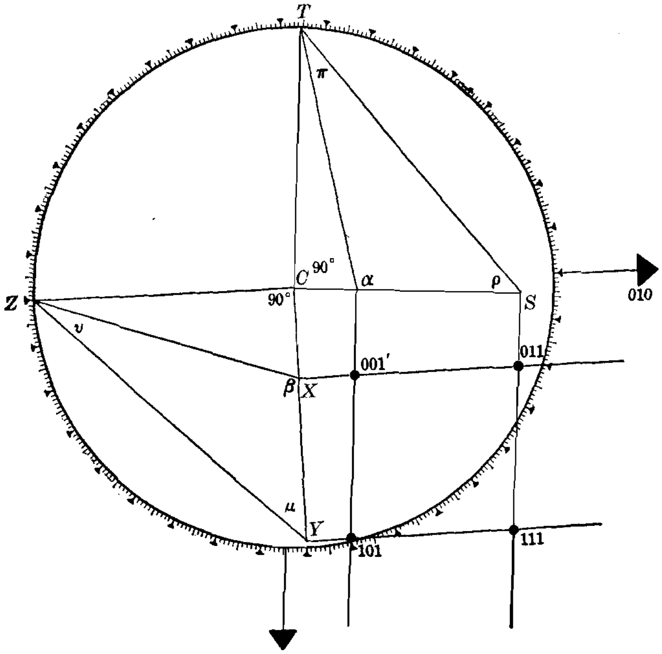
1·93c. By dividing these numbers by 1·55 we get the intercepts expressed in terms of the length of the b axis, considering that as 1·0. The intercepts then become 0·71a, 1b, 1·24c. When these are compared with the axial ratio of rhodonite, $a : b : c = 1{\cdot}114 : 1 : 0{\cdot}986$, the parameters of the face are found to be $\frac{2}{3}a$, $1b$, $2c$. The indices of x are therefore 321.

227. To determine, by plotting, the axial elements of a triclinic crystal having given the gnomonic projection of its forms. To illustrate this problem it is assumed that the positions of the poles of the faces, (100), (010), (001), (101), (011) and (111) on rhodonite are known, see Fig. 376. If this figure is compared with the stereographic projection of the same forms given in Fig. 371, it will be seen that the angle between the zones (100)-(101)-(001) and (100)-(111)-(011) is equal to π, that between the zones (100)-(111)-(011) and (100)-(110)-(010) is equal to ρ, between (010)-(011)-(001) and (010)-(111)-(101) is equal to ν and between (010)-(111)-(101) and (010)-(110)-(100) is equal to μ. The method by which the angles between these various zones may be measured was explained in Art. 42, p. 43, and is illustrated by the construction of Fig. 376. From these angles triangles can be readily constructed to give the lengths of the a and c axes in terms of the b axis, with its length taken as equal to 1·0.

376



228. To determine, by plotting, the indices of the forms of a triclinic crystal, having given the position of other poles upon the gnomonic projection. The method for the solution of this problem is similar to that already described under the previous systems. The difference lies in the fact that the lines of reference upon which are plotted the intercepts of the lines drawn to them from the poles of the faces make oblique angles with each other. These reference lines are taken as the zonal lines (001)-(101) and (001)-(011) and the intercepts from which the indices are determined are measured from the pole of (001). A study of the gnomonic projection of axinite, Fig. 367, will illustrate this problem.

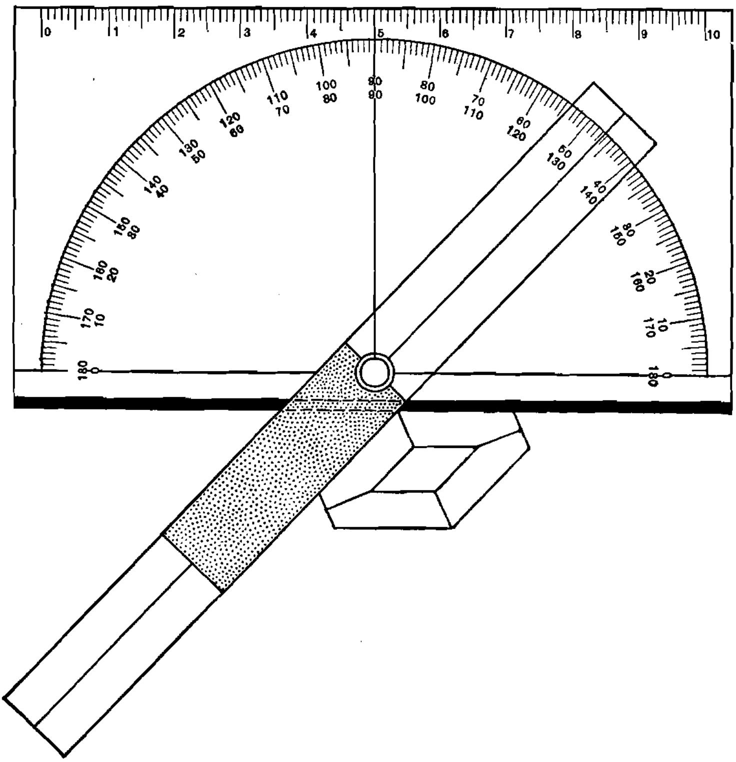
MEASUREMENT OF THE ANGLES OF CRYSTALS

229. Contact-Goniometers. — The interfacial angles of crystals are measured by means of instruments which are called *goniometers*.

The simplest form is the contact- or hand-goniometer one form of which is represented in Fig. 377.

This contact-goniometer consists of a card on which is printed a semicircular arc graduated to half degrees at the center of which is fastened a celluloid arm which may be turned to any desired position. The method of use of the goniometer is illustrated in Fig. 377. The bottom of the card and

377



Penfield Contact Goniometer, Model B

the blackened end of the celluloid arm are brought in as accurate contact as possible with the two crystal faces, the angle between which is desired. Care must be taken to see that the plane of the goniometer is at right angles to the edge of intersection between the two faces. Another model of the contact-goniometer, Fig. 378, has two arms swiveled together and separate from the graduated arc. The crystal angle is obtained by means of the arms and then the angle between them measured by placing them upon the graduated arc. This latter type is employed in cases where the crystal lies in such a position as to prevent the use of the former.*

* These simple types of contact-goniometers were devised by S. L. Penfield and can be obtained by addressing the Mineralogical Laboratory of the Sheffield Scientific School of Yale University, New Haven, Ct.

The contact-goniometer is useful in the case of large crystals and those whose faces are not well polished; the measurements with it, however, are

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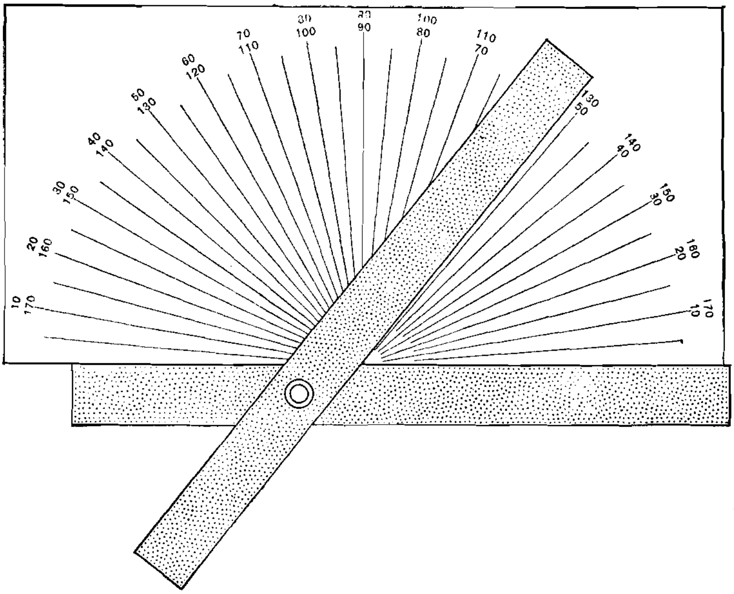

Penfield Contact Goniometer, Model A

seldom accurate within a quarter of a degree. In the finest specimens of crystals, where the faces are smooth and lustrous, results far more accurate may be obtained by means of a different instrument, called the reflecting goniometer.

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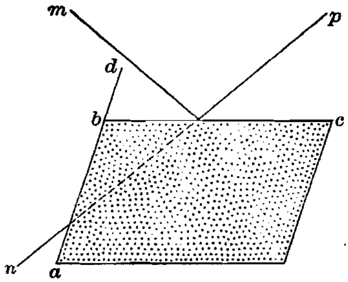
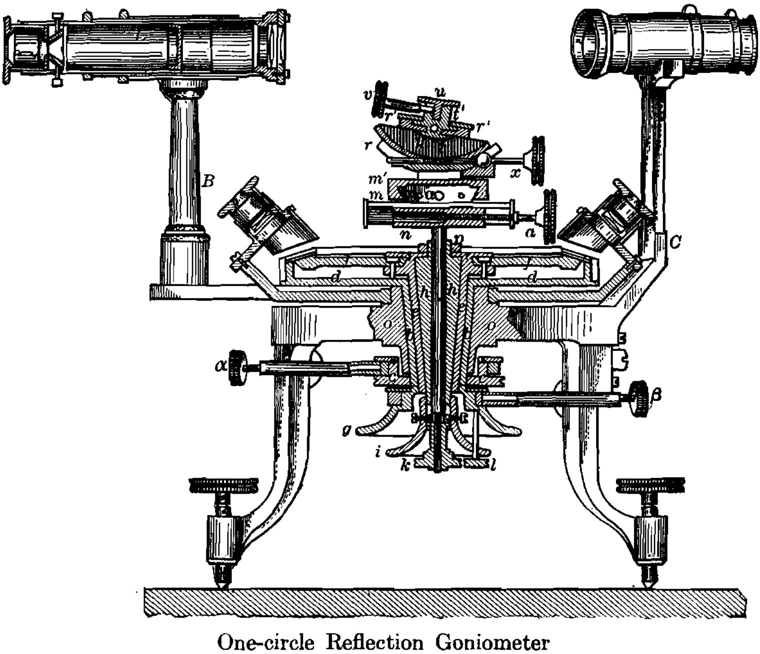


230. Reflecting Goniometer. — This type of instrument was devised by Wollaston in 1809. It has undergone extensive modifications and improvements since that time. Only the perfected forms that are in common use to-day will be described.

The principle underlying the construction of the reflecting goniometer will be understood by reference to the figure (Fig. 379), which represents a section of a crystal, whose angle, *abc*, between the faces *ab*, *bc*, is required. Let the eye be placed at *P* and the point *M* be a source of light. The eye at *P*, looking at the face of the crystal, *bc*, will observe a reflected image of *m*, in the direction of *Pn*. The crystal may now be so changed in its position that the same image is seen reflected by the next face and in the same direction, *Pn*. To effect this, the crystal must be turned around, until *abd* has the present direction of *bc*. The angle *dbc* measures, therefore, the number of degrees through which the crystal must be turned; it may be measured by attaching the crystal to a graduated circle, which turns with the crystal. This angle is the supplement of the interior angle between the two faces, or in other words is the *normal angle*, or angle between the two poles (see Art. **43**, p. 44). The reflecting goniometer hence gives directly the angle needed on the system of Miller here followed.

231. Horizontal Goniometer. — A form of reflecting goniometer well adapted for accurate measurements is shown in Fig. 380. The particular form of instrument here figured * is made by Fuess.

380



One-circle Reflection Goniometer

The instrument stands on a tripod with leveling screws. The central axis, *o*, has within it a hollow axis, *b*, with which the plate, *d*, turns, carrying the verniers and also the observing telescope, the upright support of which is shown at *B*. Within *b* is a second hollow axis, *e*, which carries the graduated circle, *f*, above, and which is turned by the screw-head, *g*; the tangent screw, *α*, serves as a fine adjustment for the observing telescope, *B*, the screw, *c*, being for this purpose raised so as to bind *b* and *e* together. The tangent screw, *β*, is a fine adjustment for the graduated circle. Again, within *e* is the third axis, *h*, turned by the screw-head, *i*, and within *h* is the central rod, which carries the support for the crystal, with the adjusting and centering contrivances mentioned below. This rod can be raised or lowered by the screw, *k*,

* The figure here used is from the catalogue of Fuess.

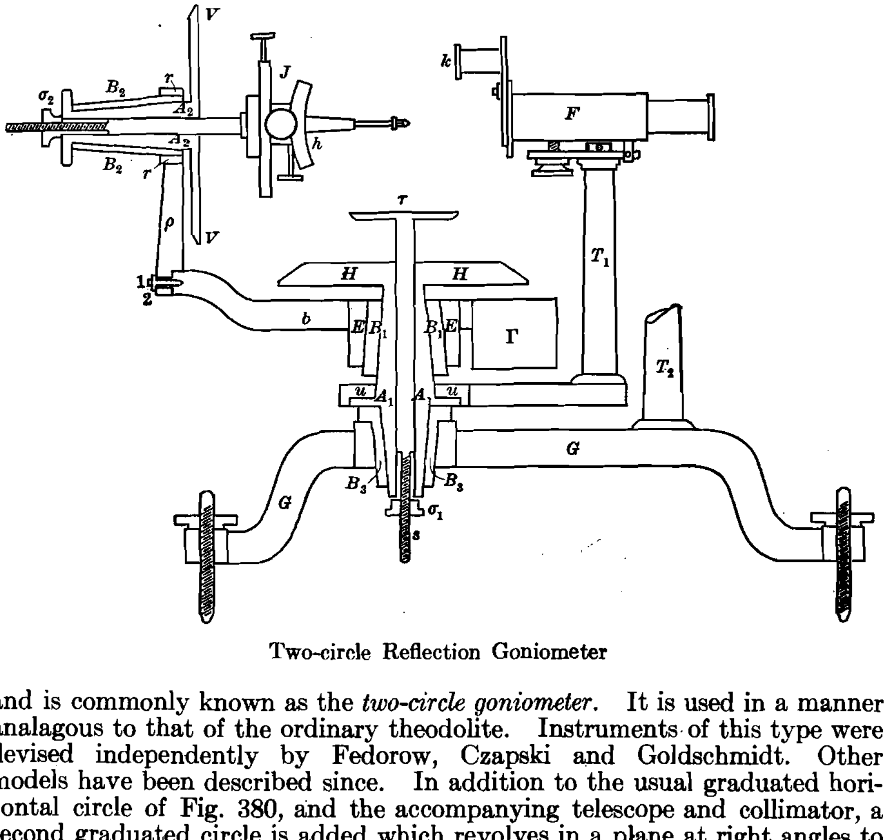
so as to bring the crystal to the proper height — that is, up to the axis of the telescope; when this has been accomplished, the clamp at *p*, turned by a set-key, binds *s* to the axis, *h*. The movement of *h* can take place independently of *g*, but after the crystal is ready for measurement these two axes are bound together by the set-screw, *l*. The signal telescope is supported at *C*, firmly attached to one of the legs of the tripod. The crystal is mounted on the plate, *u*, with wax, the plate is clamped by the screw, *v*. The *centering* apparatus consists of two slides at right angles to each other (one of these is shown in the figure) and the screw, *a*, which works it; the end of the other corresponding screw is seen at *a'*. The *adjusting* arrangement consists of two cylindrical sections, one of them, *r*, shown in the figure, the other at *r'*; the cylinders have a common center. The circle on *f* is graduated to degrees and quarter degrees, and the vernier gives the readings to 30″.

A brilliant source of light is placed behind the collimator tube which is at the top of the support *C*. Openings of various size and character are provided at the rear end of this tube in order to modify the size and shape of the beam of light that is to be reflected from the crystal faces. The most commonly used opening is one made by placing two circular disks nearly in contact with each other leaving between them an hour-glass shaped figure. The telescope tube *L* is provided with several removable telescopes with lenses which have different angular breadths and magnifying powers and hence are suitable for observing faces varying in size and degree of polish. At the front of the tube *L* there is a lens which is so pivoted that it may be thrown into or out of the axis of the telescope. When this lens lies in the axis of the tube it converts the telescope into a low-power microscope with which the crystal may be observed. Without this lens the telescope has a long-distance focus and only the beam of light reflected from the crystal face can be seen.

The method of use of the instrument is briefly as follows. The little plate *u* is removed and upon it is fastened by means of some wax the crystal to be measured. The faces of the zone that is to be measured should be placed as nearly as possible vertical to the surface of this plate. It will usually facilitate the subsequent adjustment if a prominent face in this zone be placed so that it is parallel to one of the edges of the plate *u*. This plate with the attached crystal is then fastened in place by the screw *v*. During the preliminary adjustments of the crystal the small lens in front of the tube *L* is placed in its axis and the crystal observed through the microscope thus formed. It is usually better also to make these first adjustments outside the dark room in daylight. By means of the screw-head *k* the central post is raised or lowered until the center of the crystal lies in the plane of the telescope. Next by means of the two sliding tables controlled by the screw-heads *a* and *a'* the crystal is adjusted so that the edge over which the angle is to be measured coincides with the axis of the instrument. This adjustment is most easily accomplished by turning the central post of the instrument until one of these sliding plates lies at right angles to the telescope and then by turning its screw-head bring the intersection in question to coincide with the vertical cross-hair of the telescope tube. Then turn the post until the other plate lies at right angles to the telescope and make a similar adjustment. Then in a similar manner by means of the tipping screws *x* and *y* bring the intersection between the faces to a position parallel with the vertical cross-hair of the telescope. By a combination of these adjustments this edge should be made to coincide with the vertical cross-hair and to remain stationary while the crystal is revolved upon the central post of the instrument. Next the instrument is taken into the dark room and a light placed behind the collimator tube, and the crystal turned until one of the faces is seen through the tube *L* to be brightly illuminated. Then the little lens in the front of this tube is raised and the reflection of the beam of light, or *signal* as it is called, should lie in the field. If the preliminary adjustments were accurate the horizontal cross-hair will bisect this signal. In the majority of cases, however, further slight adjustments will be necessary. Before the angles between the faces can be measured their various signals must all be bisected by the horizontal cross-hair. When these conditions are fulfilled each signal in turn is brought into place so that it is bisected also by the vertical cross-hair and its angular position read by means of the graduated scale and vernier. The difference between the angles for two faces gives the normal angle between them. In making these readings care must be taken that the plate on which the graduated circle is engraved is turned with the central post. In order to do this only the screw-head *g* must be used unless, as is wise, the two screw-heads *i* and *g* have been previously clamped together by means of *l*. For the accurate adjustment of the signals on the vertical cross-hair the tangent screw *β* is used. In making a record of the angles measured it is important to note accurately the face from which each signal is derived and the character of the signal. It is frequently helpful to make a sketch of the outlines of the different faces and number or letter them.

232. Theodolite-Goniometer. — A form of goniometer * having many practical advantages and at present in wide use has two independent circles

381



Two-circle Reflection Goniometer

and is commonly known as the *two-circle goniometer.* It is used in a manner analagous to that of the ordinary theodolite. Instruments of this type were devised independently by Fedorow, Czapski and Goldschmidt. Other models have been described since. In addition to the usual graduated horizontal circle of Fig. 380, and the accompanying telescope and collimator, a second graduated circle is added which revolves in a plane at right angles to the first. Fig. 381, after Goldschmidt, gives a cross-sectional view of one of

* Fedorow, Universal or Theodolit-Goniometer, Zs. Kryst., **21**, 574, 1893; **22**, 229, 1893; Czapski, Zeitschr. f. Instrumentenkunde, 1, 1893; Goldschmidt, Zs. Kryst., **21**, 210, 1892; **24**, 610, 1895; **25**, 321, 538, 1896; **29**, 333, 589, 1898. On the method of Goldschmidt, see Palache, Am. J. Sc., **2**, 279, 1896; Amer. Mineral, **5**, No. 2, *et seq.*, 1920. A simplified form of the theodolite-goniometer is described by Stöber, Zs. Kryst., **29**, 25, 1897; **54**, 442.

the earlier machines devised by him. It will serve to illustrate the essential features of the instrument.

The crystal to be measured is attached at the end of the axis (h) of the vertical circle and so adjusted by means of suitable centering and tipping devices that a given plane, called the polar plane, is normal to this axis and lies directly over the axis of the horizontal circle. In using the instrument, instead of directly measuring the interfacial angles of the crystal, the position of each face is determined independently of the others by the measurement of its angular co-ordinates, or what might be called its latitude and longitude. These co-ordinates are the angles (ϕ and ρ of Goldschmidt) measured, respectively, in the vertical and horizontal circles from an assumed pole and meridian, which are fixed, in most cases, by the symmetry of the crystal. In practice the crystal is usually so mounted that its prismatic zone is perpendicular to the vertical circle. A plane at right angles to this zone, *i.e.*, the basal plane in the first four systems, is known as the polar plane and its position when reflecting the signal into the telescope establishes the zero position for the horizontal circle. The position of a pinacoid, usually the 010 plane, in the prism zone establishes the zero position for the vertical circle. For example, with an orthorhombic crystal, for the pyramid 111, the angle ϕ (measured on the vertical circle) is equal to 010 $\wedge$ 110 and ρ (measured on the horizontal circle) is equal to 001 $\wedge$ 111.

Goldschmidt has shown that this instrument is directly applicable to the system of indices and methods of calculation and projection adopted by him, which admit of the deducing of the elements and symbols of a given crystal with a minimum of labor and calculation.* Fedorow has also shown that this instrument, with the addition of the appliances devised by him, can be most conveniently used in the crystallographic and optical study of crystals.

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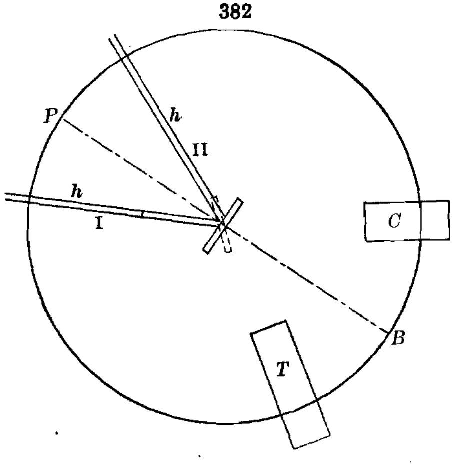


The following hints as to the methods of using this instrument may prove helpful.

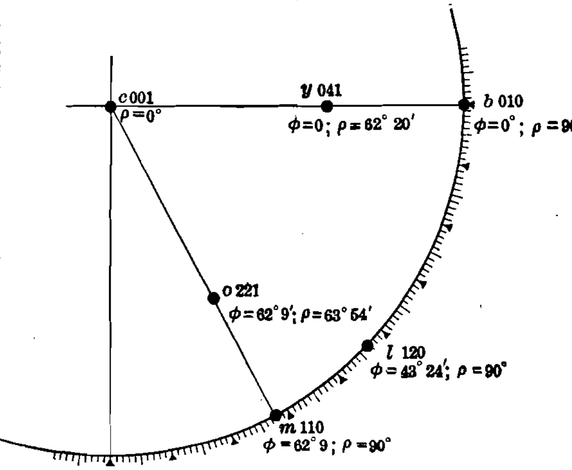
The telescope and collimator tube are placed at some convenient angle to each other (usually about 70°) and then clamped in position. The next step is to find the polar position of the horizontal circle, *i.e.*, the position at which a crystal plane lying at right angles to the axis of the vertical circle will throw the reflected beam of light on to the cross-hairs of the telescope. Obviously the plane under these conditions must be normal to the bisector of the angle between the axes of the collimator and telescope, the line B–P, Fig. 382. The method by which this polar position is found is as follows: Some reflecting surface is mounted upon the end of the post h, Figs. 381, 382, making some small inclined angle to the plane normal to that post. Then by turning the instrument in both the hori-

* See Goldschmidt's Krystallographische Winkeltabellen (432 pp., Berlin, 1897). This gives the angles required by his system for all known species. See also Zs. Kryst., **29**, 361, 1898. The same author's atlas der Krystallformen, 1913 *et seq.*, is a monumental work giving all previously published crystal figures together with a discussion of the forms observed upon them.

zontal and vertical planes this surface is brought into the proper position to reflect the signal into the telescope, see position I, Fig. 382. The horizontal angle of this position is noted. Then the vertical circle is turned through an angle of 180°. This brings the reflecting surface into the position indicated by the dotted lines in the figure. In order to again bring this surface back to its reflecting position the vertical circle with the post h must be moved in the horizontal plane until the position II is reached. The horizontal reading of this position is also noted. The angle midway between these two readings is the polar position desired. That is, when the post h lies in the direction of the broken line P–B a plane normal to its axis would reflect a beam of light from the collimator into the telescope. This position constitutes the zero position of the horizontal circle from which the ρ angles are measured.

The method used to adjust a crystal upon the instrument so that it will occupy the proper position for measurement will vary with the character of the crystal. A few illustrations follow. 1. *If the crystal has a basal plane at right angles to a prism zone.* The crystal is mounted upon the post h so that the faces of the prism zone lie as nearly as possible parallel to the axis of the post or the basal plane as nearly as possible normal to it. Then the instrument is moved until the reading of the horizontal circle agrees with the polar position already determined. Then by means of the tipping screws the crystal is moved until the reflection from the basal plane is centered upon the cross-hairs of the telescope. If the adjustments have been accurately made the signal will remain stationary while the vertical circle is revolved. Next the horizontal circle is moved through an angle of 90°. This will bring the reflections from the faces of the prism zone into the telescope. If the pinacoid 010 is present the vertical circle is turned until the reflected signal from this face falls on the horizontal cross-hair. The reading of the vertical circle under these conditions establishes the position of the meridian from which the ϕ angles are measured. If the pinacoid 010 is not present it is usually possible to determine its theoretical position from the position of other faces in the prism zone or in the zone between 010 and 100. 2. *If there is no basal plane present upon the crystal but a good prism zone.* Under these circumstances the horizontal circle is turned until it is exactly 90° away from its determined polar angle and then the crystal adjusted by means of the tipping screws until the signals from the faces of the prism zone all fall on the vertical cross-hair as the vertical circle of the goniometer is turned. 3. *If neither basal plane or prism zone is available but there are two or more faces present which are equally inclined to a theoretical basal plane.* First adjust the crystal as nearly as possible in the proper position and then obtaining reflections from these faces note the horizontal circle reading in each case. Take an average of these readings and adding or subtracting this angle from the polar angle of the horizontal scale place the instrument in this position. Then by tipping the crystal try to bring it into such a position that all of these faces will successively reflect the signal into the telescope as the vertical circle is turned. The operation may have to be repeated two or three times before the final adjustment is made. If the angle between the inclined faces and the theoretical base is known the instrument can be set in the proper position at once and the crystal brought into adjustment very quickly. Other problems will arise in practice but their solution will be along similar lines to those suggested above. It may frequently happen that more than one method of adjustment may be used with a given crystal. In that case the faces giving the best reflections should be used. It should be emphasized that the preliminary adjustment of the crystal is of supreme importance since all measurements of the co-ordinates of the different faces depend upon it. It is wise to check the adjustment in all possible ways before making the measurements.

383



After these adjustments have been completed the crystal is turned about both the horizontal and vertical planes so that each face upon it successively reflects the signal into the telescope. The horizontal and vertical readings are made in each case. The forms present can then be readily plotted in either the stereographic or gnomonic projections. Fig. 383 shows how the forms of a simple crystal of topaz could be plotted in the stereographic projection from the ϕ and ρ angles obtained from it — the two circle goniometer measurements. For each face the vertical circle angle, ϕ, is plotted on the divided circle, the position of $b(010)$ giving the zero point while the horizontal circle angle is plotted on a radial line from the center of the projection, the position of $c(001)$ giving its zero point.

COMPOUND OR TWIN CRYSTALS

233. Twin Crystals. — Twin crystals are those in which one or more parts regularly arranged are in reverse position with reference to the other part or parts. They often appear externally to consist of two or more crystals symmetrically united, and sometimes have the form of a cross or star. They also exhibit the composition in the reversed arrangement of part of the faces,

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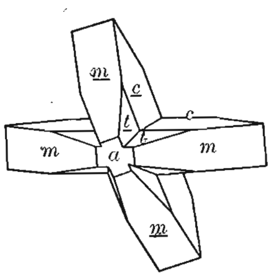

Thenardite

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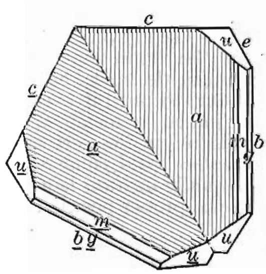

Columbite

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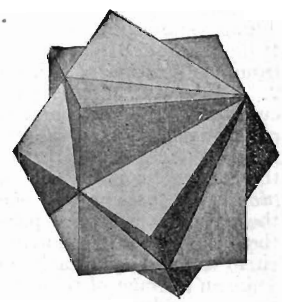

Fluorite

in the striæ of the surface, and in re-entering angles; in certain cases the compound structure can only be surely detected by an examination in polarized light. The above figures (Figs. 384–386) are examples of typical kinds of twin crystals, and many others are given on the pages following.

To illustrate the relation of the parts in a twin crystal, Figs. 387, 388 are given. Fig. 387 shows a regular octahedron divided into halves by a plane parallel to an octahedral face. If now the lower half be supposed to be revolved 180° about an axis normal to this plane, the twinned octahedron of Fig. 388 results. This is a common type of twin in the isometric system, and the method here employed to describe the position of the parts of the crystal to one another is applicable to nearly all twins.

234. Distinction between Twinning and Parallel Grouping. — It is important to understand that crystals, or parts of crystals, so grouped as to occupy parallel positions with reference to each other — that is, those whose similar faces are parallel — are not called twins; the term is applied only where the crystals or parts of them are united in their reversed position in accordance with some deducible mathematical law. Thus Fig. 389, which represents a cluster of partial crystals of analcite, is a case of *parallel grouping* simply (see Art. **252**); but Fig. 407 illustrates twinning, and this is