystals the point of view is supposed to be at an infinite distance, and it follows from this that all lines which are parallel on the crystal appear *parallel* in the drawing.

In the second place, in all ordinary cases, it is the complete ideal crystal which is represented, that is, the crystal with its full geometrical symmetry as explained on pp. 10 to 13 (cf. note on p. 13).

In general, drawings of crystals are made, either by constructing the figure upon a projection of its crystal axes, using the intercepts of the different faces upon the axes in order to determine the directions of the edges or by constructing the figure from the gnomonic (or stereographic) projection of the crystal forms. Both of these methods have their advantages and disadvantages. By drawing the crystal figure by the aid of a projection of its crystal axes the symmetry of the crystal and the relations of its faces to the axes are emphasized. In many cases, however, drawing from a projection of the poles of the crystal faces is simpler and takes less time. The student should be able to use both methods and consequently both are described below.

DRAWING OF CRYSTALS UPON PROJECTIONS OF THEIR CRYSTAL AXES

Projection of the Axes

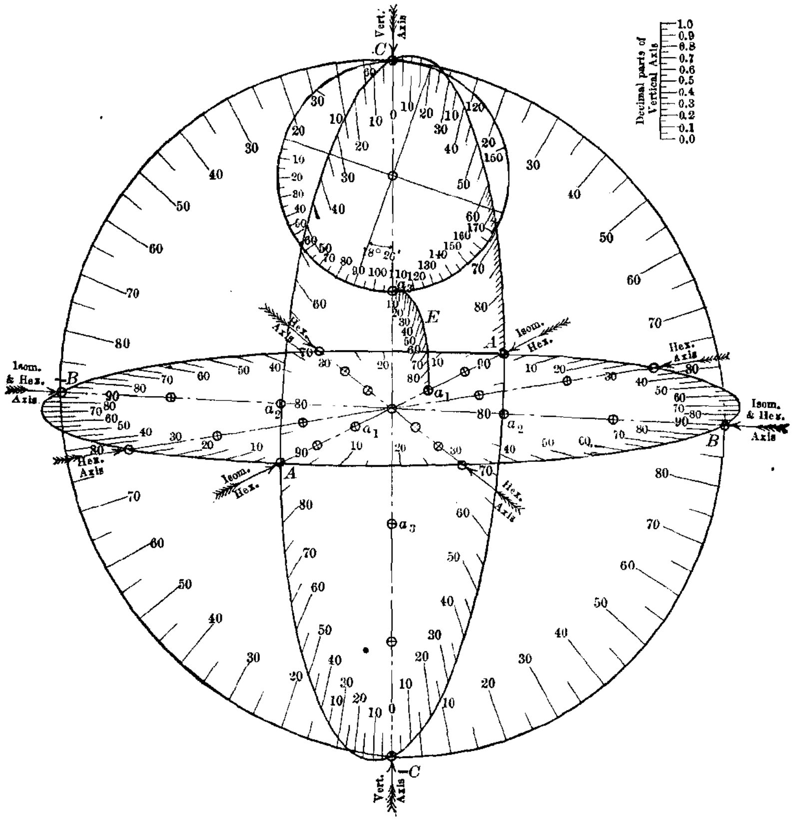
The projection of the particular axes required is obviously the first step in the process. These axes can be most easily obtained by making use of the Penfield Axial Protractor, illustrated in Fig. 1030.* The customary directions of the axes for the isometric, tetragonal, orthorhombic and hexagonal systems are given on the protractor and it is a simple matter, as explained below, to determine the directions of the inclined axes of the monoclinic and triclinic systems. Penfield drawing charts giving the projection of the isometric axes, which are easily modified for the tetragonal and orthorhombic systems, and of the hexagonal axes, (see Figs. 1031, 1032) are also quite convenient.

Isometric System. — The following explanation of the making of the projection of the isometric axes has been taken largely from Penfield's description.†

Figure 1033 will make clear the principles upon which the projection of the isometric axes are based. Figure 1033A is an orthographic projection (a *plan*, as seen from above) of a cube in two positions, one, *a b c d*, in what may be called normal position, the other, *A B C D*, after a revolution of 18° 26′ to the left about its vertical axis. The broken-dashed lines throughout represent the axes. Figure 1033B is likewise an orthographic projection of a cube in the position *A B C D* of A, when viewed from in front, the eye or point of vision being on a level with the crystal. In the position chosen, the apparent width of the side face *B C B′ C′* is one-third that of the front face *A B A′ B′*, this being dependent upon the angle of revolution 18° 26′, the tangent of which is equal to $\frac{1}{3}$. To construct the angle 18° 26′, draw a perpendicular at any point on the horizontal line, *X* − *Y*, figure 1033A as at *o*, make *op* equal one-third *Oo*, and join *O* and *p*. The next step in the construction is to change from orthographic to clinographic projection. In order to give crystal figures the appearance of solidity it is supposed that the eye or point

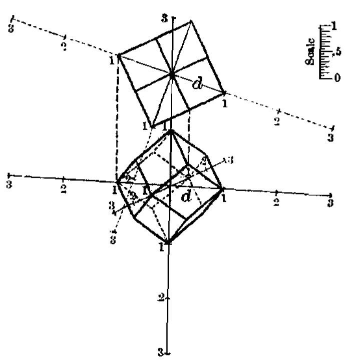
* The various Penfield crystal drawing apparatus may be obtained from the Mineralogical Laboratory of the Sheffield Scientific School of Yale University, New Haven, Conn.
† On Crystal Drawing; Am. J. Sc., **19**, 39, 1905.

1030



Protractor for plotting crystallographic axes; one-third natural size (after Penfield)

1031



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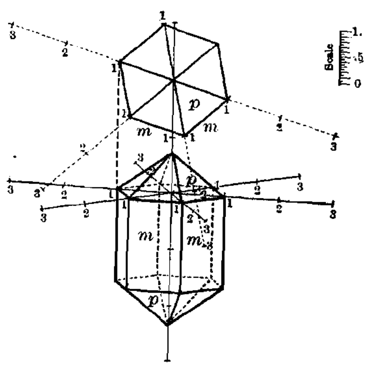
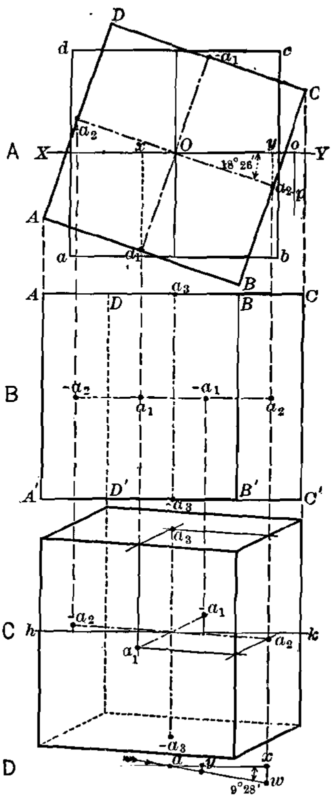


Scheme of the engraved axes of the isometric and hexagonal systems, one-sixth natural size (after Penfield)

of vision is raised, so that one looks down at an angle upon the crystal; thus, in the case under consideration, figure 1033C, the top face of the cube comes into view. The position of the crystal, however, is not changed, and the plane upon which the projection is made remains vertical. From A it may be seen that the positive ends of the axes a_1 and a_2 are forward of the line XY, the distances a_1x and $a_2\ y$ being as 3 : 1. In B it must be imagined, and by the aid of a model it may easily be seen, that the extremities of these same axes are to the front of an imaginary vertical plane (the projection of XY above) passing through the center of the crystal, the distance being the same as $a_1\ x$ and $a_2\ y$ of the plan. In D the distance ax is drawn the same length as a_1x of the plan, and the amount to which it is supposed that the eye is raised, indicated by the arrow, is such that a, instead of being projected horizontally to x, is projected at an inclination of 9° 28′ from the horizontal to w, the distance xw being one-sixth of ax; hence the angle 9° 28′ is such that its tangent is $\frac{1}{6}$. Looking down upon a solid at an angle, and still making the projection on a vertical plane, may be designated as *clinographic projection;* accordingly, to plot the axes of a cube in clinographic projection in conformity with figures A, B and D draw the horizontal construction line hk, figure C, and cross it by four perpendiculars in vertical alignment with the points a_1, $-a_1$ and a_2, $-a_2$ of figures A and B. Then determine the extremities of the first, a_1, $-a_1$ axis by laying off distances equal to xw of figure D, or one-sixth $a_1\ x$ of figure A, locating them below and above the horizontal line hk. The line a_1, $-a_1$ is thus the projection of the first, or front-to-back axis. In like manner determine the extremities of the second axis, a_2, $-a_2$, by laying off distances equal to one-third xw of figure D, or one-sixth a_2y of figure A, plotted below and above the line hk. The line a_2, $-a_2$ is thus the projection of the second, or right-to-left axis. It is important to keep in mind that in clinographic projection there is no foreshortening of vertical distances. In figure C the axis a_2, $-a_2$ is somewhat, and a_1, $-a_1$ much foreshortened, yet both represent axes of the same length as the vertical, a_3, $-a_3$.

1033

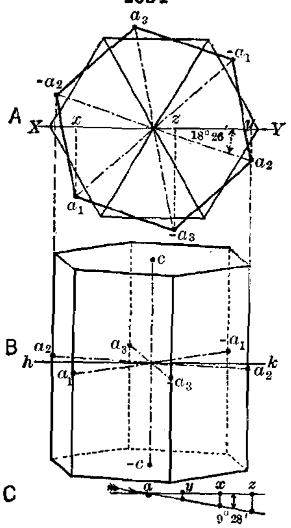


Development of the axes of the isometric system in orthographic and clinographic projection (after Penfield)

It is wholly a matter of choice that the angle of revolution shown in figure 1033A is 18° 26′, and that the eye is raised so as to look down upon a crystal at an angle of 9° 28′ from the horizontal, as indicated by figure 1033D. Also it is evident that these angles may be varied to suit any special requirement. As a matter of fact, however, the angles 18° 26′ and 9° 28′ have been well chosen and are established by long usage, and practically all the figures in clinographic projection, found in modern treatises on crystallography and mineralogy, have been drawn in accordance with them.

Tetragonal and Orthorhombic Systems. — The projection of tetragonal and orthorhombic axes can be easily obtained from the isometric axes by modifying the lengths of the various axes to conform to the axial ratio of the desired crystal. For instance with zircon the vertical axis has a relative length of $c = 0.64$ in respect to the equal lengths of the horizontal axes. By taking 0.64 of the unit length of the vertical axis of the isometric projection the crystal axes for a zircon figure are obtained. The Penfield axial charts all give *decimal parts of the unit length of the isometric vertical axis, so that any proportion of* this length can be found at once. In the orthorhombic system the lengths of both the a and c axes must be modified. The desired point upon the c axis can be obtained as described above. In the case of the a axis the required point can be found by some simple method of construction. If, as is the case in the Penfield charts, a plan of the unforeshortened horizontal axes is given in a top view, the desired length can be laid off directly upon the a axis in this orthographic projection by means of the decimal scale and then projected vertically down upon its clinographic projection. Or the proper distance can be laid off on the vertical axis and then by means of a line drawn from this point parallel to a line joining the extremities of the c and a axes of the isometric projection the proper proportional part of the a axis can be determined by intersection.

1034



Development of the axes of the hexagonal system in orthographic and clinographic projection (after Penfield)

Hexagonal System. — For projecting the hexagonal axes exactly the same principles may be made use of as were employed in the construction of the isometric axes. Figure 1034A is an orthographic projection, *a plan*, of a hexagonal prism in two positions, one of them, a_1, a_2, etc., after a revolution of 18° 26′ from what may be called normal position. In figure 1034B the extremities of the horizontal axes of A have been projected down upon the horizontal construction line hk, and a_1, a_2 and $-a_3$ which are forward of the line XY in A are located below the line hk in the clinographic projection, the distances from hk being one-sixth of a_1x, a_2y and $-a_3z$ of A. Figure 1034C is a scheme for getting the distances which the extremities of the axes are dropped. The vertical axis in 1034B has been given the same length as the axes of the plan.

Monoclinic System. — In the case of the monoclinic axes the inclination and length of the a axis must be determined in each case. The axial chart, Fig. 1030, can be most conveniently used for this purpose. The ellipse in the figure, lettered A, C, – A, – C gives the trace of the ends of the a and c axes as they are revolved in the A — C plane. To find, therefore, the inclination of the a axis it is only necessary to lay off the angle β by means of the graduation given on this ellipse. The unit length of the a axis may be determined in various ways. The plan of the axes given at the top of the chart may be used for this purpose. Fig. 1035 will illustrate the method of procedure as applied in the case of orthoclase, where β = 64° and a = 0.66. The foreshortened length of the a axis is determined as indicated and then this length can be projected vertically downward upon the inclined a axis, the direction of which has been previously determined as described above.

Triclinic System. — In the construction of triclinic axes the inclination of the a axis and its length are determined in exactly the same manner as described in the preceding paragraph in the case of the monoclinic system. The direction of the b axis is determined as follows. The vertical plane of the b and c axes is revolved about the c axis through such an angle as will conform to the angle between the pinacoids 100 and 010. Care must be taken to note whether this plane is to be revolved toward the front or toward the back. If the angle between the normals to 100 and 010 is greater than 90° the right hand end of this plane is to be revolved toward the front. Figure 1036, which is a simplified portion of the axial chart, shows the necessary construction in order to obtain the direction of the b axis in the case of rhodonite in which 100 ∧ 010 = 94° 26′ and α = 103° 18′. The plane of the b–c axes will pass through the point p which is 94° 26′ from $-a$. To locate the point b', which is the point where the b axis would emerge from the sphere, draw through the point p two or more chords from points where the vertical ellipses of the chart cross the horizontal ellipse, as lines a–p. $-a$–p. b–p, in figure 1036. Then from points on these same vertical ellipses which are 13° 18′ below the horizontal plane draw chords parallel to the first series as x–x', y–y', z–z'. The point where these three chords meet determines the position of b' and a line from this point drawn through the center of the

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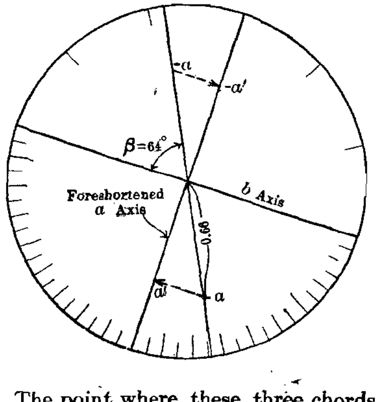

chart determines the direction of the b axis, since it lies in the proper vertical plane and makes the angle α, 103° 18′, with the c axis. The foreshortened length of the b axis can be determined by the use of the orthographic projection of the a and b axes at the top of the chart in exactly the same manner as described under the monoclinic system and the point thus determined may be projected vertically downward upon the line of the b axis of the clinographic projection as already determined. It must be remembered, however, that the position of the b axis in the orthographic projection must conform to the position of the plane of the b and c axes or in the case of rhodonite have its right hand end at an angle of 94° 26′ with the negative end of the projection of the a axis.

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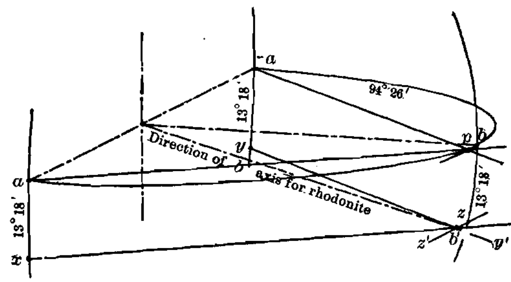

Drawing of Crystal Figures by Aid of Projections of their Axes. — In order to determine in the drawing the direction of any edge between two crystal faces it is necessary to establish two points, both of which shall be common to these two faces. A line connecting two such points will obviously have the desired direction. The positions of these points is commonly established by use of the linear or Quendstedt projection as explained in the following paragraphs, which have been taken almost verbatim from Penfield's description of the process.

The principle upon which the linear projection is based is very simple: *Every face of a crystal (shifted if necessary, but without change of direction) is made to intersect the vertical axis at UNITY, and then its intersection with the horizontal plane, or the plane of the a and b axis is indicated by a line.* For instance if a given face has the indices 111 it is clear that its linear projection would be a line passing through $1a$ and $1b$, since the face under these conditions will also pass through $1c$. If, however, the indices of the face are 112 it will only pass through $1/2\ c$ when it passes through $1a$ and $1b$. In order to fulfill, therefore, the requirements of the linear projection that the plane should pass through $1c$ the indices must be multiplied by two and then under these conditions the line in which the plane intercepts the horizontal plane, or in other words the linear projection of the face, will pass through $2a$ and $2b$. In the case of a prism face with the indices 110, its linear projection will be a line having the same direction as a line joining $1a$ and $1b$ but passing through the point of intersection of these axes, since a vertical plane such as a prism can only pass through $1c$ when it also includes the c axis and so must have its linear projection pass through the point of intersection of the three axes.

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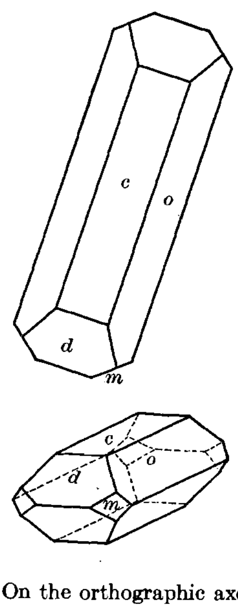


When it is desired to find the direction of an edge made by the meeting of any two faces, the lines representing the linear projection of the faces are first drawn, and the point where they intersect is noted. Thus a point common to both faces is determined, which is located in the plane of the a and b axes. A second point common to the two faces is *unity* on the vertical axis, and a line from this point to where the lines of the linear projection intersect gives the desired direction.

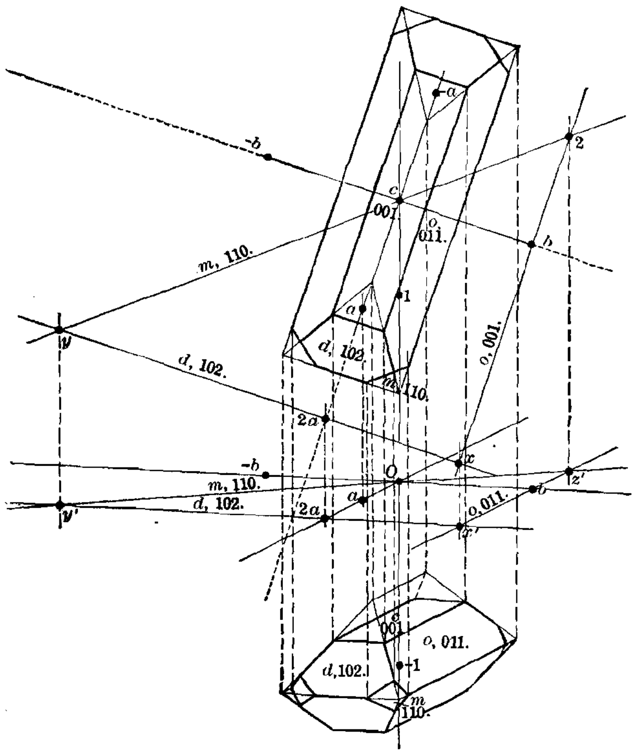
A simple illustration, chosen from the orthorhombic system, will serve to show how the linear projection may be employed in drawing. The example is a combination of barite, such as is shown in figure 1037. The axial ratio of barite is as follows:

$$a : b : c = 0{\cdot}8152 : 1 : 1{\cdot}3136$$

The forms shown in the figure and the symbols are, base c (001), prism m (110), brachydome o (011) and macrodome d (102).

Figure 1038 represents the details of construction of the orthographic and clinographic projections shown in figure 1037. On the orthographic axes the axial lengths a and b are located, the vertical axis c being foreshortened to a point at the center. On the clinographic axes, centered at O, the ends of the axes a and b are located by dropping perpendiculars from corresponding points above, and the length of the vertical axis 1·316 is laid off above and below O by means of the scale of decimal parts, at points marked *1* and -1 in the figure. The lines of the linear projection needed for the two sets of axes are as follows: For the brachydome o, 011, the lines xz and $x'z'$, through b parallel to the a axis : For the macrodome d, 102

1038



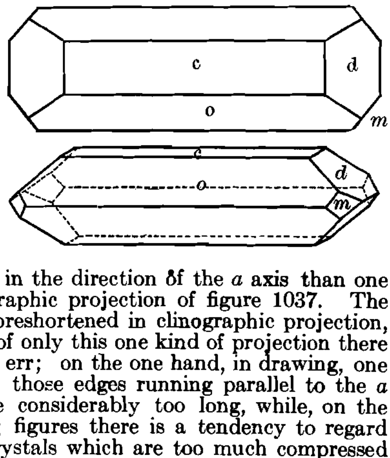
($2a : \infty b : c$), the lines xy and $x'y'$, through $2a$ parallel to the b axis: The prism m (110) is parallel to the vertical axis, hence in order that such a plane shall satisfy the conditions of the linear projection and pass through *unity on the vertical axis*, it must be considered as shifted (without change of direction) until it passes through the center: Its linear projection therefore is represented by the lines yz and $y'z'$, parallel to the directions $1a$ to $1b$ on the two sets of axes. Since a linear projection is made on the plane of the a and b axes, the intersection of any face with the base (001) has the same direction as the line representing its linear projection. It is well to note that the intersections x, y and z and x', y' and z' are in vertical alignment with one another.

Concerning the drawing of figure 1038, it is a simple matter to proportion the general outline of the barite crystal in orthographic projection. The direction of the edge between d, 102, and o, 011, is determined by finding the point x, where the lines of the linear projection of d and o intersect, and drawing the edge parallel to the direction from x to the center c. The intersection of the prism m, 110, with d and o is a straight line, parallel to the direction $1a$ to $1b$ or y to z. To construct the clino raphic figure, at some convenient point beneath the axes the horizontal middle edges of the crystal may be drawn parallel to the a and b axes, their lengths and intersections being determined by carrying down perpendiculars from the orthographic projection above. The intersection between d, 102, and o, 011, is determined by finding the point x' of the linear projection and drawing the edge parallel to the direction from x' to 1 (*unity*) on the vertical axis, while the corresponding direction below is parallel to the direction x' to -1. The size of the prism m, 110, and its intersections with d and o may all be determined by carrying down perpendiculars from the orthographic projection above, but it is well to control the directions by means of the linear projection: The edges between m, 110, and d, 102; and m, 110, and o, 011, are parallel respectively to the directions y' to 1 and z' to 1. Having completed a figure, a copy free from construction lines may be had by placing the drawing over a clean sheet of paper and puncturing the intersections of all edges with a needle-point: An accurate tracing may then be made on the lower paper.

Should it happen that the linear projection made on the plane of the a and b axes gives intersections far removed from the center of the figure, a linear projection may be made on the clinographic axes either on the plane of the a and c or b and c axes, supposing that the faces pass, respectively, through *unity* on the b or the a axes.

Importance of an Orthographic in connection with a Clinographic Projection. — Many students, on commencing the study of crystallography, fail to derive the benefit they should from the figures given in text-books. Generally clinographic projections are given almost exclusively, with perhaps occasional basal or orthographic projections, and beginners find it hard to reconcile many of the figures with the appearance of the models and crystals which they are intended to represent. For example, given only the clinographic projection of barite, figure 1037, it takes considerable training and knowledge of the projection employed to gain from the figure a correct idea of the proportions of the crystal which it actually represents. This may be shown by comparing figures 1037 and 1039, which represent the same crystal, drawn one with the a, the other with the b axis to the front. It is seen from figure 1039 that the crystal is far longer in the direction of the a axis than one would imagine from inspection of only the clinographic projection of figure 1037. The front or a axis is much foreshortened in clinographic projection, consequently by the use of only this one kind of projection there is a two-fold tendency to err; on the one hand, in drawing, one is inclined to represent those edges running parallel to the a axis by lines which are considerably too long, while, on the other hand, in studying figures there is a tendency to regard them as representing crystals which are too much compressed in the direction of the a axis. By using orthographic in connection with clinographic projections these tendencies are overcome. Having in mind the proportions of a certain crystal, or having at hand a model, it is easy to construct an orthographic projection in which the a and b axes are represented with their true proportions; then the construction of a clinographic projection of correct proportions follows as a comparatively simple matter. Without an orthographic projection it would have been a difficult task to have constructed the clinographic projection of figure 1039 wth the proportions of the intercepts upon the a and b axes the same as in figure 1037, while with the orthographic projection orientated as in figure 1039 it was an easy matter. A combination of the two projections is preferable in many cases and from the two figures a proper conception of the development of the crystal may be had.

1039



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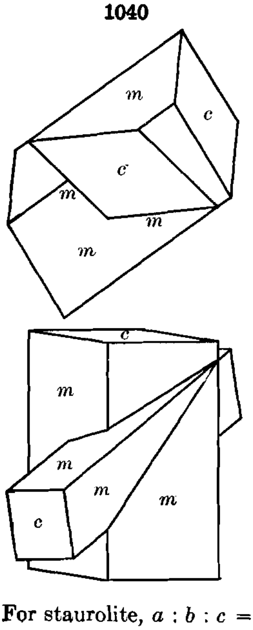
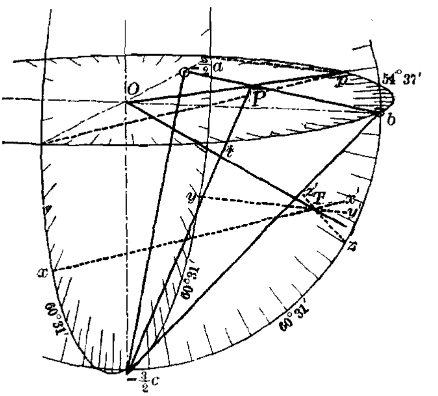


Drawing of Twin Crystals. — The axial protractor furnishes a convenient means for plotting the axes of twin crystals. The actual operation will differ with different problems but the general methods are the same. The two examples given will illustrate these methods.

(1). *To plot the axes for the staurolite twin shown in Fig. 1040.* In this case the twinning plane is parallel to the crystal face $\bar{2}3\bar{2}$ which has the axial intercepts of $-3/2a$, b, $-3/2c$. For staurolite, $a : b : c = 0{\cdot}473 : 1 : 0{\cdot}683$, while the ϕ and ρ angles of the twinning plane are, $\phi = 010 \wedge \bar{2}30 = 54^\circ 37'$ and $\rho = 00\bar{1} \wedge \bar{2}3\bar{2} = 60^\circ 31'$. To insure accuracy in plotting, the full lengths of the axes of the protractor have been regarded as unity. The first step is to locate on the clinographic projection the position of the twinning plane, $\bar{2}3\bar{2}$.

1041



This is shown in Fig. 1041 as the triangle from $-3/2a$ to b to $-3/2c$. The next step is to find the position of the twinning axis which will be normal to this plane. The coordinates of this twinning axis are given by the ϕ and ρ angles quoted above. The point p which is 54° 37′ back from the pole to 010 or b marks the place where the normal to the prism face $\bar{2}30$ would emerge from the sphere. The normal to $\bar{2}3\bar{2}$, which is the twinning axis will emerge on the meridian that runs through the point p and at such a distance below it that it will make the angle 60° 31′ with the negative end of the c axis. Chords are drawn to p from the points where the a and b axes meet the equator of the sphere and then chords parallel to these are drawn from the points x, y and z which are in each case 60° 31′ from the point where the negative end of the c axis cuts the spherical surface. The common meeting point of these chords T marks the place where the twinning axis pierces the spherical surface. The next step is to determine the point t at which the twinning axis cuts the twinning plane. The line OPp is by construction at right angles to the line connecting $-3/2a$ and $1b$. Therefore a vertical plane which is normal to the twinning plane would intersect that plane in the line connecting $-3/2c$ and P. The twinning axis OT would lie in this plane also. Consequently the point t, where OT and $-3/2c$–P intersect would lie both on the twinning axis and in the twinning plane. In order to make the method of construction clearer Fig. 1042 is given. Here the twinning axis is repeated from Fig. 1041. The twin position of the crystal is to be found by revolving it from its normal position through an arc of 180°, using the twinning axis as the axis of revolution. This will turn the twinning plane about upon the point t as a pivot and so transpose the points $-3/2a$, b and $-3/2c$ to points equidistant from it in an opposite position. By drawing lines through t and laying off equal distances beyond that point the new points $-3/2A$, B and $-3/2C$ will be obtained. These points lie upon the three axes in their twin position and so determine their directions.

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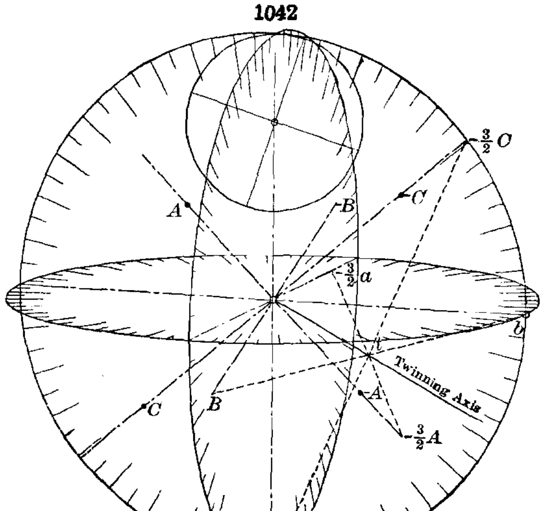

The plotting of the twin axes in the top view follows similar methods. In order to make the construction learer a separate figure, Fig. 1043, is given. The line O–t is laid off at an angle of 54° 37′ to the b axis. Upon this line the point t is found by projection upward from the clinographic view below. This point t then becomes the point around which the axes are revolved 180° to their twin positions. The figure shows clearly the methods of construction and the directions of the axes for the twin.

Upon the twin axes found in this way the portion of the crystal in twin position is drawn in exactly the same manner as if it was in the normal position.

(2). *To plot the axes for the calcite twin shown in Fig. 1044.* In this case it was desired to represent a scalenohedron twinned upon the rhombohedron *f* (02$\bar{2}$1) and so drawn that the twinning plane should be vertical and have the position of *b* (010) of an orthorhombic crystal. The angle from *c* (001) to *f* (02$\bar{2}$1) equals 63° 7′. In order to make the face *f* vertical, the vertical axis must be inclined at an angle of 26° 53′, or the angle between the *c* axes of the two individuals composing the twin would be double this or 53° 46′. These relations are shown in Fig 1045. As indicated in Fig. 1046 the position of these axes, *c* and *C* in the figure, are easily obtained at inclinations of 26° 53′ by use of the graduation of the vertical ellipse that passes through *B* and −*B*. The points *X*,*X*′ and *Y*,*Y*′ indicate the intersections with this same eclipse of the two planes containing the a_1, a_2 and a_3 axes in their respective inclined positions, the angles −*BX*, *BX*′, and *BY* and −*BY*′ being in each case equal to 26° 53′. In order to have the twinning plane occupy a position parallel to the 010 plane of an orthorhombic crystal it is necessary to revolve the axes so

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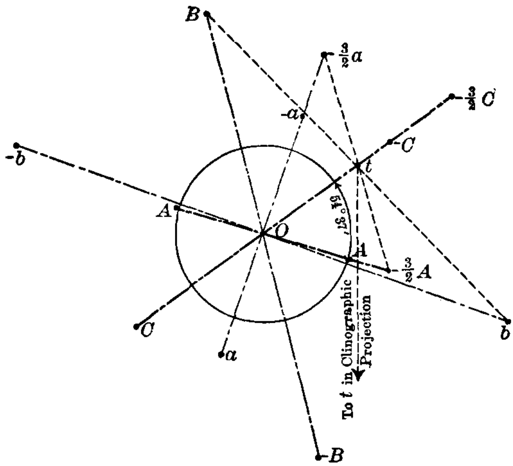

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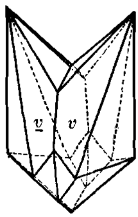

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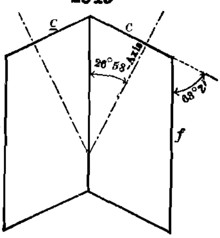

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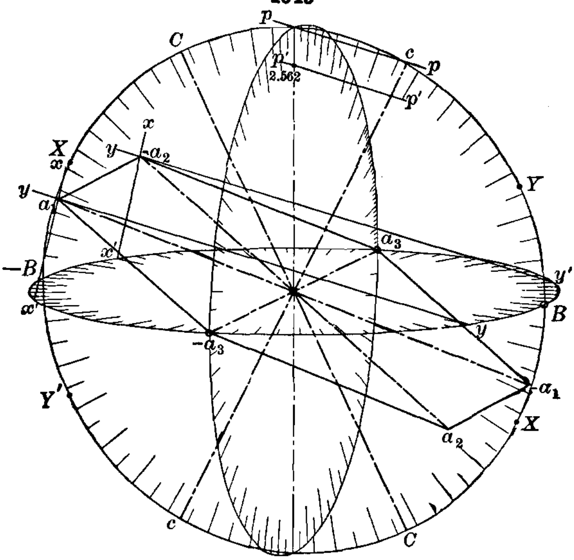


that one of the *a* hexagonal axes shall coincide with the position of the *a* axis of the orthorhombic system, as $-a_3$, a_3 in Fig. 1046. The two other hexagonal axes corresponding to the axis *c* must therefore lie in a plane which includes $-a_3$, a_3 and the points *X* and *X*′ and have such positions that they will make angles of 60° with $-a_3$, a_3. The construction necessary to determine the ends of these axes is as follows: Draw the two chords lettered x–x' through points that are 60° from $-a_3$ and a_3 and parallel to the direction of a chord that would pass through $-B$ and X. In a similar way draw the two chords y–y' through the second pair of points that are 60° from $-a_3$ and a_3, parallel to the direction of a chord that would pass through the points B and X. The intersections of these two sets of chords determine the points a_1 and $-a_2$ which are the ends of these respective axes. The hexagon shown in the figure connects the ends of the a_1, a_2 and a_3 axes that lie in a plane perpendicular to the axis c. The set of axes that belong to the axis C are to be found in a similar way. The length of the vertical axis is to be obtained by multiplying that of calcite, $c = 0{\cdot}854$, by three and laying off on the vertical line the length obtained or 2·562. This is transferred to the twin axis c by drawing the line p'–p' parallel to the line p–p. The desired figure of the calcite twin is to be drawn upon these two sets of inclined axes.

Drawing Crystals by use of the Stereographic and Gnomonic Projections

The following explanation of the methods of drawing crystals from the projections of their forms has been taken with only minor modifications from Penfield's description.*

1. Use of the Stereographic Projection

In explaining the method, a general example has been chosen; the construction of a drawing of a crystal of axinite, of the triclinic system. Figure 1047A represents a stereographic projection of the ordinary forms of axinite, m (110), a (100), M (1$\bar{1}$0), p (111), r (1$\bar{1}$1) and s (201). As shown by the figure, the *first meridian*, locating the position of 010, has been chosen at 20° from the horizontal direction SS'.

Figure 1047B is a *plan*, or an orthographic projection of an axinite crystal, as it appears when looked at in the direction of the vertical axis. It may be derived from the stereographic projection in a simple manner, as follows: — The direction of the parallel edges made by the intersections of the faces in the zone m, s, r, m', A, is parallel to a tangent at either m or m', and this direction may be had most easily by laying a straight edge from m to m' and, by means of a 90° triangle, transposing the direction to B, as shown by the construction.

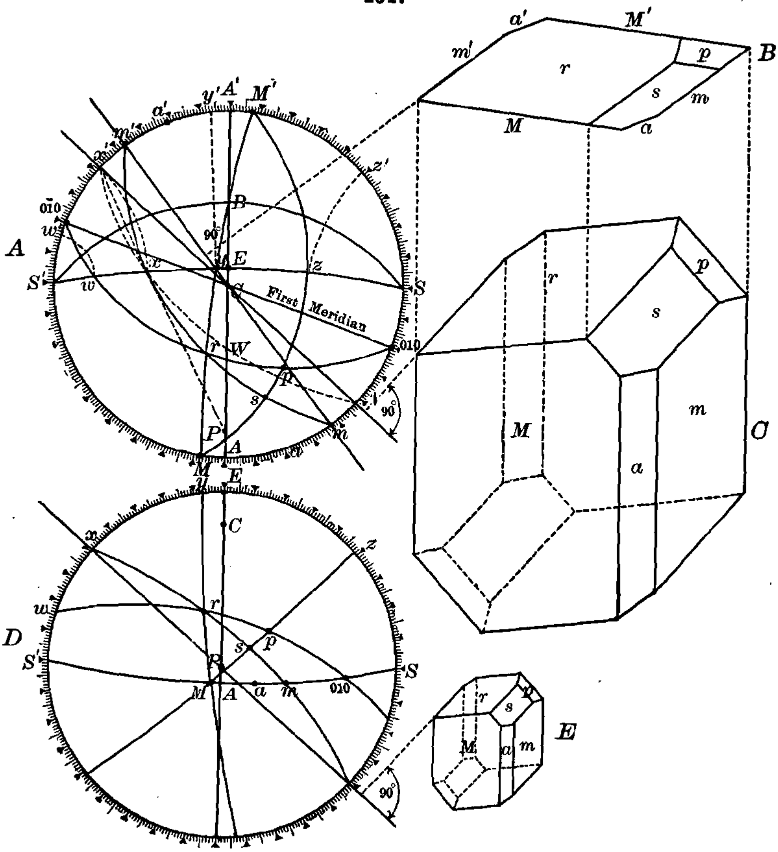
The construction of C, which may be called a *parallel-perspective* view, may next be explained: It is not a clinographic projection like the usual crystal drawings from axes, but an orthographic projection, made on a plane intersecting the sphere, represented by the stereographic projection, A, along the great circle SES'; the distance EC being 10°. The plane on which a drawing is to be made may, of course, have any desired inclination or position, but by making the distance CE equal 10° and taking the first meridian at 20° from S, almost the same effects of plan and parallel perspective are produced as in the conventional method of drawing from axes, where the eye is raised 9° 28′ and the crystal turned 18° 26′.

The easiest way to explain the construction of C from A is to imagine the sphere, represented by the stereographic projection, as revolved 80° about an axis joining S and S', or until the great circle SES' becomes horizontal. After such a revolution, the stereographic projection shown in A would appear as in D, and the parallel-perspective drawing, E, could then be derived from D in exactly the same manner as B was derived from A. This is, for example, because the great circle through m, s and r, D, intersects the graduated circle at x, where the pole of a vertical plane in the same zone would fall, provided one were present; hence the intersection of such a surface with the horizontal plane, and, consequently, the direction of the edges of the zone, would be parallel to a tangent at x: In other words, E is a *plan* of a crystal in the position represented by the stereographic projection, D. Although not a difficult matter to transpose the poles of a stereographic projection so as to derive D from A, it takes both time and skill to do the work with accuracy, and it is not at all necessary to go through the operation. To find the direction of the edges of any zone in C, for example m s r, note first in A the point x, where the great circles m s r and SES' cross. During the supposed revolution of 80° about the axis SS', the pole x follows the arc of a small circle and falls finally at x' (the same position as x of D) and a line at right angles to a diameter through x', as shown by the construction, is the desired direction for C. Similarly for the zones pr, MrM' and $MspM'$, their intersections with SES' at w, y and z are transposed by the revolution of 80° to w', y' and z'. The transposition of the poles w, x, y and z, A, to w', x', y' and z' may easily be accomplished

* Am. J. Sc., **21**, 206, 1906.

in the following ways: — (1) By means of the Penfield transparent, small-circle protractor (Fig. 68, p. 39) the distances of w, x, y and z from either S or S' may be determined and the corresponding number of degrees counted off on the graduated circle. (2)

1047

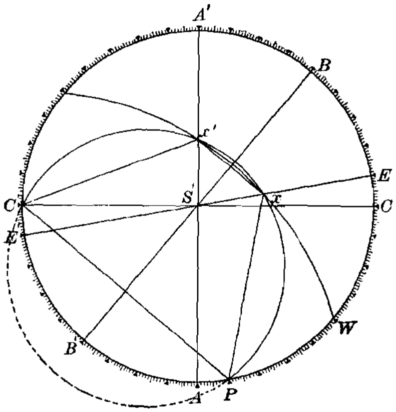


Development of a plan and parallel-perspective figure of axinite, triclinic system from a stereographic projection (after Penfield)

Find first the pole P of the great circle SES', where P is 90° from E or 80° from C, and is located by means of a stereographic scale or protractor (Fig. 62, p. 35): A straight line drawn through P and x will so intersect the graduated circle at x', that $S'x$ and $S'x'$ are equal in degrees. The reason for this is not easily comprehended from A, but if it is imagined that the projection is revolved 90° about an axis AA', so as to bring S' at the center, the important poles and great circles to be considered will appear as in figure 1048, where P and C' are the poles, respectively, of the great circles $ES'E'$ and $AS'A'$, and x is $41\frac{1}{4}°$ from S' as in figure 1047A. It is evident from the symmetry of figure 1048 that a plane surface touching at C', P and x will so intersect the great circle $AS'A'$ that the distances $S'x$ and $S'x'$ are equal. Now a plane passing through C', P, x and x', if extended, would intersect the sphere as a small circle, shown in the figure, but since this circle passes through C', which in figure 1047A is the pole of the stereographic projection (antipodal to C), it will be projected in figure A as a straight line, drawn through P and x, since the intersections upon the plane of projection of all planes that pass through the point of vision of the projection will appear as straight lines. (3) In figure 1048 B is located midway between E

and A', $BS'B'$ is a great circle, and W, 40° from C, is its pole: It is now evident from the symmetry of the figure that a great circle through W and x so intersects the great circle $AS'A'$, that the distances $S'x$ and $S'x'$ are equal. Transferring the foregoing relations to figure 1047A, W, 40° from C, is the pole of the great circle SBS', and a great circle drawn through W and x falls at x'. However, it is not necessary to draw the great circle through W and x to locate the point x' on the graduated circle: By centering the Penfield transparent great circle protractor, (Fig. 67, p. 39) at C, and turning it so that W and x fall on the same great circle, the point x may be transposed to x', and other points, w', y' and z', would be found in like manner.

1048



The three foregoing methods of transposing x to x', z to z', etc., are about equally simple, and it may be pointed out that, supplied with transparent stereographic protractors, and having the poles of a crystal plotted in stereographic projection, it is only necessary to draw the great circle SES' and to locate one point, either W or P, in order to find the directions needed for a parallel-perspective drawing, corresponding to figure 1047C. Thus, with only a great circle protractor, the great circle through the poles of any zone may be traced, and its intersection with SES' noted and spaced off with dividers from either S or S'; then the great circle through the intersection just found and W is determined, and where it falls on the divided circle noted, when the desired direction may be had by means of a straight edge and 90° triangle, as already explained.

2. Drawing of Twin Crystals by Use of the Stereographic Projection

In the great majority of cases the drawing of twin crystals can be most advantageously accomplished by the use of a stereographic projection of their forms. It is only necessary first to prepare a projection showing the poles of the faces in the normal and twin positions and then follow the methods outlined above. The preparation of the desired projection may, however, need some explanation. An illustrative example is given below taken from an article by Ford and Tillotson on some Bavenno twins of orthoclase.*

According to the Baveno law of twinning the n (021) face becomes the twinning plane and as the angle $c \wedge n = 44° 56\ 1/2'$ the angle between c and c' (twin position) becomes 89° 53′. For the purposes of drawing it is quite accurate enough to assume that this angle is exactly 90° and that accordingly the c face of the twin will occupy a position parallel to that of the b face of the normal individual.

Fig. 1049 shows the forms observed of the crystals both in normal and in twin positions, the faces in twin position being indicated by open circles and a prime mark (′) after their respective letters, while the zones in twin position are drawn in dashed lines. Starting out with the forms in normal position, the first face to transpose is the base c. This form, from the law of the twinning, will be transposed to c' where it occupies the same position as b of the normal individual, and it necessarily follows that b itself in being transposed will come to b' at the point where the normal c is located.

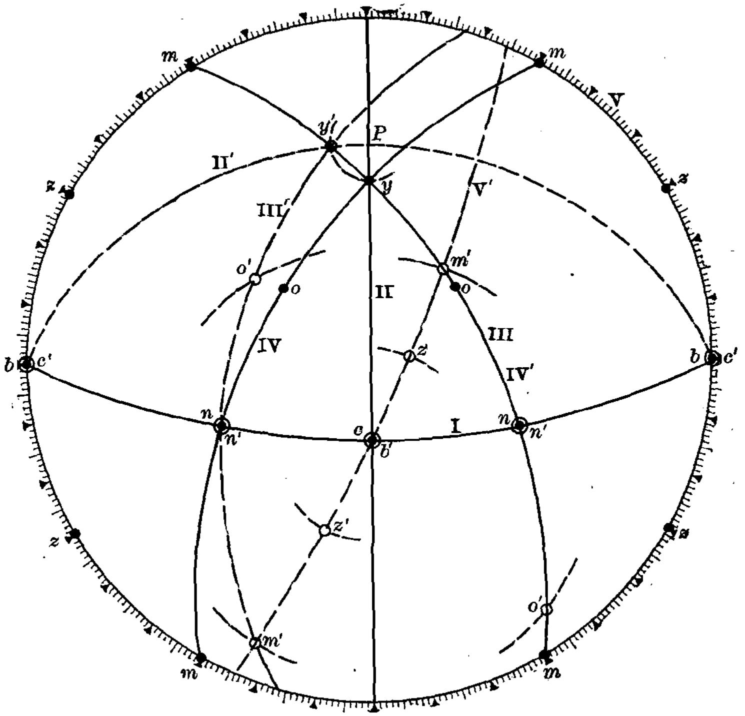
In turning therefore the crystal to the left from normal to twin position, the fades c and b travel along the great circle I through an arc of 90° until they reach their respective twin positions. We have, in other words, revolved the crystal 90° to the left about an axis which is parallel to the faces of the zone I. The pole of this axis is located on the stereographic projection at 90° from the great circle I and falls on the straight line II, another great circle which intersects zone I at right angles. This pole P is readily located by the stereographic protractor on the great circle II at 90° from c. The problem then is to revolve the poles of the faces from their normal positions about the point P to the left and through an arc of 90° in each case.

During the revolution the poles of the n faces remain on the great circle I and as the angle $n \wedge n = 90°$, the location of their poles when in twin position is identical with that of

* Am. J. Sc., **26**, 149, 1908.

the normal position and n' falls on top of n. We can now transpose the great circle II from its normal to its twin position, since P remains stationary during the revolution and we have determined the twin position of c. The dashed arc II′ gives the twin position of the

1049



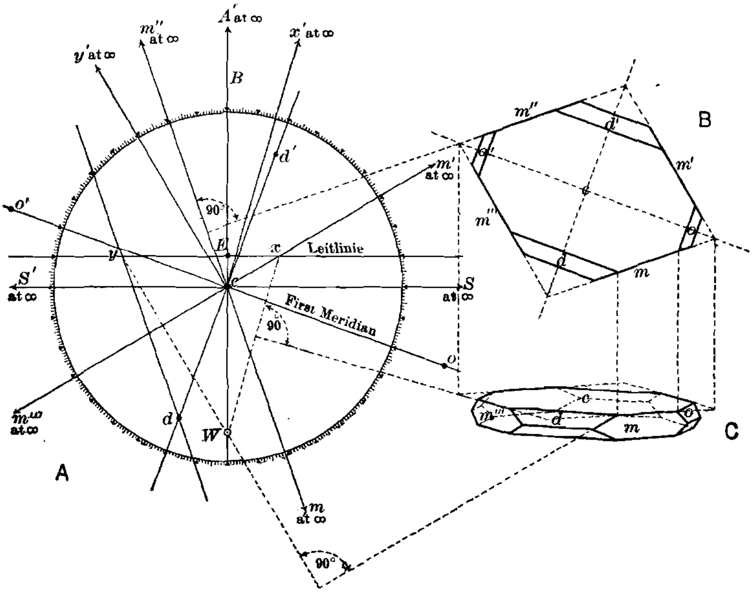
great circle II. The twin position of y must lie on arc II′ and can be readily located at y', the intersection of arc II′ with a small circle about P having the radius $P \wedge y$. It is now possible to construct the arc of the zone III in its transposed position III′, for we have two of the points, y' and n' of the latter, already located. By the aid of the Penfield transparent great circle protractor the position of the arc of the great circle on which these two points lie can be determined. On this arc, III′, o' and m' must also lie. Their positions are most easily determined by drawing arcs of small circles about b' with the required radii, $b \wedge o = 63° 8'$, $b \wedge m = 59° 22\ 1/2'$ and the points at which they intersect arc III′ locate the position of the poles o' and m'. At the same time the corresponding points on IV′ may be located, it being noted that IV′ and III are the same arc. But one other form remains to be transposed, the prism z. We have already b' and m' located and it is a simple matter with the aid of the great circle protractor to determine the position of the great circle upon which they lie. Then a small circle about b' with the proper radius, $b \wedge z = 29° 24'$, determines at once by its intersections with this arc the position of the poles of the z faces.

It may be pointed out that if it should be desired to make use of the methods of the gnonomic projection for the drawing of the figures as described below, the stereographic projection of the forms may be readily transformed into a gnomonic projection by doubling the angular distance from the center of the projection to each pole by the use of the stereographic protractor, Fig. 62, p. 35.

3. Use of the Gnomonic Projection

As an illustration, the method of drawing a simple combination of barite has been chosen. The forms shown in figure 1050 are c (001), m (110), o (011) and d (102). The location of the poles in the gnomonic projection is shown in A, where, as in figure 1047A, the *first* *meridian* is taken at 20° from the horizontal direction SS'. The poles of the prism m and the locations of S and S' (compare figure 1047A) fall in the gnomonic projection at infinity. In any plan, such as figure 1050B, the direction of an edge made by the meeting of two faces

1050



is at right angles to a line joining the poles of the faces, shown in figures A and B by the direction at 90° to the line joining m'' and c.

The parallel-perspective view, 1050C, is an orthographic projection (compare figures 1047A and C) drawn on a plane passing through S and S', and intersecting the sphere on which the gnomonic projection is based as a great circle passing through E, figure 1050A, and drawn parallel to SS', the distance cE being 10°: This great circle is called by Goldschmidt the *Leitlinie*. To find such intersections as between m''' and c, and m and d, figure C, note, as in figure 1047A, where the great circles through the poles of the faces intersect the *Leitlinie;* thus, the one through m''' and c at x, and that through m and d (through d parallel to $m\ m''$, since m and m'' are at infinity) at y. Next imagine the points x and y transposed as in figure 1047A to x' and y', which latter points, however, are located at infinity: This transposition is done by locating first the so-called *Winkelpunkt, W*, of Goldschmidt, 40° from c in figure 1050A, and as in figure 1047A, 90° from a point B, which is an equal number of degrees from E and A' (compare figure 1048). Of the three methods given above for transposing x and y to x' and y', the third may be easily applied in the gnomonic projection. Great circles, or straight lines, through W and x and W and y, figure 1050A, if continued to infinity, would determine x' and y', which is accomplished by drawing lines parallel to Wx and Wy through the center. It is not necessary, however, to draw the lines Wx and Wy, nor the parallel lines through the center; all that is needed to find the directions of the edges $m''' \wedge c$ and $m \wedge d$ is to lay a straight edge from W to x, respectively W to y, and with a 90° triangle transpose the directions to C, as indicated in the drawings. The principles are exactly the same as worked out for the interrelations of figures 1047A and C. As in the case of the stereographic projection, it is evident that, given the poles of a crystal plotted in the gnomonic projection, it would be necessary to draw only one line, the *Leitlinie*, and to locate one point, the *Winkelpunkt, W*, in order to find all possible directions for a plan and parallel-perspective views, corresponding to figures 1050B and C.