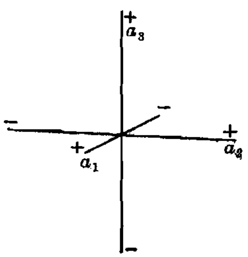
I. ISOMETRIC SYSTEM

(Regular or Cubic System)

50. The Isometric System embraces all the forms which are referred to three axes of equal lengths and at right angles to each other. Since these axes are mutually interchangeable it is customary to designate them all by the letter a. When properly orientated (*i.e.* placed in the commonly accepted position for study) one of these axes has a vertical position and of the two which lie in the horizontal plane, one is perpendicular and the other parallel to the observer. The order in which the axes are referred to in giving the relations of any face to them is indicated in Fig. 85 by lettering them a_1, a_2 and a_3. The positive and negative ends of each axis are also shown.

85



Isometric Axes

There are five classes here included; of these the normal class,* which possesses the highest degree of symmetry for the system and, indeed, for all crystals, is by far the most important. Two of the other classes, the pyritohedral and tetrahedral, also have numerous representatives among minerals.

1. NORMAL CLASS (1). GALENA TYPE

(Hexoctahedral or Holohedral Class)

51. Symmetry. — The symmetry of each of the types of solids enumerated in the following table, as belonging to this class, and of all their combinations, is as follows.

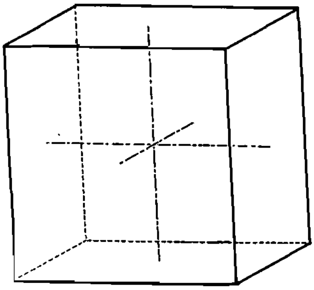
Axes of Symmetry. There are three principal axes of tetragonal symmetry which are coincident with the crystallographic axes and are sometimes known as the cubic axes since they are perpendicular to the faces of the cube. There are three diagonal axes of trigonal symmetry which emerge in the middle of the octants formed by the cubic axes. These are known as the octahedral axes since they are perpendicular to the faces of the octahedron. Lastly there are six diagonal axes of binary symmetry which bisect the plane angles made by the cubic axes. These are perpendicular to the faces of the dodecahedron and are known as the dodecahedral axes. These symmetry axes are shown in the Figs. 86–88.

Planes of Symmetry. There are three principal planes of symmetry which are at right angles to each other and whose intersections fix the posi-

* It is called *normal,* as before stated, since it is the most common and hence by far the most important class under the system; also, more fundamentally, because the forms here included possess the highest grade of symmetry possible in the system. The cube is a possible form in each of the five classes of this system, but although these forms are alike geometrically, it is only the cube of the normal class that has the full symmetry as regards molecular structure which its geometrical shape suggests. If a crystal is said to belong to the isometric system, without further qualification, it is to be understood that it is included here. Similar remarks apply to the normal classes of the other systems.

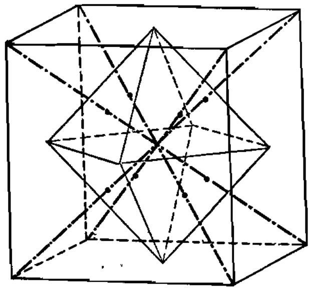
tion of the crystallographic axes, Fig. 89. In addition there are six diagonal planes of symmetry which bisect the angles between the principal planes, Fig. 90.

86



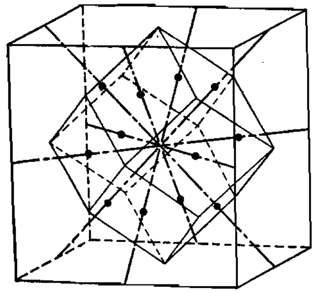
Axes of Tetragonal Symmetry (Cubic Axes)

87



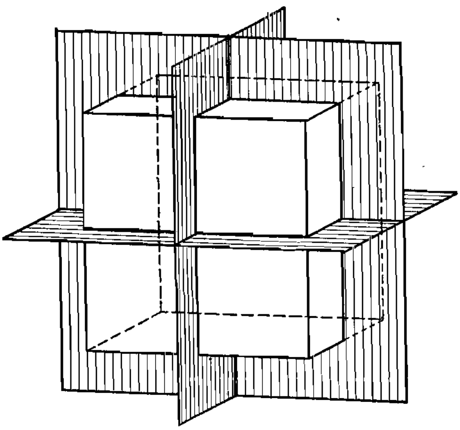
Axes of Trigonal Symmetry (Octahedral Axes)

88



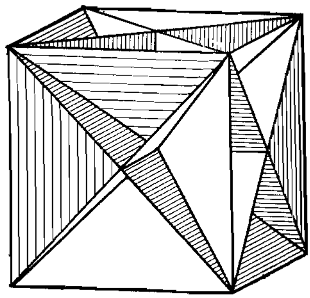
Axes of Binary Symmetry (Dodecahedral Axes)

89



Principal Symmetry Planes

90

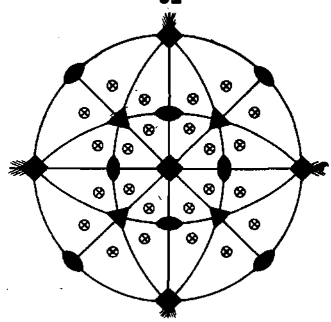


Diagonal Symmetry Planes

The accompanying stereographic projection (Fig. 91), constructed in accordance with the principles explained in Art. **44,** shows the distribution of the faces of the general form, *hkl* (hexoctahedron) and hence represents clearly the symmetry of the class. Compare also the projections given later.

52. Forms. — The various possible forms belonging to this class, and possessing the symmetry defined, may be grouped under seven types of solids. These are enumerated in the following table, commencing with the simplest.

91



Symmetry of Normal Class, Isometric System

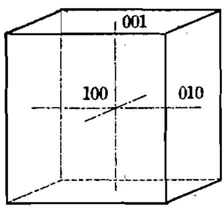
		Indices		
1. Cube	(100)			
2. Octahedron	(111)			
3. Dodecahedron	(110)			
4. Tetrahexahedron	(*hk*0) as, (310);	(210);	(320), etc.	
5. Trisoctahedron	(*hhl*) as, (331);	(221);	(332), etc.	
6. Trapezohedron	(*hll*) as, (311);	(211);	(322), etc.	
7. Hexoctahedron	(*hkl*) as, (421);	(321),	etc.	

Attention is called to the letters uniformly used in this work and in Dana's System of Mineralogy (1892) to designate certain of the isometric forms.* They are:

Cube: *a*.
Octahedron: *o*.
Dodecahedron: *d*.
Tetrahexahedrons: $e = 210$; $f = 310$; $g = 320$; $h = 410$.
Trisoctahedrons: $p = 221$; $q = 331$; $r = 332$; $\rho = 441$.
Trapezohedrons: $m = 311$; $n = 211$; $\beta = 322$.
Hexoctahedrons: $s = 321$; $t = 421$.

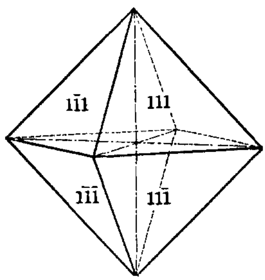
53. Cube. — The *cube*, whose general symbol is (100), is shown in Fig. 92. It is bounded by six similar faces, each parallel to two of the axes. Each face is a square, and the interfacial angles are all 90°. The faces of the cube are parallel to the principal or axial planes of symmetry.

92



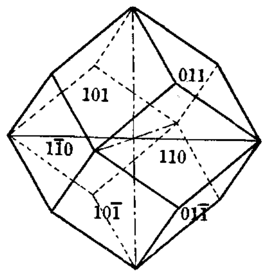
Cube

93



Octahedron

94



Dodecahedron

54. Octahedron. — The *octahedron*, shown in Fig. 93, has the general symbol (111). It is bounded by eight similar faces, each meeting the three axes at equal distances. Each face is an equilateral triangle with plane angles of 60°. The normal interfacial angle, $(111 \wedge 1\bar{1}1)$, is 70° 31′ 44″.

55. Dodecahedron. — The *rhombic dodecahedron*,† shown in Fig. 94, has the general symbol (110). It is bounded by twelve faces, each of which meets two of the axes at equal distances and is parallel to the third axis. Each face is a rhomb with plane angles of $70\frac{1}{2}$° and $109\frac{1}{2}$°. The normal interfacial angle is 60°. The faces of the dodecahedron are parallel to the six auxiliary, or diagonal, planes of symmetry.

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* The usage followed here (as also in the other systems) is in most cases that of Miller (1852).

† The *dodecahedron* of the crystallographer is this form with rhombic shaped faces commonly found on crystals of garnet. Geometricians recognize various solids bounded by twelve similar faces; of these the regular (pentagonal) dodecahedron is the most important. In crystallography this solid is impossible though the *pyritohedron* approximates to it. (See Art. 68.)

It will be remembered that, while the forms described are designated respectively by the symbols (100), (111), and (110), each face of any one of the forms has its own indices. Thus for the *cube* the six faces have the indices

100, 010, 001, $\bar{1}00$, $0\bar{1}0$, $00\bar{1}$.

For the *octahedron* the indices of the eight faces are:

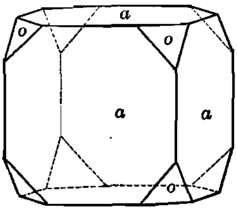
Above	111,	$\bar{1}11$,	$\bar{1}\bar{1}1$,	$1\bar{1}1$;
Below	$11\bar{1}$,	$\bar{1}1\bar{1}$,	$\bar{1}\bar{1}\bar{1}$,	$1\bar{1}\bar{1}$.

For the *dodecahedron* the indices of the twelve faces are:

110,	$\bar{1}10$,	$\bar{1}\bar{1}0$,	$1\bar{1}0$,
101,	$\bar{1}01$,	$\bar{1}0\bar{1}$,	$10\bar{1}$,
011,	$0\bar{1}1$,	$0\bar{1}\bar{1}$,	$01\bar{1}$.

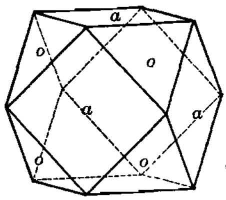
These should be carefully studied with reference to the figures (and to models), and also to the projections (Figs. 125, 126). The student should become thoroughly familiar with these individual indices and the relations to the axes which they express, so that he can give at once the indices of any face required.

95



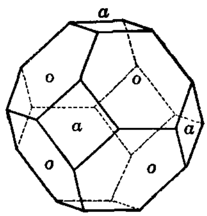
Cube and Octahedron

96



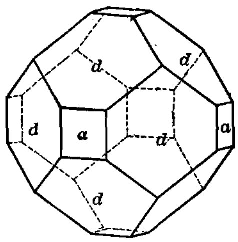
Cube and Octahedron

97



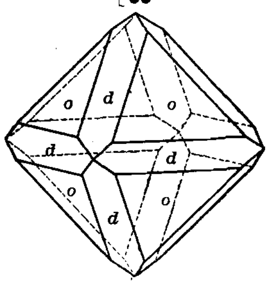
Octahedron and Cube

98



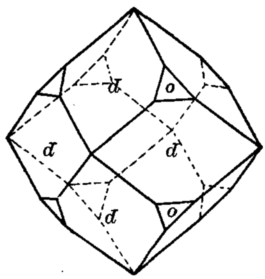
Dodecahedron and Cube

99



Octahedron and Dodecahedron

100

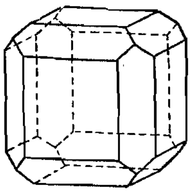


Dodecahedron and Octahedron

56. Combinations of the Cube, Octahedron, and Dodecahedron. — Figs. 95, 96, 97 represent combinations of the cube and octahedron; Figs. 98, 101 of the cube and dodecahedron; Figs. 99, 100 of the octahedron and dodecahedron; finally, Figs. 102, 103 show combinations of the three forms. The predominating form, as the cube in Fig. 95, the octahedron in Fig. 97, etc., is usually said to be *modified* by the faces of the other forms. In Fig. 96 the cube and octahedron are sometimes said to be "in equilibrium," since the faces of the octahedron meet at the middle points of the edges of the cube.

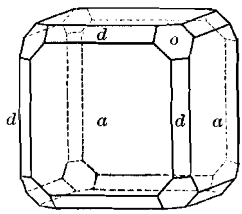
It should be carefully noticed, further, that the octahedral faces replace the solid angles of the cube, as regular triangles equally inclined to the adjacent cubic faces, as shown in Fig. 95. Again, the square cubic faces replace the six solid angles of the octahedron, being equally inclined to the adjacent octahedral faces (Fig. 97). The faces of the dodecahedron *truncate* * the twelve similar edges of the cube, as shown in Fig. 101. They also truncate the twelve edges of the octahedron (Fig. 99). Further, in Fig. 98 the cubic faces replace the six tetrahedral solid angles of the dodecahedron, while the octahedral faces replace its eight trihedral solid angles (Fig. 100).

101



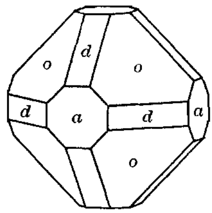
Cube and Dodecahedron

102



Cube, Octahedron and Dodecahedron

103



Octahedron, Cube and Dodecahedron

The normal interfacial angles for adjacent faces are as follows:

Cube on octahedron,	*ao*. 100 ∧ 111	= 54° 44′ 8″.
Cube on dodecahedron,	*ad*, 100 ∧ 110	= 45° 0′ 0″.
Octahedron on dodecahedron,	*od*, 111 ∧ 110	= 35° 15′ 52″.

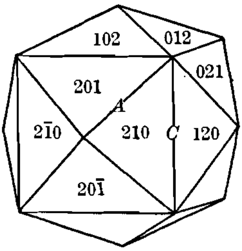
57. As explained in Art. **18** actual crystals always deviate more or less widely from the ideal solids figured, in consequence of the unequal development of like faces. Such crystals, therefore, do not satisfy the *geometrical* definition of right symmetry relatively to the three principal and the six auxiliary planes mentioned on p. 53 but they do conform to the conditions of crystallographic symmetry, requiring like angular position for similar faces. Again, it will be noted that in a combination form many of the faces do not actually meet the axes within the crystal, as, for example, the octahedral face *o* in Fig. 95. It is still true, however, that this face would meet the axes at equal distances if produced; and since the *axial ratio* is the essential point in the case of each form, and the *actual lengths* of the axes are of no importance, it is not necessary that the faces of the different forms in a crystal should be referred to the same actual axial lengths. The above remarks will be seen to apply also to all the other forms and combinations of forms described in the pages following.

58. Tetrahexahedron. — The *tetrahexahedron* (Figs. 104, 105, 106) is bounded by twenty-four faces, each of which is an isosceles triangle. Four of these faces together occupy the position of one face of the cube (hexahedron) whence the name commonly applied to this form. The general symbol is (*hk*0), hence each face is parallel to one of the axes while it meets the other two axes at unequal distances which are definite multiples of each other. There are two kinds of edges, lettered *A* and *C* in Fig. 104; the interfacial angle of either edge is sufficient to determine the symbol of a given form (see below). The angles of some of the common forms are given on a later page (p. 63).

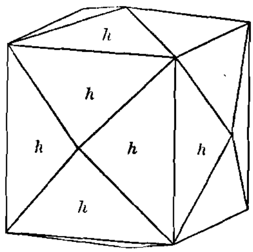
* The words *truncate, truncation,* are used only when the modifying face makes equal angles with the adjacent similar faces.

There may be a large number of tetrahexahedrons, as the ratio of the intercepts of the two axes, and hence of h to k varies; for example, (410), (310), (210), (320), etc. The form (210) is shown in Fig. 104; (410) in Fig. 105, and (530) in Fig. 106. All the tetrahexahedrons fall in a zone with a cubic face and a dodecahedral face. As h increases relatively to k the form approaches the cube (in which $h : k = \infty : 1$ or $1 : 0$), while as it diminishes and becomes more and more nearly equal to k in value it approaches the dodecahedron; for which $h = k$. Compare Fig. 105 and Fig. 106; also Figs. 125, 126. The special symbols belonging to each face of the tetrahexahedron should be carefully noted.

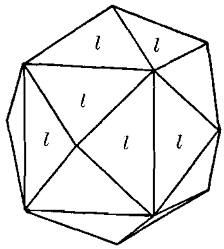
104



105

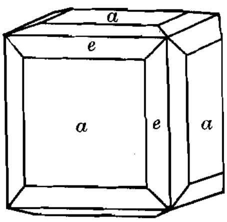


106



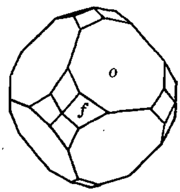
Tetrahexahedrons

107



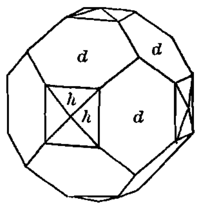
Cube and Tetrahexahedron

108



Octahedron and Tetrahexahedron

109



Dodecahedron and Tetrahexahedron

The faces of the tetrahexahedron bevel * the twelve similar edges of the cube, as in Fig. 107; they replace the solid angles of the octahedron by four faces inclined on the edges (Fig. 108; $f = 310$), and also the tetrahedral solid angles of the dodecahedron by four faces inclined on the faces (Fig. 109; $h = 410$).

59. Trisoctahedron. — The *trisoctahedron* (Fig. 110) is bounded by twenty-four similar faces; each of these is an isosceles triangle, and three together occupy the position of an octahedral face, whence the common name. Further, to distinguish it from the trapezohedron (or tetragonal trisoctahedron), it is sometimes called the *trigonal trisoctahedron.* There are two kinds of edges, lettered A and B in Fig. 110, and the interfacial angle corresponding to either is sufficient for the determination of the special symbol.

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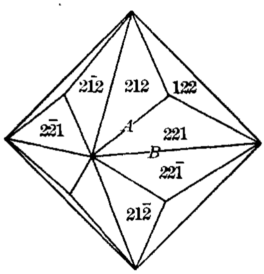
* The word *bevel* is used when two like faces replace the edge of a form and hence are inclined at equal angles to its adjacent similar faces.

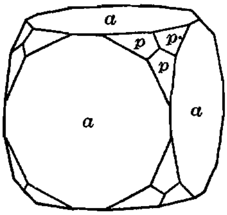
The general symbol is (*hhl*); common forms are (221), (331), etc. Each face of the trisoctahedron meets two of the axes at a distance less than unity and the third at the unit length, or (which is an identical expression *) it meets two of the axes at the unit length and the third at a distance greater than unity. The indices belonging to each face should be carefully noted. The normal interfacial angles for some of the more common forms are given on a later page.

110



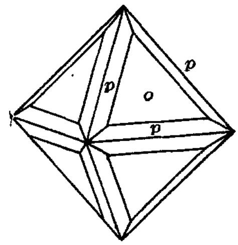
Trisoctahedron

111



Cube and Trisoctahedron

112



Octahedron and Trisoctahedron

60. Trapezohedron. — The *trapezohedron* † (Figs. 113, 114) is bounded by twenty-four similar faces, each of them a quadrilateral or trapezium. It also bears in appearance a certain relation to the octahedron, whence the name, sometimes employed, of *tetragonal trisoctahedron.* There are two kinds of edges, lettered *B* and *C*, in Fig. 113. The general symbol is *hll*; common forms are (311), (211), (322), etc. Of the faces, each cuts an axis at a distance less than unity, and the other two at the unit length, or (again, an identical expression) one of them intersects an axis at the unit length and the other two at equal distances greater than unity. The indices belonging to each face should be carefully noted. The normal interfacial angles for some of the common forms are given on a later page. Another name for this form is *icositetrahedron.*

61. The combinations of these forms with the cube, octahedron, etc., should be carefully studied. It will be seen (Fig. 111) that the faces of the trisoctahedron replace the solid angles of the cube as three faces equally inclined on the *edges;* this is a combination which has not been observed on crystals. The faces of the trapezohedron appear as three equal triangles equally inclined to the cubic faces (Fig. 115).

Again, the faces of the trisoctahedron bevel the edges of the octahedron, Fig. 112, while those of the trapezohedron are triangles inclined to the faces at the extremities of the cubic axes, Fig. 119; *m*(311). Still again, the faces of the trapezohedron *n*(211) truncate the edges of the dodecahedron (110), as shown in Fig. 118; this can be proved to follow at once from the zonal

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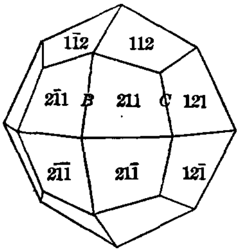
* Since $\frac{1}{2}a : \frac{1}{2}b : \frac{1}{1}c = 1a : 1b : 2c$. The student should read again carefully the explanations in Art. **35.**

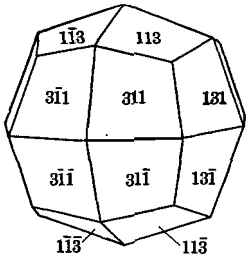
† It will be seen later that the name trapezohedron is also given to other solids whose faces are trapeziums conspicuously to the tetragonal trapezohedron and the trigonal trapezohedron.

relations (Arts. **45**, **46**), cf. also Figs. 125, 126. The position of the faces of the form *m*(311), in combination with *o*, is shown in Fig. 119; with *d* in Fig. 120.

113



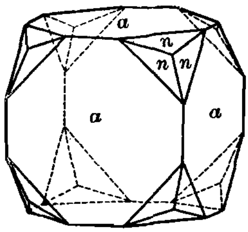
114



Trapezohedrons

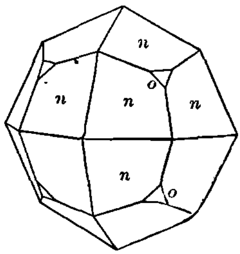
It should be added that the trapezohedron *n*(211) is a common form both alone and in combination; *m*(311) is common in combination. The trisoctahedron alone is rarely met with, though in combination (Fig. 112) it is not uncommon.

115



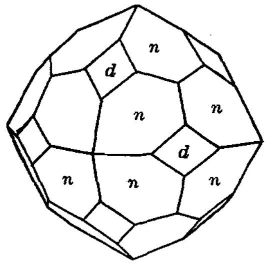
Analcite. Cube and Trapezohedron

116



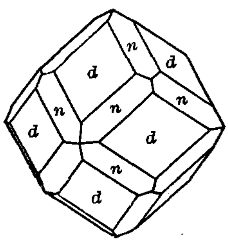
Analcite. Trapezohedron and Octahedron

117



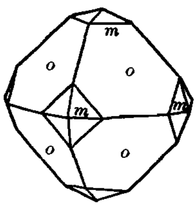
Garnet. Trapezohedron and Dodecahedron

118



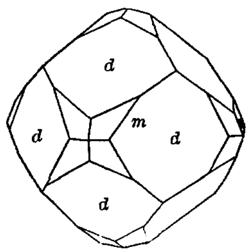
Garnet. Dodecahedron and Trapezohedron

119



Spinel. Octahedron and Trapezohedron

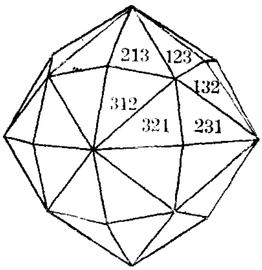
120



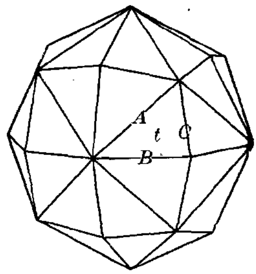
Magnetite. Dodecahedron and Trapezohedron

62. Hexoctahedron. — The *hexoctahedron*, Figs. 121, 122, is the general form in this system; it is bounded by forty-eight similar faces, each of which is a scalene triangle, and each intersects the three axes at unequal distances. The general symbol is (hkl); common forms are $s(321)$, shown in Fig. 121, and $t(421)$, in Fig. 122. The indices of the individual faces, as shown in Fig. 121 and more fully in the projections (Figs. 125, 126), should be carefully studied.

121

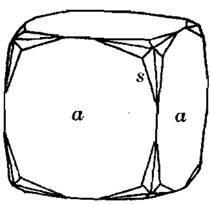


122



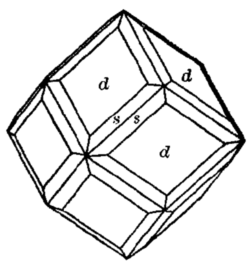
The hexoctahedron has three kinds of edges lettered A, B, C (longer, middle, shorter) in Fig. 122; the angles of two of these edges are needed to fix the symbol unless the zonal relation can be made use of. In Fig. 124 the faces of the hexoctahedron bevel the dodecahedral edges, and hence for this form $h = k + l$; the form s has the special symbol (321). The hexoctahedron alone is a very rare form, but it is seen in combination with the cube (Fig. 123, fluorite) as six small faces replacing each solid angle. Fig. 124 is common with garnet.

123



Fluorite Cube and Hexoctahedron

124



Garnet Dodecahedron and Hexoctahedron

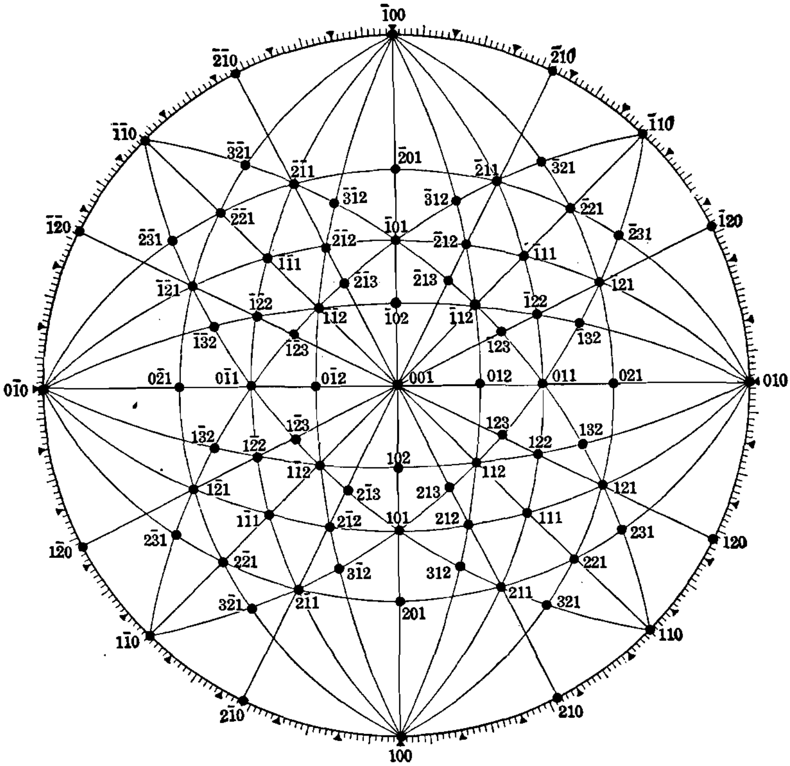
64. Pseudo-symmetry in the Isometric System. — Isometric forms, by development in the direction of one of the cubic axes, simulate tetragonal forms. More common, and of greater interest, are forms simulating those of rhombohedral symmetry by extension, or by flattening, in the direction of an octahedral axis. Both these cases are illustrated later. Conversely, certain rhombohedral forms resemble an isometric octahedron in angle.

65. Stereographic and Gnomonic Projections. — The stereographic projection, Fig. 125, and gnomonic projection, Fig. 126, show the positions of the poles of the faces of the cube (100), octahedron (111), and dodecahedron (110); also the tetrahexahedron (210), the trisoctahedron (221), the trapezohedron (211), and the hexoctahedron (321).

The *student should study this projection* carefully, noting the symmetry marked by the zones 100, 001, $\bar{1}00$, and 100, 010, $\bar{1}00$; also by 110, 001, $\bar{1}\bar{1}0$; $1\bar{1}0$, 001, $\bar{1}10$; 010, 101, $0\bar{1}0$; 010, $\bar{1}01$, $0\bar{1}0$. Note further that the faces of a given form are symmetrically distributed about a cubic face, as 001; a dodecahedral face, as 101; an octahedral face, as 111.

Note further the symbols that belong to the individual faces of each form, comparing the projections with the figures which precede.

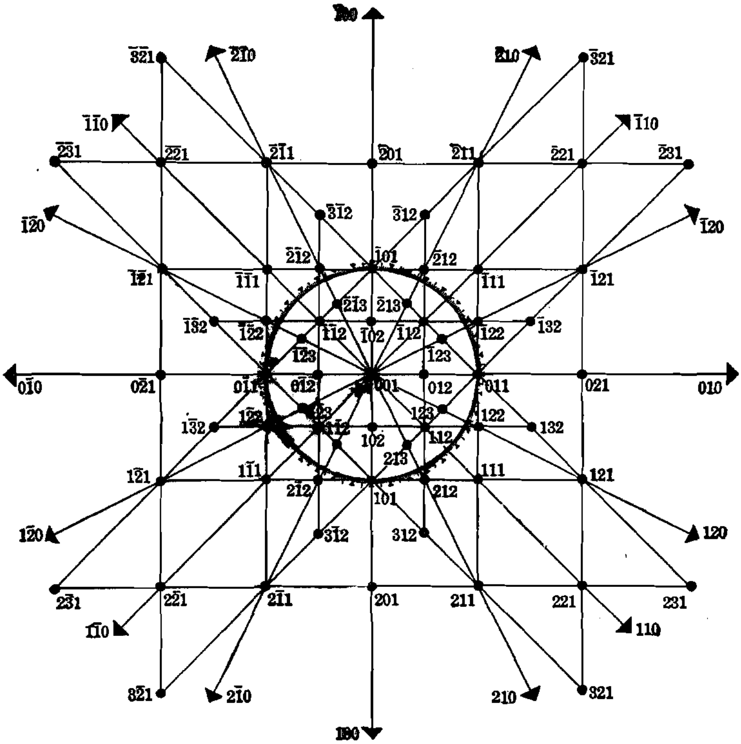
125



Stereographic Projection of Isometric Forms (Cube (100), Octahedron (111), Dodecanedron (110), Tetrahexahedron (210), Trisoctahedron (221), Trapezohedron (211), Hexoctahedron (321))

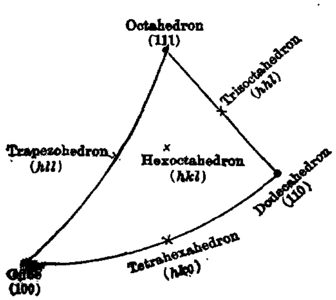
Finally, note the prominent *zones of planes;* for example, the zone between two cubic faces including a dodecahedral face and the faces of all possible tetrahexahedrons. Again, the zones from a cubic face (as 001) through an octahedral face (as 111) passing through the trisoctahedrons, as 113, 112, 223, and the trapezohedrons 332, 221, 331, etc. Also the zone from one dodecahedral face, as 110, to another, as 101, passing through 321, 211, 312, etc. At the same time compare these zones with the same zones shown on the figures already described. A study of the relations illustrated in Fig. 127 will be found useful. From it is seen that any crystal face falling in the zone between the cube and dodecahedron must belong to a tetrahexahedron; any face falling in the zone between the cube and octahedron must belong to a trapezohedron; and any face falling in the zone between the octahedron and dodecahedron must belong to a trisoctahedron, further any face falling outside these three zones must belong to a hexoctahedron.

126

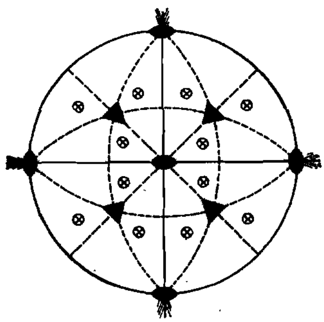


Gnomonic Projection of Isometric Forms (Cube (100), Octahedron (111), Dodecahedron (110), Tetrahexahedron (210), Trisoctahedron (221), Trapezohedron (211), Hexoctahedron (321))

127



128



Symmetry of Pyritohedral class

66. Angles of Common Isometric Forms.*

TETRAHEXAHEDRONS.

	Edge A	Edge C	Angle on	Angle on
Cf. Fig. 104.	210 ∧ 201, etc.	210 ∧ 120, etc.	a(100)	o(111)
410	19° 45′	61° 55¾′	14° 2¼′	45° 33¾′
310	25 50½	53 17¾	18 26	43 5¼
520	30 27	46 23¾	21 48	41 22
210	36 52¼	36 52¼	26 34	39 14
530	42 40	28 4¼	30 57¾	37 37
320	46 11¼	22 37¼	33 41½	36 48½
430	50 12½	16 15½	36 52¼	36 4¼
540	52 25¾	12 40¾	38 39½	35 45½

TRISOCTAHEDRONS.

	Edge A	Edge B	Angle on	Angle on
Cf. Fig. 110.	221 ∧ 212, etc.	221 ∧ 22$\bar{1}$, etc.	a(100)	o(111)
332	17° 20½′	50° 28¾′	50° 14¼′	10° 1½′
221	27 16	38 56½	48 11	15 47½
552	33 33½	31 35¼	47 7½	19 28¼
331	37 51¾	26 31½	46 30½	22 0
772	40 59	22 50⅔	46 7½	23 50½
441	43 20½	20 2¾	45 52	25 14½

TRAPEZOHEDRONS.

	Edge B	Edge C	Angle on	Angle on
Cf. Fig. 113.	211 ∧ 2$\bar{1}$1, etc	211 ∧ 121, etc.	a(100)	o(111)
411	27° 16′	60° 0′	19° 28¼	35° 15¾′
722	30 43¼	55 50¾	22 0	32 44
311	35 5¾	50 28¾	25 14¼	29 29¾
522	40 45	43 20½	29 29¾	25 14¼
211	48 11½	33 33½	35 15¾	19 28¼
322	58 2	19 45	43 18¾	11 25¼

HEXOCTAHEDRONS.

	Edge A	Edge B	Edge C	Angle on	Angle on
Cf. Fig. 121.	321 ∧ 312, etc.	321 ∧ 32$\bar{1}$, etc.	321 ∧ 231, etc.	a(100)	o(111)
421	17° 45¼′	25° 12½′	35° 57′	29° 12¼′	28° 6½′
531	27 39½	19 27¾	27 39½	32 18¾	28 33¾
321	21 47¼	31 0¼	21 47¼	36 42	22 12½
432	15 5½	43 36¼	15 5½	42 1¾	15 13½
431	32 12¼	22 37¼	15 56½	38 19¾	25 4

2. PYRITOHEDRAL CLASS (2). PYRITE TYPE

(Dyakisdodecahedral or Pentagonal Hemihedral Class)

67. Typical Forms and Symmetry. — The typical forms of the pyritohedral class are the *pyritohedron*, or pentagonal dodecahedron, Figs. 129, 130, and the *diploid*, or dyakisdodecahedron, Fig. 135. The symmetry of these forms, as of the class as a whole, is as follows: The three crystallographic axes are axes of binary symmetry only; there are also four diagonal axes of trigonal symmetry coinciding with the octahedral axes. There are but three planes of symmetry; these coincide with the planes of the crystallographic axes and are parallel to the faces of the cube.

The stereographic projection in Fig. 128 shows the distribution of the faces of the general form (*hkl*), diploid, and thus exhibits the symmetry of the class. This should be carefully compared with the corresponding pro-

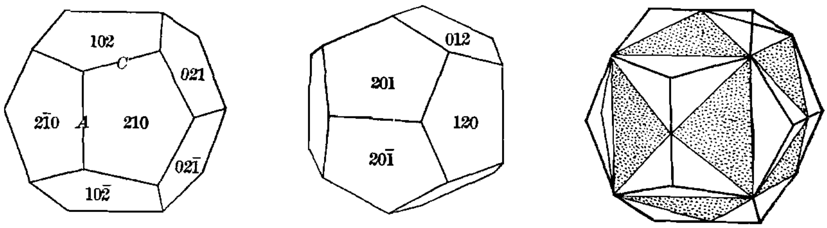
* A fuller list is given in the Introduction to Dana's System of Mineralogy, 1892, pp. xx–xxiii.

jection (Fig. 91) for the normal class, so that the lower grade of symmetry here present be thoroughly understood. In studying the forms described and illustrated in the following pages, this matter of symmetry, especially in relation to that of the normal class, should be continually before the mind.

It will be observed that the faces of both the pyritohedron (Fig. 129) and the diploid (Fig. 135) are arranged in parallel pairs, and on this account these forms have been sometimes called *parallel hemihedrons.* Further, those authors who prefer to describe these forms as cases of hemihedrism call this type parallel-faced hemihedrism or pentagonal hemihedrism.

68. Pyritohedron. — The *pyritohedron* (Fig. 129) is so named because it is a typical form with the common species, pyrite. It is a solid bounded by twelve faces, each of which is a pentagon, but with one edge (*A*, Fig. 129) longer than the other four similar edges (*C*). It is often called a pentagonal dodecahedron, and indeed it resembles closely the regular dodecahedron of geometry, in which the faces are regular pentagons. This latter form is, however, an impossible form in crystallography.

129 130 131



Pyritohedrons

Showing Relation between Pyritohedron and Tetrahexahedron

The general symbol is (*hk*0) or like that of the tetrahexahedron of the normal class. Hence each face is parallel to one of the axes and meets the other two axes at unequal distances. Common forms are (410), (310), (210), (320), etc. Besides the positive pyritohedron, as (210), there is also the complementary negative form * shown in Fig. 130; the symbol is here (120). Other common forms are (250), (230), (130), etc.

The positive and negative pyritohedrons together embrace twenty-four faces, having the same position as the twenty-four like faces of the tetrahexahedron of the normal class. The relation between the tetrahexahedron and the pyritohedron is shown in Fig. 131, where the alternate faces of the tetrahexahedron (indicated by shading) are extended to form the faces of the pyritohedron.

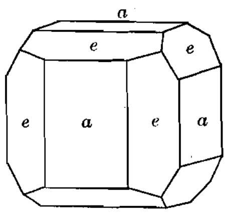
69. Combinations. — The faces of the pyritohedron replace the edges of the cube as shown in Fig. 132; this resembles Fig. 101 but here the faces make unequal angles with the two adjacent cubic faces. On the other hand, when the pyritohedron is modified by the cube, the faces of the latter truncate the longer edges of the pentagons.

---

* The negative forms in this and similar cases have sometimes distinct letters, sometimes the same as the positive form, but are then distinguished by a subscript accent, as *e*(210) and *e*, (120).

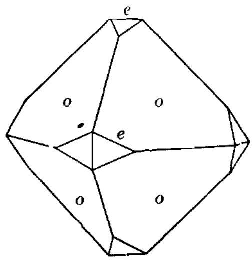
Fig. 133 shows the combination of the pyritohedron and octahedron, and in Fig. 134 these two forms are equally developed. The resulting combination bears a close similarity to the *icosahedron*, or regular twenty-faced solid, of geometry. Here, however, of the twenty faces, the eight octahedral are equilateral triangles, the twelve others belonging to the pyritohedron are isosceles triangles.

132



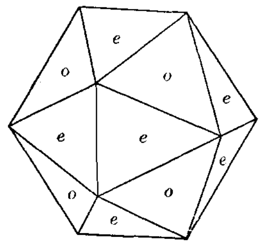
Cube and Pyritohedron

133



Octahedron and Pyritohedron

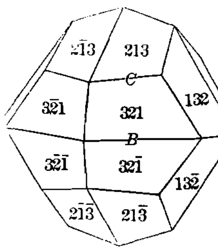
134



Octahedron and Pyritohedron

70. Diploid. — The *diploid* is bounded by twenty-four similar faces, each meeting the axes at unequal distances; its general symbol is hence (*hkl*), and common forms are *s*(321), *t*(421), etc. The form (321) is shown in Fig. 135; the symbols of its faces, as given, should be carefully studied. As seen in the figure, the faces are quadrilaterals or trapeziums; moreover, they are grouped in pairs, hence the common name diploid. It is also sometimes called a dyakisdodecahedron.

135



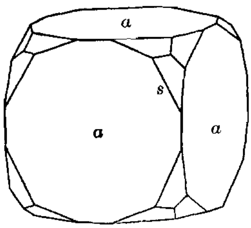
Diploid

The complementary negative form bears to the positive form of Fig. 135 the same relation as the negative to the positive pyritohedron. Its faces have the symbols 312, 231, 123, in the front octant, and similarly with the proper negative signs in the others. The positive and negative forms together obviously embrace all the faces of the hexoctahedron of the normal class. The diploid can be considered to be derived from the hexoctahedron by the extension of the alternate faces of the latter and the omission of the remaining faces, exactly as in the case of the pyritohedron and tetrahexahedron (Art. **68**).

In Fig. 136 the positive diploid is shown in combination with the cube. Here the three faces replace each of its solid angles. This combination form resembles that of Fig. 111, but the three faces are here unequally inclined upon two adjacent cubic faces. Other combinations of the diploid with the cube, octahedron, and pyritohedron are given in Figs. 137 and 138.

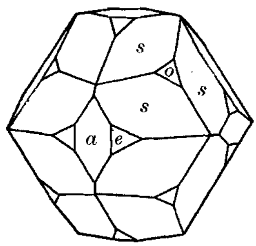
71. Other Forms. — If the pyritohedral type of symmetry be applied to planes each parallel to two of the axes, it is seen that this symmetry calls for six of these, and the resulting form is obviously a cube. This cube cannot be distinguished geometrically from the cube of the normal class, but it has its own characteristic molecular symmetry. Corresponding to this it is common to find cubes of pyrite with fine lines (striations) parallel to the alternate edges, as indicated in Fig. 139. These are due to the partial development of pyritohedral faces (210). On a normal cube similar striations, if present, must be parallel to both sets of edges on each cubic face.

136



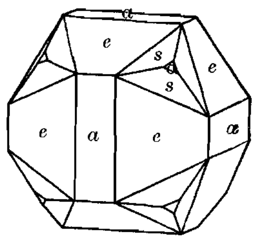
Cube and Diploid

137



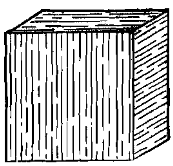
Cube, Octahedron and Diploid

138



Cube, Diploid and Pyritohedron

139



Pyrite. Striated Cube

Similarly to the cube, the remaining forms of this pyritohedral class, namely, (111), (110), (*hhl*), (*hll*), have the same geometrical form, respectively, as the octahedron, dodecahedron, the trisoctahedrons and trapezohedrons of the normal class. In molecular structure, however, these forms are distinct, each having the symmetry described in Art. **67**.

72. Angles. — The following tables contain the angles of some common forms.

PYRITOHEDRONS.

Cf. Fig. 129.	Edge A $210 \wedge 2\bar{1}0$, etc.	Edge C $210 \wedge 102$, etc.	Angle on *a*(100)	Angle on *o*(111)
410	28° 4¼′	76° 23½′	14° 2¼′	45° 33¾′
310	36 52¼	72 32½	18 26	43 5¼
520	43 36¼	69 49¾	21 48	41 22
210	53 7¾	66 25¼	26 34	39 14
530	61 55¾	63 49¼	30 57¾	37 37
320	67 22¾	62 30¾	33 41½	36 48½
430	73 44½	61 19	36 52¼	36 4¼
540	77 19¼	60 48¼	38 39½	35 45½
650	79 36¾	60 32½	39 48¼	35 35¾

DIPLOIDS.

Cf. Fig. 135.	Edge A $321 \wedge 3\bar{2}1$, etc.	Edge B $321 \wedge 32\bar{1}$, etc.	Edge C $321 \wedge 213$, etc.	Angle on *a*(100)	Angle on *o*(111)
421	51° 45¼′	25° 12½′	48° 11½′	29° 12¼′	28° 6½′
532	58 14½	37 51¾	35 20	35 47¾	20 30¾
531	60 56½	19 27¾	19 27¾	32 18¾	28 33¾
851	63 36¾	12 6	53 55¼	32 30¾	31 34
321	64 37½	31 0¼	38 12¼	36 42	22 12½
432	67 42½	43 36¼	26 17½	42 1¾	15 13½
431	72 4¾	22 37¼	43 3	38 19¾	25 4

3. TETRAHEDRAL CLASS (3). TETRAHEDRITE TYPE

(Hextetrahedral, Tetrahedral Hemihedral Class)

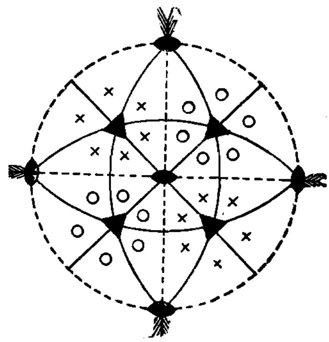
73. Typical Forms and Symmetry. — The typical form of this class, and that from which it derives its name, is the *tetrahedron*, shown in Figs.

141, 142. There are also three other distinct forms, shown in Figs. 149, 150, 151.

The symmetry of this class is as follows. There are three axes of binary symmetry which coincide with the crystallographic axes. There are also four diagonal axes of trigonal symmetry which coincide with the octahedral axes. There are six diagonal planes of symmetry. There is no center of symmetry.

The stereographic projection (Fig. 140) shows the distribution of the faces of the general form (*hkl*), hextetrahedron, and thus exhibits the symmetry of the class. It will be seen at once that the like faces are all grouped in the *alternate octants*, and this will be seen to be characteristic of all the forms peculiar to this class. The relation between the symmetry here described and that of the normal class must be carefully studied.

140

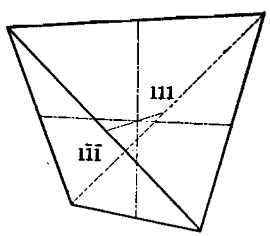


Symmetry of Tetrahedral Class

In distinction from the pyritohedral forms whose faces were in parallel pairs, the faces of the tetrahedron and the analogous solids are inclined to each other, and hence they are sometimes spoken of as *inclined hemihedrons*, and the type of so-called hemihedrism here illustrated is then called inclined or tetrahedral hemihedrism.

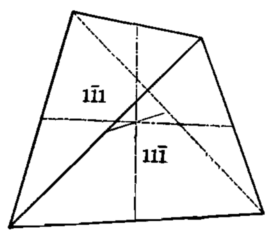
74. Tetrahedron. — The tetrahedron,* as its name indicates, is a four-faced solid, bounded by planes meeting the axes at equal distances. Its general symbol is (111), and the four faces of the positive form (Fig. 141) have the symbols 111, $\bar{1}\bar{1}1$, $1\bar{1}\bar{1}$, $\bar{1}1\bar{1}$. These correspond to four of the faces of the octahedron of the normal class (Fig. 93). The relation between the two forms is shown in Fig. 143.

141



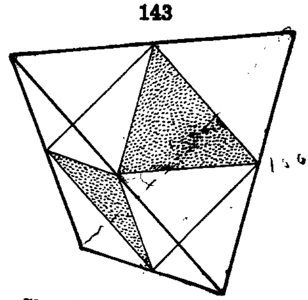
Positive Tetrahedron

142



Negative Tetrahedron

143



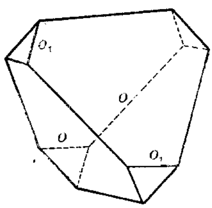
Showing Relation between Octahedron and Tetrahedron

Each of the four faces of the tetrahedron is an equilateral triangle; the (normal) interfacial angle is 109° 29′ 16″. The tetrahedron is the regular triangular pyramid of geometry, but crystallographically it must be so placed that the axes join the middle points of opposite edges, and one axis is vertical.

* This is one of the five regular solids of geometry, which include also the cube, octahedron, the regular pentagonal dodecahedron, and the icosahedron; the last two, as already noted, are impossible forms among crystals.

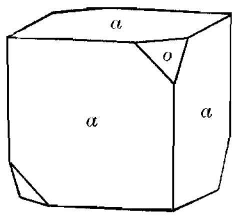
There are two possible tetrahedrons: the positive tetrahedron (111), designated by the letter *o*, which has already been described, and the negative tetrahedron, having the same geometrical form and symmetry, but the indices of its four faces are $\bar{1}11$, $1\bar{1}1$, $11\bar{1}$, $\bar{1}\bar{1}\bar{1}$. This second form is shown in Fig. 142; it is usually designated by the letter $o_{,}$. These two forms are, as stated above, identical in geometrical shape, but they may be distinguished in many cases by the tests which serve to reveal the molecular structure, particularly the etching-figures; also in many cases by pyro-electricity (see under boracite, p. 306), Art. **438.** It is probable that the positive and negative tetrahedrons of sphalerite (see that species) have a constant difference in this particular, which makes it possible to distinguish them on crystals from different localities and of different habit.

144



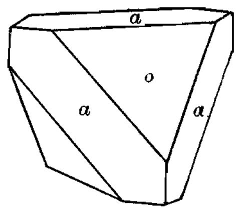
Positive and Negative Tetrahedrons

145



Cube and Tetrahedron

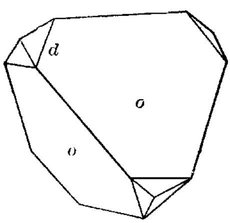
146



Tetrahedron and Cube

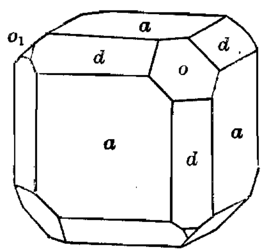
If both tetrahedrons are present together, the form in Fig. 144 results. This is geometrically an octahedron when the two forms are equally developed, but crystallographically it is always only a combination of two unlike forms, the positive and negative tetrahedrons, which can be distinguished as already noted.

147



Tetrahedron and Dodecahedron

148

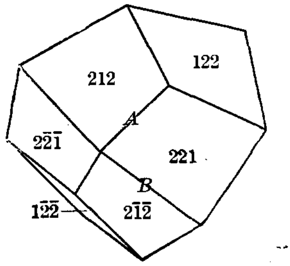


Boracite. Cube, Dodecahedron with Positive and Negative Tetrahedrons

The tetrahedron in combination with the cube replaces the alternate solid angles as in Fig. 145. The cube modifying the tetrahedron truncates its edges as shown in Fig. 146. The normal angle between adjacent cubic and tetrahedral faces is 54° 44′. In Fig. 147 the dodecahedron is shown modifying the positive tetrahedron, while in Fig. 148 the cube is the predominating form with the positive and negative tetrahedrons and dodecahedron.

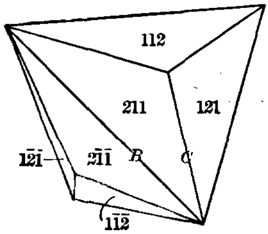
75. Other Typical Forms. — There are three other distinct types of solids in this class, having the general symbols (*hhl*), (*hll*), and (*hkl*). The first of these is shown in Fig. 149; here the symbol is (221). There are twelve faces, each a quadrilateral, belonging to this form, distributed as determined by the tetrahedral type of symmetry. They correspond to twelve of the faces of the trisoctahedron, namely, all those falling in alternate octants. This type of solid is sometimes called a *tetragonal tristetrahedron*, or a deltoid dodecahedron. It does not occur alone among crystals, but its faces are observed modifying other forms

149



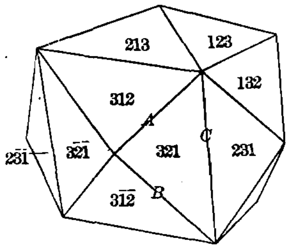
Tetragonal Tristetrahedron

150



Trigonal Tristetrahedron

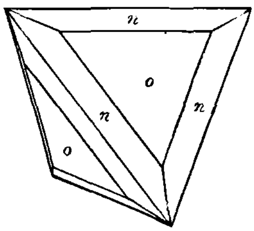
151



Hextetrahedron

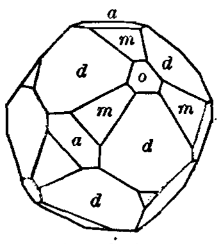
There is also a complementary negative form, corresponding to the positive form, related to it in precisely the same way as the negative to the positive tetrahedron. Its twelve faces are those of the trisoctahedron which belong to the other set of alternate octants.

152



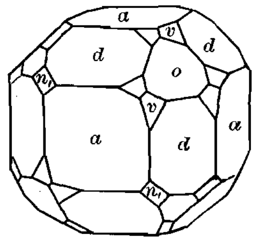
Tetrahedrite

153



Sphalerite

154



Boracite

Another form, shown in Fig. 150, has the general symbol (*hll*), here (211); it is bounded by twelve like triangular faces, distributed after the type demanded by tetrahedral symmetry, and corresponding consequently to the faces of the alternate octants of the form (*hll*) — the trapezohedron — of the normal class. This type of solid is sometimes called a *trigonal tristetrahedron* or trigondodecahedron.* It is observed both alone and in combination,

* It is to be noted that the tetragonal tristetrahedron has faces which resemble those of the trapezohedron (tetragonal trisoctahedron), although it is related not to this but to the trisoctahedron (trigonal trisoctahedron). On the other hand, the faces of the trigonal tristetrahedron resemble those of the trisoctahedron, though in fact related to the trapezohedron.

especially with the species tetrahedrite; it is much more common than the form (*hhl*). There is here again a complementary negative form. Fig. 152 shows the positive form *n*(211) with the positive tetrahedron, and Fig. 153 the form *m*(311) with *a*(100), *o*(111), and *d*(110). In Fig. 154, the negative form $n_{,}(2\bar{1}1)$ is present.

The fourth independent type of solids in this class is shown in Fig. 151. It has the general symbol (*hkl*), here (321), and is bounded by twenty-four faces distributed according to tetrahedral symmetry, that is, embracing all the faces of the alternate octants of the forty-eight-faced hexoctahedron. This form is sometimes called a *hextetrahedron* or *hexakistetrahedron.* The complementary negative form $(h\bar{k}l)$ embraces the remaining faces of the hexoctahedron. The positive hextetrahedron, *v*(531), is shown in Fig. 154 with the cube, octahedron, and dodecahedron, also the negative trigonal tristetrahedron $n_{,}(2\bar{1}1)$.

76. If the tetrahedral symmetry be applied in the case of planes each parallel to the two axes, it will be seen that there must be six such faces. They form a *cube* similar geometrically to the cube both of the normal and pyritohedral class but differing in its molecular structure, as can be readily proved, for example, by pyro-electricity (Art. **438**). Similarly in the case of the planes having the symbol (110), there must be twelve faces forming a rhombic dodecahedron bearing the same relation to the like geometrical form of the normal class. The same is true again of the planes having the position expressed by the general symbol (*hk*0); there must be twenty-four of them and they together form a tetrahexahedron.

In this class, therefore, there are also seven types of forms, but only four of them are geometrically distinct from the corresponding forms of the normal class.

77. Angles. — The following tables contain the angles of some common forms:

TETRAGONAL TRISTETRAHEDRONS.

	Edge A	Edge B	Angle on	Angle on
Cf. Fig. 149.	221 ∧ 212, etc.	221 ∧ $2\bar{1}\bar{2}$, etc.	*a*(100)	*o*(111)
332	17° 20½′	97° 50¼′	50° 14¼′	10° 1½′
221	27 16	90 0	48 11½	15 47½
552	33 33½	84 41	47 7½	19 28⅓
331	37 51¾	80 55	46 30½	22 0

TRIGONAL TRISTETRAHEDRONS.

	Edge B	Edge C	Angle on	Angle on
Cf. Fig. 150	211 ∧ $2\bar{1}\bar{1}$, etc.	211 ∧ 121, etc.	*a*(100)	*o*(111)
411	38° 56½′	60° 0′	19° 28¼′	35° 15¾′
722	44 0¼	55 50¾	22 0	32 44
311	50 28¾	50 28¾	25 14¼	29 29¾
522	58 59½	43 20½	29 29¾	25 14¼
211	70 31¾	33 33½	35 15¾	19 28⅓
322	86 37¾	19 45	43 18¾	11 25¼

HEXTETRAHEDRONS.

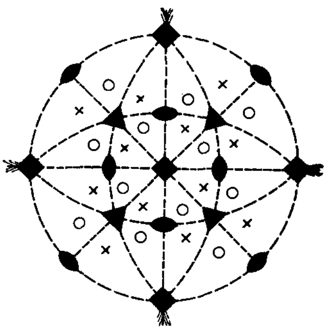
	Edge A	Edge B	Edge C	Angle on	Angle on
Cf. Fig. 151.	321 ∧ 312, etc.	321 ∧ $3\bar{1}\bar{2}$, etc.	321 ∧ 231, etc.	*a*(100)	*o*(111)
531	27° 39⅔′	57° 7⅓′	27° 39⅔′	32° 18⅔′	28° 33⅔′
321	21 47¼	69 4½	21·47¼	36 42	22 12½
432	15 5½	82 4⅓	15 5½	42 1¾	15 13½
431	32 12¼	67 22¾	15 56½	38 19¾	25 4

4. PLAGIOHEDRAL CLASS (4). CUPRITE TYPE.

(Pentagonal Icositetrahedral, Plagiohedral Hemihedral Class)

78. Typical Forms and Symmetry. — The fourth class under the isometric system is called the plagiohedral or gyroidal class because the faces of the general form (*hkl*) are arranged in spiral order. This is shown on the stereographic projection, Fig. 155, and also in Figs. 156, 157, which represent the single typical form of the class. These two complementary solids together embrace all the faces of the hexoctahedron. They are distinguished from one another by being called respectively right-handed and left-handed *pentagonal icositetrahedrons.* The other forms of the class are geometrically like those of the normal class.

155

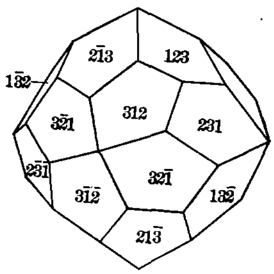


Symmetry of Plagiohedral Class

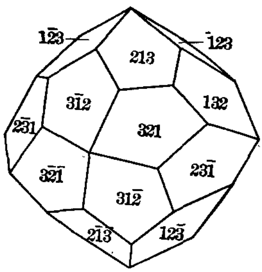
The symmetry characteristic of the class in general is as follows:

There are no planes of symmetry and no center of symmetry. There are, however, three axes of tetragonal symmetry normal to the cubic faces, four axes of trigonal symmetry normal to the octahedral faces, and six axes of binary symmetry normal to the faces of the dodecahedron. In other words, it has all the axes of symmetry of the normal class while without planes or center of symmetry.

156

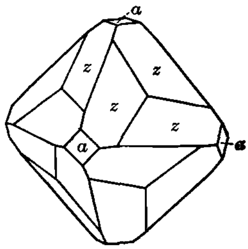


157



Right and Left-handed Pentagonal Icositetrahedrons

158



Cuprite

79. It is to be noted that the two forms shown in Figs. 156, 157 are alike geometrically, but are not superposable; in other words, they are related to one another as is a right- to a left-hand glove. They are hence said to be *enantiomorphous,* and, as explained elsewhere, the crystals belonging here may be expected to show circular light polarization. It will be seen that the complementary positive and negative forms of the preceding classes, unlike those here, may be superposed by being rotated 90° about one of the crystallographic axes. This distinction between positive and negative forms, and between right- and left-handed enantiomorphous forms, exists also in the case of the classes of several of the other systems.

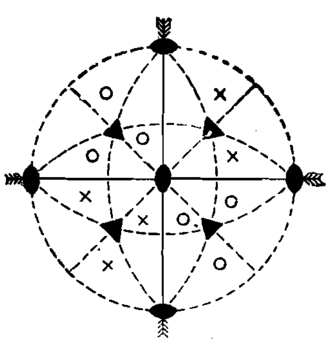
This class is rare among minerals; it is represented by cuprite, sal ammoniac, sylvite, and halite. It is usually shown by the distribution of the small modifying faces, or by the form of the etching figures. Fig. 158 shows a crystal of cuprite from Cornwall (Pratt) with the form $z(13 \cdot 10 \cdot 12)$.

5. TETARTOHEDRAL CLASS (5). ULLMANNITE TYPE.

(Tetrahedral Pentagonal Dodecahedral Class)

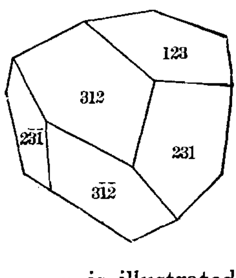
80. Symmetry and Typical Forms. — The fifth remaining possible class under the isometric system is illustrated by Fig. 160, which represents the twelve-faced solid corresponding to the general symbol (*hkl*). The distribution of its faces is shown in the projection, Fig. 159. This form is sometimes called a tetrahedral-pentagonal dodecahedron. It is seen to have one-fourth as many faces as the form (*hkl*) in the normal class, hence there are four similar solids which together embrace all the faces of the hexoctahedron. These four solids, which are distinguished as right-handed (positive and negative) and left-handed (positive and negative), are enantiomorphous, like those of Figs. 156 and 157, and hence the salts crystallizing here may be expected to also show circular polarization. The remaining forms of the class are (besides the cube and rhombic dodecahedron) the tetrahedrons, the pyritohedrons, the tetragonal and trigonal tristetrahedrons; geometrically they are like the solids of the same names already described. This class has no plane of symmetry and no center of symmetry. There are three axes of binary symmetry normal to the cubic faces, and four axes of trigonal symmetry normal to the faces of the tetrahedron.

159

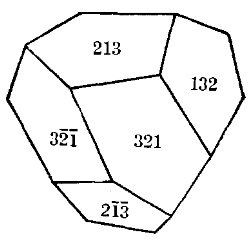


Symmetry of Tetartohedral Class

160



161



This group is illustrated by artificial crystals of barium nitrate, strontium nitrate, sodium chlorate, etc. Further, the species ullmannite, which shows sometimes pyritohedral and again tetrahedral forms, both having the same composition, must be regarded as belonging here.

Mathematical Relations of the Isometric System

81. Most of the problems arising in the isometric system can be solved at once by the right-angled triangles in the sphere of projection (Fig. 125) without the use of any special formulas.

It will be remembered that the angles between a cubic face, as 100, and the adjacent face of a tetrahexahedron, 310, 210, 320, etc., can be obtained at once, since the tangent of this angle is equal to $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, or in general $\frac{k}{h}$

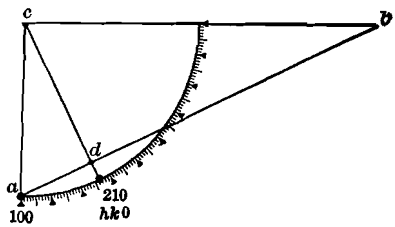
$$\tan (hk0 \wedge 100) = \frac{k}{h}.$$

162

$ac = k = 1$
$bc = h = 2$
$\angle\ adc = 90°$

$$\tan \angle\ abc = \frac{ac}{bc} = \frac{k}{h} = \frac{1}{2}$$

$$\left.\begin{array}{c}\angle\ abc \\ (100) \wedge (210)\end{array}\right\} = 26°\ 34'$$



This relation is illustrated in Fig. 162, which also shows the method of graphically determining the indices of a tetrahexahedron, the angle between one of its faces and an adjacent cube face being given.

Since all the forms of a given symbol under different species have the same angles, the tables of angles already given are very useful.

These and similar angles may be calculated immediately from the sphere, or often more simply by the formulas given in the following article.

82. Formulas. — (1) The distance of the pole of any face P(hkl) from the cubic faces is given by the following equations. Here Pa is the distance between (hkl) and (100); Pb is the distance between (hkl) and (010); and Pc that between (hkl) and (001).

These equations admit of much simplification in the various special cases, for ($hk0$), (hhl), etc.:

$$\cos^2 \text{P}a = \frac{h^2}{h^2 + k^2 + l^2}; \quad \cos^2 \text{P}b = \frac{k^2}{h^2 + k^2 + l^2}; \quad \cos^2 \text{P}c = \frac{l^2}{h^2 + k^2 + l^2}$$

(2) The distance between the poles of any two faces P(hkl) and Q(pqr) is given by the following equation, which in special cases may also be more or less simplified:

$$\cos \text{PQ} = \frac{hp + kq + lr}{\sqrt{(h^2 + k^2 + l^2)(p^2 + q^2 + r^2)}}.$$

(3) The calculation of the supplement interfacial or normal angles for the several forms may be accomplished as follows:

Trisoctahedron. — The angles A and B are, as before, the supplements of the interfacial angles of the edges lettered as in Fig. 110.

$$\cos \text{A} = \frac{h^2 + 2hl}{2h^2 + l^2}; \qquad \cos \text{B} = \frac{2h^2 - l^2}{2h^2 + l^2}.$$

For the *tetragonal-tristetrahedron* (Fig. 149), $\cos \text{B} = \frac{h^2 - 2hl}{2h^2 + l^2}.$

Trapezohedron (Fig. 113). B and C are the supplement angles of the edges as lettered in the figure.

$$\cos \text{B} = \frac{h^2}{h^2 + 2l^2}; \qquad \cos \text{C} = \frac{2hl + l^2}{h^2 + 2l^2}.$$

For the *trigonal-tristetrahedron* (Fig. 150), $\cos \text{B} = \frac{h^2 - 2l^2}{h^2 + 2l^2}.$

Tetrahexahedron (Fig. 104).

$$\cos \text{A} = \frac{h^2}{h^2 + k^2}; \qquad \cos \text{C} = \frac{2hk}{h^2 + k^2}.$$

For the *pyritohedron* (Fig. 129), $\cos A = \frac{h^2 - k^2}{h^2 + k^2}$; $\cos C = \frac{hk}{h^2 + k^2}$.

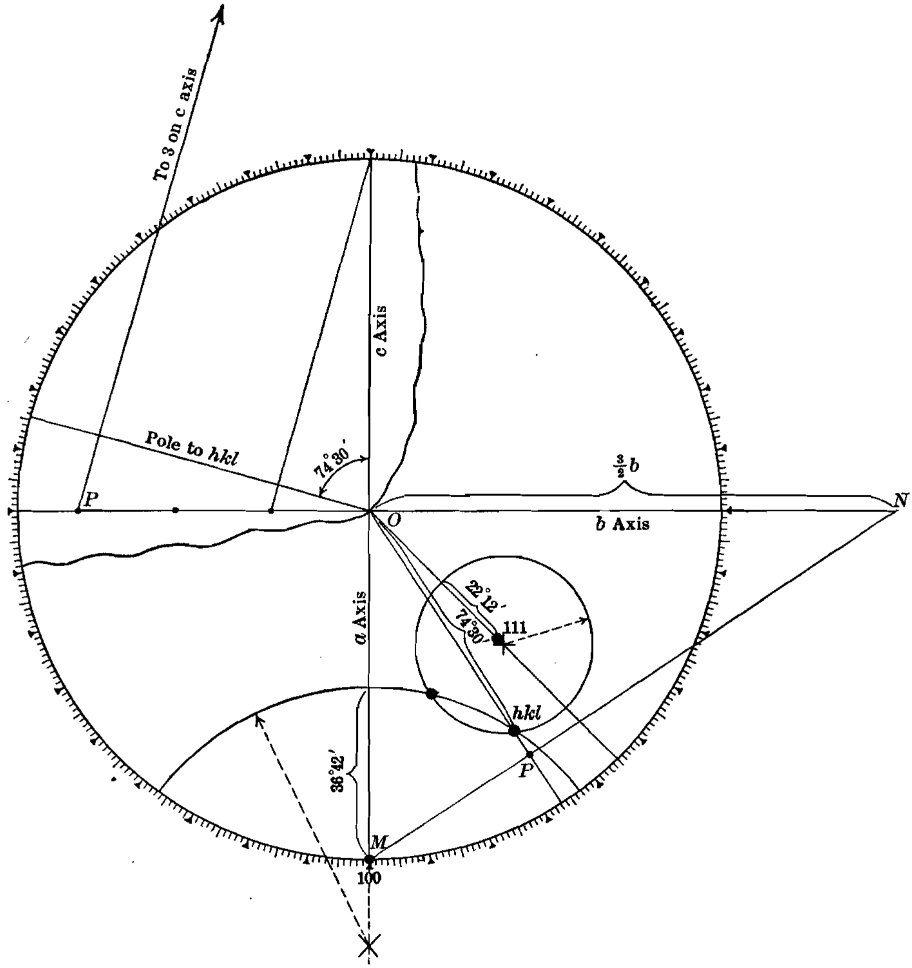
Hexoctahedron (Fig. 122).

$$\cos A = \frac{h^2 + 2kl}{h^2 + k^2 + l^2}; \quad \cos B = \frac{h^2 + k^2 - l^2}{h^2 + k^2 + l^2}; \quad \cos C = \frac{2hk + l^2}{h^2 + k^2 + l^2}.$$

For the *diploid* (Fig. 135), $\cos A = \frac{h^2 - k^2 + l^2}{h^2 + k^2 + l^2}$; $\cos C = \frac{kl + lh + hk}{h^2 + k^2 + l^2}$.

For the *hextetrahedron* (Fig. 151), $\cos B = \frac{h^2 - 2kl}{h^2 + k^2 + l^2}$.

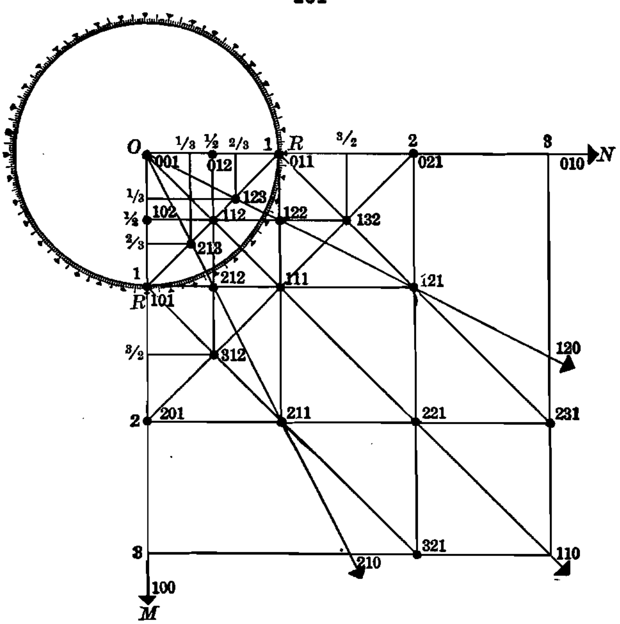
163



83. To determine the indices of any face (*hkl*) of an isometric form, given the position of its pole on the stereographic projection. As an illustrative example of this problem the hexoctahedron (321) has been taken. It is assumed that the angles 100 ∧ 321 = 36° 42′

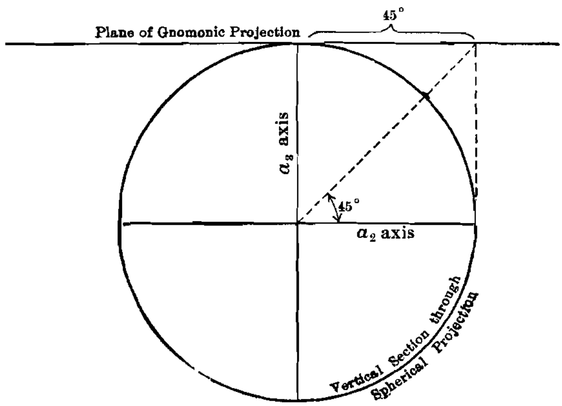
and 111 ∧ 321 = 22° 12′ are given. The methods by which the desired pole is located from these measurements have been described on page 38 and are illustrated in Fig. 163. Having located the pole (*hkl*) a line is drawn through it from the center *O* of the projection. This line *O–P* represents the intersection with the horizontal plane (which is the plane of the horizontal crystal axes, *a* and *b*) of a plane which is normal to the crystal face (*hkl*). Since two planes which are at right angles to each other will intersect a third plane in lines that are at right angles to each other, it follows that the plane of the hexoctahedral face will intersect the plane of the horizontal axes in a line at right angles to *O–P*. If, therefore, the distance *O–M* be taken as representing unity on the *a* axis and the line *M–P–N* be drawn at right angles to *O–P* the distance *O–N* will represent the intercept of the face in question upon the *b* axis. *O–N* is found in this case to be $\frac{3}{2}$ *O–M* in value. The intercepts upon the two horizontal axes are, therefore 1*a*, $\frac{3}{2}$*b*. The plotting of the intercept upon the *c* axis is shown in the upper left hand quadrant of the figure. The angular distance from *O* to the pole (*hkl*) is measured by the stereographic protractor as 74° 30′. This angle is then laid off from the line representing the *c* axis and the line representing the pole (*hkl*) is drawn. The distance *O–P* is transferred from the lower part of the figure. Then we can construct the right triangle, the vertical side of which is the *c* axis, the horizontal side is this line *O–P* (the intersection of the plane which is normal to the crystal face with the horizontal plane) and the hypothenuse is a line lying in the face and therefore at right angles to the pole of the face. This line would intersect the *c* axis at a distance equal to 3*O–M*. The same relation may be shown by starting this last line from a point on the *c* axis which is at a distance from the center of the figure equal to *O–M*. In this case the intercept on the horizontal line *O–P* would be at one third its total length. By these constructions the parameters of the face in question are shown to be 1*a*, $\frac{3}{2}$*b*, 3*c*, giving (321) as its indices.

164



84. To determine the indices of the faces of isometric forms, given the positions of their poles on the gnomonic projection. — As an illustrative example of this problem the lower right hand quadrant of the gnomonic projection of isometric forms, Fig. 126, has been taken and reproduced in Fig. 164. The lines O–M and O–N are at right angles to each other and may represent the horizontal crystallographic axes a_1 and a_2. If from each pole of the projection lines are drawn perpendicular to these two axial directions it will be seen that the intercepts made upon these lines have rational relations to each other. And since we are dealing with the isometric system in which the crystallographic axes are all alike and interchangeable with each other, it follows that the different intercepts upon O–M and O–N are identical. The distance O–R (*i.e.* the distance from the center to the 45° point of the projection) must equal the unit length of the axes. That this is true is readily seen by the consideration of Fig. 165. The intercepts of the lines drawn from the different poles to the lines O–M and O–N are found to be $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $\frac{3}{2}$, 2 and 3 times this unit distance. To find the Miller indices of any face represented, it is only necessary to

165



take the intercepts of the two lines drawn from its pole upon the two axes a_1 and a_2, place these numbers in their proper order and add a 1 as a third figure and then if necessary clear of fractions. Take for example the hexoctahedron face with indices 312. The lines drawn from its pole intercept the axes at $\frac{3}{2}a_1$ and $\frac{1}{2}a_2$, which gives the expression $\frac{3}{2}$ $\frac{1}{2}$ 1, which, again, on clearing of fractions, yields 312, the indices of the face in question. In the case of a face parallel to the vertical axis, the pole of which lies at infinity on the gnomonic projection, the indices may be obtained by taking any point on the radial line that points to the position of the pole and dropping perpendiculars to the lines representing the two horizontal axes. The relative intercepts formed upon these axes will give the first two numbers of the required indices while the third number will necessarily be 0.