

course with a known mineral, whose forms have already had indices assigned to them, the intercept that shall be considered as 1 is fixed.

If we take r as equivalent to the radius of the fundamental circle of the projection, q as equal to the chosen intercept upon the b crystallographic axis and p that upon the a axis, then the axial ratio can be derived from the following expressions:

$$\frac{b}{c} = \frac{r}{q}; \quad \frac{a}{c} = \frac{r}{p}.$$

The proof of these relationships is similar to that already given under the Tetragonal System, Art. 117, p. 93.

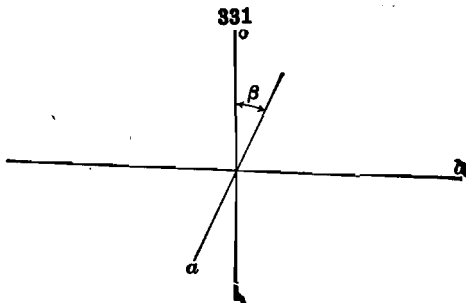
193. To determine, by plotting, the indices of a face upon an orthorhombic crystal, given the position of its pole upon the gnomonic projection and the axial ratio of the mineral. The method of construction in this case is the reverse of that given in the problem above and is essentially the same as given under the Isometric and Tetragonal Systems, Arts. 84 and 118. In the case of an orthorhombic mineral the intercepts of the perpendiculars drawn from the pole of the face to the a and b axes must be expressed in each case in terms of the unit intercept on that axis. These values, p and q , can be determined from the equations given in the preceding problem.

V. MONOCLINIC SYSTEM

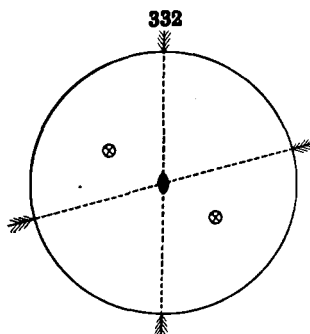
(Oblique System)

194. **Crystallographic Axes.** — The *monoclinic system* includes all the forms which are referred to three unequal axes, having one of their axial inclinations oblique.

The axes are designated as follows: the inclined or clino-axis is a ; the ortho-axis is b , the vertical axis is c . The acute angle between the axes a and c is represented by the letter β ; the angles between a and b and b and c are right angles. See Fig. 331. When properly orientated the inclined axis, a , slopes down toward the observer, the b axis is horizontal and parallel to the observer and the c axis vertical.



Crystal Axes of Orthoclase
 $a:b:c = 0.66:1:0.55$. $\beta = 64^\circ$



Symmetry of Normal Class

1. NORMAL CLASS (28). GYPSUM TYPE

(Prismatic or Holohedral Class)

195. **Symmetry.** — In the normal class of the monoclinic system there is one plane of symmetry and one axis of binary symmetry normal to it. The plane of symmetry is always the plane of the axes a and c , and the axis of symmetry coincides with the axis b , normal to this plane. The position of one axis (b) and that of the plane of the other two axes (a and c) is thus fixed by the symmetry; but the latter axes may occupy different positions in this plane. Fig. 332 shows the typical stereographic projection, projected on the plane of symmetry. Figs. 347, 348 are the projections of an actual

crystal of epidote; here, as is usual, the plane of projection is normal to the prismatic zone.

196. Forms. — The various forms * belonging to this class, with their symbols, are given in the following table. As more particularly explained later, an orthodome includes two faces only, and a pyramid four only.

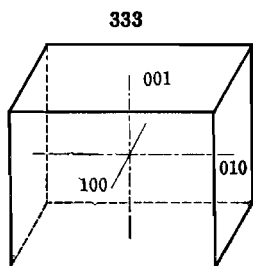
	Symbols
1. Orthopinacoid or <i>a</i> -pinacoid.....	(100)
2. Clinopinacoid or <i>b</i> -pinacoid.....	(010)
3. Base or <i>c</i> -pinacoid.....	(001)
4. Prisms.....	(<i>h</i> <i>k</i> 0)
5. Orthodomes.....	{ (<i>h</i> 0 <i>l</i>) (<i>h</i> 0 <i>l</i>)
6. Clinodomes.....	{ (0 <i>k</i> <i>l</i>) (0 <i>k</i> <i>l</i>)
7. Pyramids.....	{ (<i>h</i> <i>k</i> <i>l</i>) (<i>h</i> <i>k</i> <i>l</i>)

197. Pinacoids. — The pinacoids are the orthopinacoid, clinopinacoid, and the basal plane.

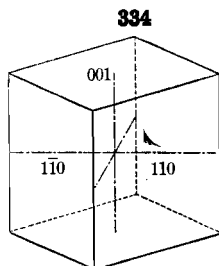
The *orthopinacoid*, (100), includes the two faces parallel to the plane of the ortho-axis *b* and the vertical axis *c*. They have the indices 100 and $\bar{1}00$. This form is designated by the letter *a*, since it is situated at the extremity of the *a* axis; it is hence conveniently called the *a*-face or *a*-pinacoid.

The *clinopinacoid*, (010), includes the two faces parallel to the plane of symmetry, that is, the plane of the clino-axis *a* and the axis *c*. They have the indices 010 and $0\bar{1}0$. The clinopinacoid is designated by the letter *b*, and is called the *b*-face or *b*-pinacoid.

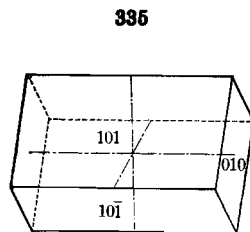
The *base* or *basal pinacoid*, (001), includes the two terminal faces, above and below, parallel to the plane of the axes *a*, *b*; they have the indices 001 and $00\bar{1}$. The base is designated by the letter *c*, and is often called the *c*-face or *c*-pinacoid. It is obviously inclined to the orthopinacoid, and the normal angle between the two faces ($100 \wedge 001$) is the acute axial angle β .



Ortho-, Clino-
and Basal Pinacoids



Prism and
Basal Pinacoid



Orthodomes
and Clinopinacoid

The *diametral prism*, formed by these three pinacoids, taken together, Fig. 333, is the analogue of the cube in the isometric system. It is bounded by three sets of unlike faces; it has four similar vertical edges; also four similar edges parallel to the axis *a*, but the remaining edges, parallel to the axis *b*, are of two sets. Of its eight solid angles there are two sets of

* On the general use of the terms pinacoid, prisms, domes, pyramids, see pp. 31, 122.

four each; the two above in front are similar to those below behind, and the two below in front to those above in behind.

198. Prisms. — The prisms are all of one type, the oblique rhombic prism. They may be divided into three classes as follows: the *unit prism*, (110), designated by the letter *m*, shown in Fig. 334; the *orthoprisms*, (*hk*0), where $h > k$, lying between *a*(100) and *m*(110), and the *clinoprisms*, (*hk*0) where $h < k$, lying between *m*(110) and *b*(010). The orthoprisms and clinoprisms correspond respectively to the macroprisms and brachyprisms of the orthorhombic system, and the explanation on p. 123 will hence make their relation clear. Common cases of these prisms are shown in the figures given later.

199. Orthodomes. — The four faces parallel to the ortho-axis *b*, and meeting the other two axes, fall into two sets of two each, having the general symbols (*h*0*l*) and (\bar{h} 0*l*). These forms are called *orthodomes*; they are strictly hemiorthodomes. For example, the unit orthodome (101) has the faces 101 and $\bar{1}0\bar{1}$; they would replace the two obtuse edges between *a*(100) and *c*(001) in Fig. 333. The other unit orthodome ($\bar{1}01$) has the faces $\bar{1}01$ and 10 $\bar{1}$, and they would replace the acute edges between *a*(100) and *c*(001). These two independent forms are shown together, with *b*(010), in Fig. 335.

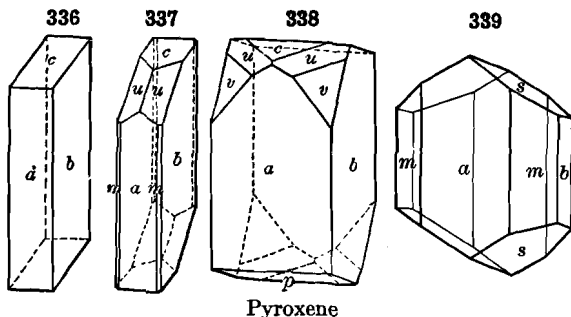
Similarly the faces 201, $20\bar{1}$ belong to the form (201), and $20\bar{1}$, 201 to the independent but complementary form ($20\bar{1}$).

200. Clinodomes. — The *clinodomes* are the forms whose faces are parallel to the inclined axis, *a*, while intersecting the other two axes. Their general symbol is hence (0*kl*) and they lie between the base (001) and the clinopinacoid (010). Each form has four faces; thus for the unit clinodome these have the symbols, 011, $0\bar{1}1$, $01\bar{1}$, 011 . The form *n*(021) in Fig. 342 is a clinodome.

201. Pyramids. — The *pyramids* in the monoclinic system are all hemipyramids, embracing four faces only in each form, corresponding to the general symbol (*hkl*). This obviously follows from the symmetry; it is shown, for example, in the fact already stated that the solid angles of the diametral prism (Fig. 333, see above), which are replaced by these pyramids, fall into two sets of four each. Thus any general symbol, as (321), includes the two independent forms (321) and ($3\bar{2}1$) with the faces

$$321, \quad 3\bar{2}1, \quad 32\bar{1}, \quad 3\bar{2}\bar{1}, \quad \text{and} \quad 321, \quad 3\bar{2}\bar{1}, \quad 32\bar{1}, \quad 3\bar{2}1.$$

The pyramids may also be divided into three classes as *unit pyramids*, (*hhl*); *orthopyramids*, (*hkl*), when $h > k$; or *clinopyramids*, (*hkl*), when $h < k$. These correspond respectively to the three prisms already named. They are analogous also to the unit pyramids, macropyramids, and brachypyramids of the orthorhombic system, and the explanation given on p. 124, should serve to make their relations clear. But it must be remembered that each general symbol embraces two forms, (*hkl*) and ($\bar{h}kl$) with four faces each, as above explained.



202. Illustrations. — Figs. 336–339 of pyroxene ($a : b : c = 1.092 : 1 : 0.589$, $\beta = 74^\circ = a(100) \wedge c(001)$) show typical monoclinic forms. Fig. 336 shows the diametral prism. Of the other forms, m is the unit prism (110); $p(\bar{1}01)$ is an orthodome; $u(111)$, $v(221)$, $s(\bar{1}11)$ are pyramids; for other figures see p. 475. Again, Figs. 340–342 represent common crystals of orthoclase ($a : b : c = 0.659 : 1 : 0.555$, $\beta = 64^\circ$). Here $z(130)$ is a prism; $x(\bar{1}01)$ and $y(\bar{2}01)$ are orthodomes; $n(021)$ is a clinodome; $o(\bar{1}11)$ a pyramid. Since (Fig. 340) c and x happen to make nearly equal angles with the vertical edge of the prism m , the combination often simulates an orthorhombic crystal.

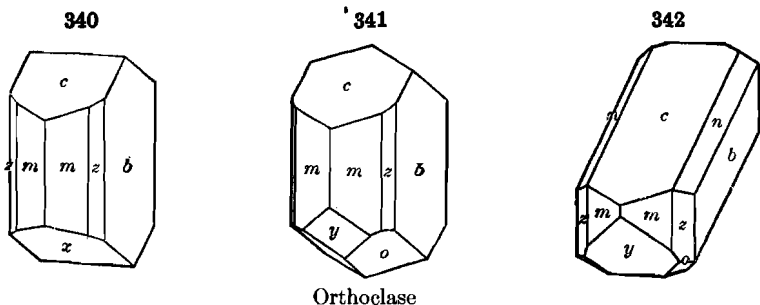
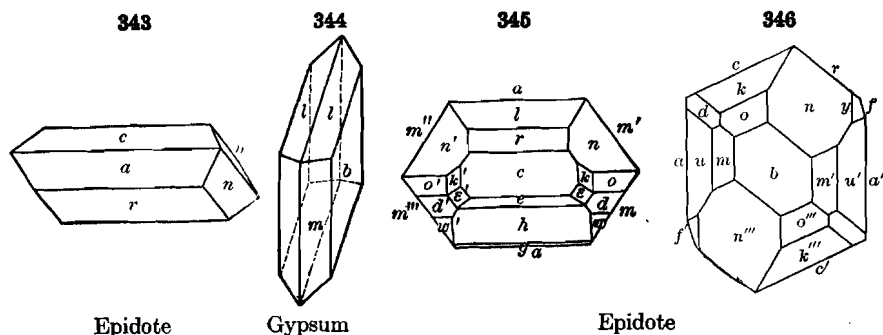


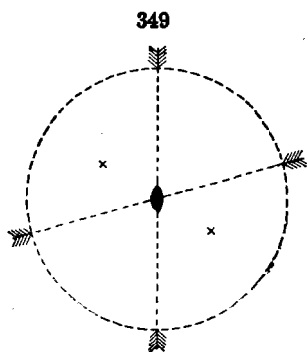
Fig. 343 shows a monoclinic crystal, epidote, prismatic in the direction of the ortho-axis; the forms are $a(100)$, $c(001)$, $r(\bar{1}01)$ and $n(\bar{1}11)$. Fig. 344 of gypsum is flattened $\parallel b(010)$; it shows the unit pyramid $l(111)$ with the unit prism $m(110)$.



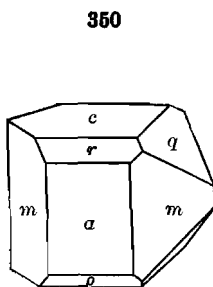
203. Projections. — Fig. 345 shows a projection of a crystal of epidote (cf. Fig. 897, p. 531) on a plane normal to the prismatic zone, and Fig. 346 one of a similar crystal on a plane parallel to $b(010)$; both should be carefully studied, as also the stereographic and gnomonic projections of the same species, Figs. 347, 348. The symbols of the prominent faces are given in the latter figures.

2. HEMIMORPHIC CLASS (29). TARTARIC ACID TYPE (*Sphenoidal Class*)

204. The *monoclinic-hemimorphic* class is characterized by a single axis



Symmetry of Hemimorphic Class



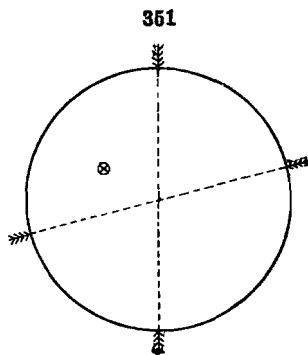
Tartaric Acid

of binary symmetry, the crystallographic axis b , but it has no plane of symmetry. It is illustrated by the stereographic projection (Fig. 349) made upon a plane parallel to $b(010)$. Fig. 350 shows a common form of tartaric acid; sugar crystals also belong here. The hemimorphic character is distinctly shown in the distribution of the clinodomes and pyramids; corresponding to this the

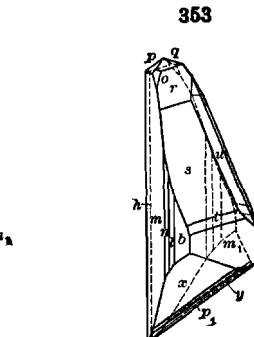
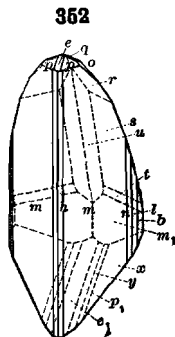
artificial salts belonging here often exhibit marked pyroelectrical phenomena.

3. CLINOHEDRAL CLASS (30). CLINOHEDRITE TYPE (*Domatic or Hemihedral Class*)

205. The *monoclinic-clinohedral* class is characterized by a single plane of symmetry, parallel to the clinopinacoid, $b(010)$, but it has no axis of symmetry. This symmetry is shown in the stereographic projection made upon a plane parallel to $b(010)$, Fig. 351. In this class, therefore, the forms parallel to the b axis, viz., $c(001)$, $a(100)$, and the orthodomes, are represented by a



Symmetry of Clinohedral Class



Clinohedrite

single face only. The other forms have each two faces, but it is to be noted that, with the single exception of the clinopinacoid $b(010)$, the faces of a given form are never parallel to each other. The name given to the class is based on this fact.

Several artificial salts belong here in their crystallization, but the only

known representative among minerals is the rare silicate, clinohedrite ($\text{H}_2\text{CaZnSiO}_6$),* a complex crystal of which is shown in two positions in Figs. 352, 353. As seen in these figures, the crystals of the group have a hemimorphic aspect with respect to their development in the direction of the vertical axis, although they cannot properly be called hemimorphic since this is not an axis of symmetry. The forms shown in Figs. 352 353 are as follows: pinacoid, $b(010)$; prisms, $m(110)$, $m_1(\bar{1}10)$, $h(320)$, $n(120)$, $l(130)$; orthodomes, $e(101)$, $e_1(\bar{1}0\bar{1})$; pyramids, $p(\bar{1}11)$, $p_1(\bar{1}\bar{1}\bar{1})$, $q(\bar{1}11)$, $r(\bar{3}31)$, $s(\bar{5}51)$, $t(771)$, $u(\bar{5}31)$, $o(\bar{1}31)$, $x(\bar{1}3\bar{1})$, $y(\bar{1}2\bar{1})$.

It is to be noted that crystals of the common species pyroxene (also of ægirite and titanite) occasionally show this habit in the distribution of their faces, but it is not certain that this may not be accidental.†

MATHEMATICAL RELATIONS OF THE MONOCLINIC SYSTEM

206. Choice of Axes. — It is repeated here (Art. 195) that the fixed position of the plane of symmetry establishes the direction of the plane of the a and c crystallographic axes and also of the axis b which is the symmetry axis and lies at right angles to this plane. The a and c axes, however, may have varying positions in the symmetry plane according to which faces are taken as the pinacoids $a(100)$ and $c(001)$, and which the unit pyramid, prism, or domes.

207. Axial and Angular Elements. — The *axial elements* are the *lengths* of the axes a and c in terms of the unit axis b , that is, the axial ratio, with also the acute angle of inclination of the axes a and c , called β . Thus for orthoclase the axial elements are:

$$a : b : c = 0.6585 : 1 : 0.5554 \quad \beta = 63^\circ 56\frac{1}{2}'.$$

The *angular elements* are usually taken as the angle $(100 \wedge 001)$ which is equal to the angle β ; also the angles between the three pinacoids 100, 010, 001, respectively, and the unit prism 110, the unit orthodome $(101 \text{ or } \bar{1}01)$ and the unit clinodome 011. Thus, again, for orthoclase, the angular elements are:

$$\begin{aligned} 001 \wedge 100 &= 63^\circ 56\frac{1}{2}', & 100 \wedge 110 &= 30^\circ 36\frac{1}{2}', \\ 001 \wedge \bar{1}01 &= 50^\circ 16\frac{1}{2}', & 001 \wedge 011 &= 26^\circ 31'. \end{aligned}$$

208. The mathematical relations connecting axial and angular elements are given in the following equations in which a , b , and c represent the unit lengths of the respective crystallographic axes.

$$a = \frac{\tan (100 \wedge 110)}{\sin \beta} \quad \text{or} \quad \tan (100 \wedge 110) = a \cdot \sin \beta; \quad (1)$$

$$c = \frac{\tan (001 \wedge 011)}{\sin \beta} \quad \text{or} \quad \tan (001 \wedge 011) = c \cdot \sin \beta; \quad (2)$$

$$\left. \begin{aligned} c &= \frac{a \cdot \tan (001 \wedge 101)}{\sin \beta - \cos \beta \cdot \tan (001 \wedge 101)} & \text{or} & \quad \tan (001 \wedge 101) = \frac{c \sin \beta}{a + c \cdot \cos \beta}, \\ c &= \frac{a \cdot \tan (001 \wedge \bar{1}01)}{\sin \beta + \cos \beta \cdot \tan (001 \wedge \bar{1}01)} & \text{or} & \quad \tan (001 \wedge \bar{1}01) = \frac{c \sin \beta}{a - c \cdot \cos \beta}. \end{aligned} \right\} \quad (3)$$

These relations may be made more general by writing in the several cases —

$$\text{in (1) } hk0 \text{ for } 110 \quad \text{and} \quad \frac{k}{h}a \text{ for } a; \quad \text{in (2) } 0kl \text{ for } 011 \quad \text{and} \quad \frac{k}{l}c \text{ for } c;$$

$$\text{in (3) } h0l \text{ for } 101 \quad \text{and} \quad \frac{h}{l}c \text{ for } c.$$

* Penfield and Foote, Am. J. Sc., 5, 289, 1898.

† See G. H. Williams, Am. J. Sc., 34, 275, 1887, 38, 115, 1889.

Also

$$\frac{c}{a} = \frac{\sin (001 \wedge 101)}{\sin (100 \wedge 101)} = \frac{\sin (001 \wedge \bar{1}01)}{\sin (\bar{1}00 \wedge \bar{1}01)},$$

and more generally

$$\frac{h}{a} \cdot \frac{c}{l} = \frac{\sin (001 \wedge h0l)}{\sin (100 \wedge h0l)} = \frac{\sin (001 \wedge \bar{h}0l)}{\sin (\bar{1}00 \wedge \bar{h}0l)}.$$

Note also that

$$\tan \phi = a \quad \text{and} \quad \tan \zeta = c,$$

where ϕ is the angle (Fig. 347) between the zone-circles (001, 100) and (001, 110); also ζ is the angle between (100, 001) and (100, 011).

All the above relations are important and should be thoroughly understood.

209. The problems which usually arise have as their object either the deducing of the axial elements, *i.e.*, the angle β and the values of a and c in terms of $b (= 1)$, from three measured angles, or the finding of any required interfacial angles from these elements or from the fundamental angles.

The simple relations of the preceding article connect the angular and axial elements, and beyond this all ordinary problems can be solved * either by the solution of spherical triangles on the sphere of projection, or by the aid of the cotangent (and tangent) relation.

It is to be noted, in the first place, that all great circles on the sphere of projection (see the stereographic projection, Fig. 347) from 010 cut the zone circle 100, 001, $\bar{1}00$ at right angles, but those from 100 cut the zone circles 010, 001, 0 $\bar{1}0$ obliquely, as also those from 001 cutting the zone circle 100, 010, $\bar{1}00$.

210. Tangent and Cotangent Relations. — The simple *tangent relation* holds good for all zones from 010 to any pole on the zone circle 100, 001, $\bar{1}00$; in other words, for the prisms, clinodomes, and also zones of pyramids in which the ratio of $h : l$ is constant (from 001 to $h0l$ or to $\bar{h}0l$). Thus it is still true, as in the orthorhombic system, that the tangents of the angles of the prisms 210, 110, 120, 130 from 100 are in the ratio of $\frac{1}{2} : 1 : 2 : 3$, or, more generally, that

$$\frac{\tan (100 \wedge hk0)}{\tan (100 \wedge 110)} = \frac{k}{h} \quad \text{or} \quad \frac{\tan (010 \wedge hk0)}{\tan (010 \wedge 110)} = \frac{h}{k}.$$

Also for the clinodomes the tangents of the angles of 012, 011, 021 from 001 are in the ratio of $\frac{1}{2} : 1 : 2$, etc. A similar relation holds for the tangents of the angles of pyramids in the zones mentioned, as 121, 111, 212, etc.

For zones other than those mentioned as from 100 to a clinodome, or from 001 to a prism, the more general *cotangent formula* given in Art. 49 must be employed. This relation is simplified for certain common cases.

For any zone starting from 001, as the zone 001, 100, or 001, 110, or 001, 210, etc.; if two angles are known, *viz.*, the angles between 001 and those two faces in the given zone which fall (1) in the zone 010, 101, and (2) in the prismatic zone 010, 100; then the angle between 001 and any other face in the given zone can be calculated.

Thus,

$$\begin{array}{lll} \text{Let } 001 \wedge 101 = PQ & \text{and} & 001 \wedge 100 = PR, \\ \text{or " } 001 \wedge 111 = PQ & \text{"} & 001 \wedge 110 = PR, \\ \text{or " } 001 \wedge 212 = PQ & \text{"} & 001 \wedge 210 = PR, \text{ etc.} \end{array}$$

Then for these, or any similar cases, the angle (PS) between 001 and any face in the given zone (as 201, or 221, or 421, etc., or in general $h0l$, hhl , etc.) is given by the equation

$$\frac{\cot PS - \cot PR}{\cot PQ - \cot PR} = \frac{l}{h}.$$

For the corresponding zones from 001 to $\bar{1}00$, to $\bar{1}10$, to $\bar{2}10$, etc., the expression has the same value; but here

$$\begin{array}{lll} PQ = 001 \wedge \bar{1}01, & PR = 001 \wedge \bar{1}00, & PS = 001 \wedge \bar{h}0l. \\ \text{or} & 001 \wedge \bar{1}11, \text{ etc.,} & 001 \wedge \bar{1}10, \text{ etc.,} & 001 \wedge \bar{h}hl, \text{ etc.} \end{array}$$

*. The general formulas, from which it is possible to calculate directly the angles between any face and the pinacoids, or the angle between any two faces whatever, are so complex as to be of little value.

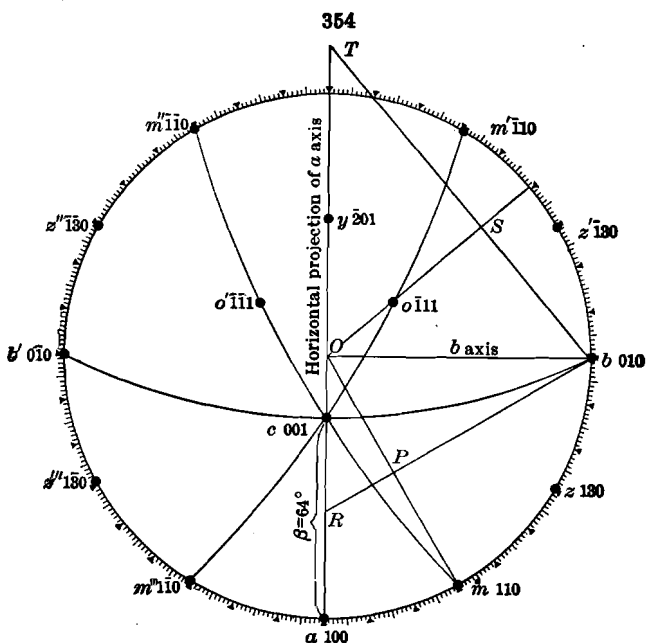
If, however, 100 is the starting-point, and

$$\begin{array}{ll} 100 \wedge 101 = PQ & 100 \wedge 001 = PR, \\ \text{or } 100 \wedge 111 = PQ, & 100 \wedge 011 = PR, \text{ etc.,} \end{array}$$

then the relation becomes

$$\frac{\cot PS - \cot PR}{\cot PQ - \cot PR} = \frac{h}{l}$$

211 To determine, by plotting, the axial elements of a monoclinic crystal, given the stereographic projection of its forms. As an example of this problem it is assumed that an orthoclase crystal similar to the one shown in Fig. 341 has been measured and the poles of its faces located on the stereographic projection, Fig. 354. The inclination of the a axis or the angle β is given directly by measuring, by means of the stereographic protractor, the angular distance between the poles of $a(100)$ and $c(001)$. In the present case the $a(100)$ form does not actually occur on the crystal. β is measured as 64° . If the base is not present upon the crystal it will be usually possible to locate its position by means of some zone circle on which it must lie. In the present case the great circle of the zone of $m'(110)$, $o(111)$, $m''(1\bar{1}0)$ will cross the front to back line (zone of the orthodomes) at the point of the pole to the base.



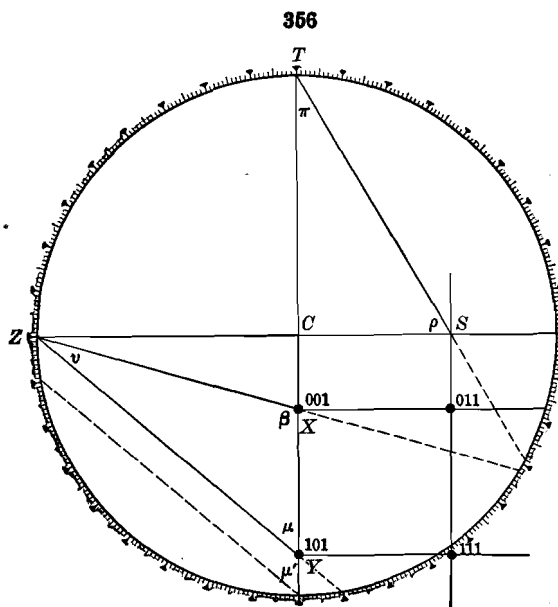
Determination of Axial Elements of Orthoclase from Stereographic Projection

The ratio between the lengths of the a and b axes can be readily determined from the position of the pole, $m(110)$. Draw the radial line $O-P$ from the center of the projection to $m(110)$. From the end of the b axis draw a line at right angles to $O-P$. This represents the intersection of the prism face with the horizontal plane and the distance $O-R$ gives the intercept of the prism upon the horizontal projection of the a axis. The distance $O-R$ therefore is not the unit length of the a axis but is that distance foreshortened somewhat because of the inclination of that axis. The construction by which the true length of the a axis is obtained is shown in Fig. 355. The line $R-O-S-T$ represents the horizontal projection of the a axis upon which the distance $O-R$ is transferred from Fig. 354. As the prism face is vertical its intercept upon the a axis can be found by dropping a perpendicular from R to intersect the line which represents the a axis. The inclination of this last

the angle from the center of the projection to 001. This can be measured directly by means of the gnomonic tangent scale. Then construct the triangles CST and XYZ. The angles ρ and π , and ν and ν are measured. This can most easily be done by means of the divided circle and the fact that an angle at the circumference of a circle is measured by one half its subtended arc. The following relations will then yield the axial ratio.

$$\frac{b}{c} = \frac{\sin \rho}{\sin \pi}; \quad \frac{a}{c} = \frac{\sin \nu}{\sin \gamma}.$$

For the proof of these relations see the explanation of the more general case under the triclinic system, Art. 227, p. 152.



Determination of Axial Elements of Pyroxene from Gnomonic Projection

214. To determine, by plotting, the indices of a face on a monoclinic crystal, having given the position of its pole upon the gnomonic projection. There is no essential difference between the orthorhombic and monoclinic systems in the determination of indices from the gnomonic projection. The intercepts of perpendiculars from the poles of the faces upon the front to back and left to right zonal lines running through the pole of $c(001)$ give directly the first two numbers of the indices. The gnomonic projection of the epidote crystal already given (Fig. 348) will serve to illustrate this problem.

VI. TRICLINIC SYSTEM

(Anorthic System)

215. Crystallographic Axes. — The *triclinic system* includes all the forms which are referred to three unequal axes with all their intersections oblique. When orientated in the customary manner one axis has a vertical posi-