II. TETRAGONAL SYSTEM

85. THE TETRAGONAL SYSTEM includes all the forms which are referred to three axes at right angles to each other of which the two horizontal axes are equal to each other in length and interchangeable and the third, the vertical axis, is either shorter or longer. The horizontal axes are designated by the letter *a*; the vertical axis by *c* (see Fig. 166). The length of the vertical axis expresses properly the axial ratio of $a : c$, *a* being uniformly taken as equal to unity. The axes are orientated and their opposite ends designated by plus and minus signs exactly as in the case of the Isometric System.

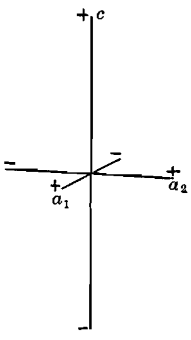
Seven classes are embraced in this system. Of these the normal class is common and important among minerals; two others have several representatives, and another a single one only. It may be noted that in four of the classes the vertical axis is an axis of tetragonal symmetry; in the remaining three it is an axis of binary symmetry only.

1. NORMAL CLASS (6). ZIRCON TYPE

(Ditetragonal Bipyramidal or Holohedral Class)

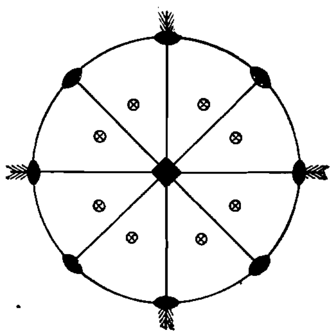
86. Symmetry. — The forms belonging to the normal class of the tetragonal system (cf. Figs. 170 to 192) have one principal axis of tetragonal symmetry (whence name of the system) which coincides with the vertical crystallographic axis, *c*. There are also four horizontal axes of binary symmetry, two of which coincide with the horizontal crystallographic axes while the other two are diagonal axes bisecting the angles between the first two.

166



Axes of Tetragonal Mineral, Octahedrite $a : c = 1 : 1{\cdot}78$

167



Symmetry of Normal Class Tetragonal System

Further they have one principal plane of symmetry, the plane of the horizontal crystallographic axes. There are also four vertical planes of symmetry which pass through the vertical crystallographic axis *c* and make angles of 45° with each other. Two of these latter planes include the horizontal crystallographic axes and are known as axial planes of symmetry. The other two are known as diagonal planes of symmetry.

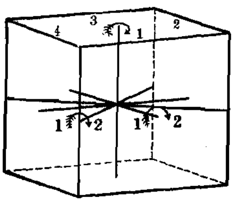
The axes and planes of symmetry are shown in Figs. 168 and 169.

The symmetry and the distribution of the faces of the general form, *hkl*, is shown in the stereographic projection, Fig. 167.

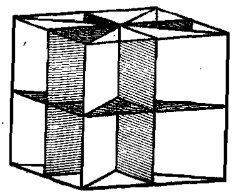
87. Forms. — The various possible forms under the normal class of this system are as follows:

		Symbols
1.	Base or basal pinacoid	(001)
2.	Prism of the first order	(110)
3.	Prism of the second order	(100)
4.	Ditetragonal prism	(*hk*0) as, (310); (210); (320), etc.
5.	Pyramid of the first order	(*hhl*) as, (223); (111); (221), etc.
6.	Pyramid of the second order	(*h*0*l*) as, (203); (101); (201), etc.
7.	Ditetragonal pyramid	(*hkl*) as, (421); (321); (122), etc.

168



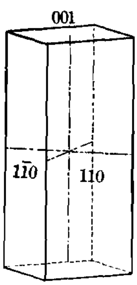
169



Symmetry of Normal Class, Tetragonal System

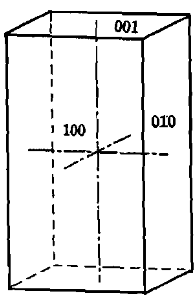
88. Base or Basal Pinacoid. — The *base* is that form which includes the two similar faces which are parallel to the plane of the horizontal axes. These faces have the indices 001 and $00\bar{1}$ respectively; it is an "open form," as they do not inclose a space, consequently this form can occur only in combination with other forms. Cf. Figs. 170–173, etc. This form is always lettered *c* in this work.

170



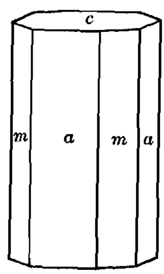
First Order Prism

171



Second Order Prism

172



First and Second Order Prisms

89. Prisms. — Prisms, in systems other than the isometric, have been defined to be forms whose faces are parallel to the vertical axis (*c*) of the crystal, while they meet the two horizontal axes; in this system the four-faced form whose planes are parallel both to the vertical and one horizontal axis is also called a prism. There are hence three types of prisms here included.

90. Prism of First Order. — The *prism of the first order* includes the four faces which, while parallel to the vertical axis, meet the horizontal axes at equal distances; its general symbol is consequently (110). It is a *square prism*, with interfacial angles of 90°. It is shown in combination with the base in Fig. 170. It is uniformly designated by the letter *m*. The indices of its faces, taken in order, are 110, $\bar{1}10$, $\bar{1}\bar{1}0$, $1\bar{1}0$.

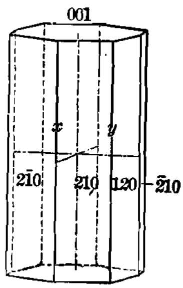
91. Prism of Second Order. — The *prism of the second order* shown* in combination with the base in Fig. 171 includes the four faces which are parallel at once to the vertical and to a horizontal axis; it has, therefore, the general symbol (100). It is a *square prism* with an angle between any two adjacent faces of 90°. It is uniformly designated by the letter *a*, and its faces, taken in order, have the indices 100, 010, $\bar{1}00$, $0\bar{1}0$.

It will be seen that the combination of this form with the base is the analogue of the cube of the isometric system.

The faces of the prism of the first order truncate the edges of the prism of the second order and *vice versa*. When both are equally developed, as in Fig. 172, the result is a regular eight-sided prism, which, however, it must be remembered, is a combination of *two* distinct forms.

It is evident that the two prisms described do not differ geometrically from one another, and furthermore, in a given case, the symmetry of this class allows either to be made the first order, and the other the second order, prism according to the position assumed for the horizontal axes. If on crystals of a given species both forms occur together equally developed (or, on the other hand, separately on different crystals) and without other faces than the base, there is no means of telling them apart unless by minor characteristics, such as striations or other markings on the surface, etchings, etc.

173



Ditetragonal Prism

92. Ditetragonal Prism. — The *ditetragonal prism* is the form which is bounded by eight similar faces, each one of which is parallel to the vertical axis while meeting the two horizontal axes at unequal distances. It has the general symbol (*hk*0). It is shown in Fig. 173, where (*hk*0) = (210). The successive faces have here the indices 210, 120, $\bar{1}20$, $\bar{2}10$, $\bar{2}\bar{1}0$, $\bar{1}\bar{2}0$, $1\bar{2}0$, $2\bar{1}0$.

In Fig. 185 a combination is shown of this form (y = 310) with the second order prism, the edges of which it bevels. In Fig. 189 (h = 210) it bevels the edges of the first order prism *m*. In Fig. 190 (l = 310) it is combined with both orders of prisms.

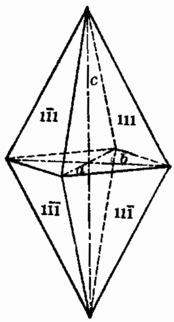
93. Pyramids. — There are three types of pyramids in this class, corresponding, respectively, to the three prisms which have just been described.

* In Figs. 170–173 the dimensions of the form are made to correspond to the assumed length of the vertical axis (here c = 1·78 as in octahedrite) used in Fig. 177. It must be noted, however, that in the case of actual crystals of these forms, while the tetragonal symmetry is usually indicated by the unlike physical character of the face *c* as compared with the faces *a*, *m*, etc., in the vertical prismatic zone, no inference can be drawn as to the relative length of the vertical axis. This last can be determined only when a pyramid is present; it is fixed for the species when a particular pyramid is chosen as fundamental or unit form, as explained later.

As already stated, the name *pyramid* is given (in systems other than the isometric) to a form whose planes meet all three of the axes; in this system the form whose planes meet the axis c and one horizontal axis while parallel to the other is also called a pyramid. The pyramids of this class are strictly double pyramids (*bipyramids* of some authors).

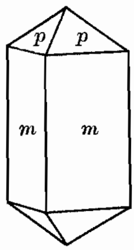
94. Pyramid of First Order. — A *pyramid of the first order*, is a form whose eight similar faces intersect the two horizontal axes at equal distances and also intersect the vertical axis. It has the general symbol (hhl). It is a *square pyramid* with equal interfacial angles over the terminal edges, and the faces replace the horizontal, or basal, edges of the first order prism and the solid angles of the second order prism. If the ratio of the vertical to the horizontal axis for a given first order pyramid is the assumed axial ratio for the species, the form is called the *fundamental form*, and it has the symbol (111) as in Fig. 174. The indices of its faces me oned in order are: Above 111, $\bar{1}11$, $\bar{1}\bar{1}1$, $1\bar{1}1$; below $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$, $1\bar{1}\bar{1}$.

174



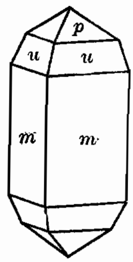
First Order Pyramid

175



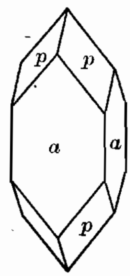
Zircon, First Order Prism and Pryamid

176



Zircon, First Order Prism and Pyramids

177



Apophyllite, Second Order Prism and First Order Pyramid

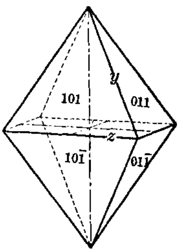
Obviously the angles of the first order pyramid, and hence its geometrical aspect, vary widely with the length of the vertical axis. In Figs. 174 and 182 the pyramids shown have in both cases the symbol (111) but in the first case (octahedrite) $c = 1.78$, while in the second (vesuvianite), $c = 0.64$.

For a given species there may be a number of second order pyramids, varying in position according to the ratio of the intercepts upon the vertical and horizontal axes. Their symbols, passing from the base (001) to the unit prism (110), may thus be (115), (113), (223), (111), (332), (221), (441), etc. In the general symbol of these forms (hhl), as h diminishes, the form approximates more and more nearly to the base (001), for which $h = 0$; as h increases, the form passes toward the first order prism. In Fig. 176 two pyramids of this order are shown, p(111) and u(331).

95. Pyramid of Second Order. — The *pyramid of the second order* is the form, Fig. 178, whose faces are parallel to one of the horizontal axes, while meeting the other two axes. The general symbol is ($h0l$). These faces replace the basal edges of the second order prism (Fig. 179), and the solid angles of the first order prism (cf. Fig. 180). It is a *square pyramid* since its basal section is a square, and the interfacial angles over the four terminal edges, above and below, are equal. The successive faces of the form (101) are as follows: Above 101, 011, $\bar{1}01$, $0\bar{1}1$; below $10\bar{1}$, $01\bar{1}$, $\bar{1}0\bar{1}$, $0\bar{1}\bar{1}$.

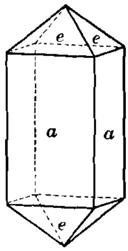
If the ratio of the intercepts on the horizontal and vertical axes is the assumed axial ratio of the species, the symbol is (101), and the form is designated by the letter *e*. This ratio can be deduced from the measurement of either one of the interfacial angles (*y* or *z*, Fig. 178) over the terminal or basal edges, as explained later. In the case of a given species, a number of second

178



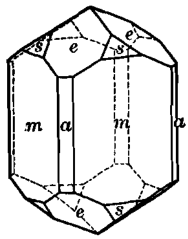
Second Order Pyramid

179



Second Order Prism and Pyramid

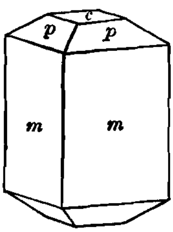
180



Rutile, First and Second Order Prisms and Pyramids

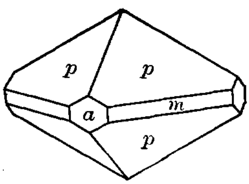
order pyramids may occur, varying in the ratio of the axes *a* and *c*. Hence there is possible a large number of such forms whose symbols may be, for example, (104), (103), (102), (101), (302), (201), (301), etc. Those mentioned first come nearest to the base (001), those last to the second order prism (100); the base is therefore the limit of these pyramids (*h*0*l*) when $h = 0$, and the second order prism (100) when $h = 1$ and $l = 0$. Fig. 186 shows the three second order pyramids *u*(105), *e*(101), *q*(201).

181



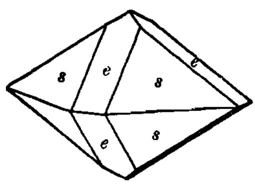
Vesuvianite
First Order Prism, Pyramid and Base

182



Vesuvianite
First Order Pyramid and First and Scond Order Prisms

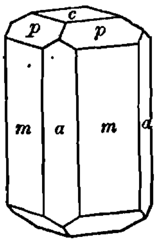
183



Cassiterite
First and Second Order Pyramids

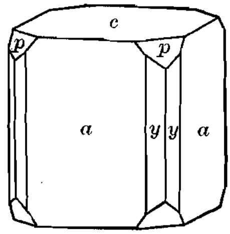
A second order pyramid truncating the pyramidal edges of a given first order pyramid as in Fig. 183 has the *same* ratio as it for *h* to *l*. Thus (101) truncates the terminal edge of (111); (201) of (221), etc. This is obvious because each face has the same position as the corresponding edge of the other form (see Fig. 183, when $s = 111$ and $e = 101$; also Figs. 186, 191, where $r = 115$, $u = 105$). Again, if a first order pyramid truncates the pyramidal edges of a given second order pyramid, its ratio for *h* to *l* is *half* that of the other form; that is, (112) truncates the pyramidal edges of (101); (111) of (201), etc. This relation is exhibited by Fig. 186, where $p(111)$ truncates the edges of $q(201)$. In both cases the zonal equations prove the relations stated.

184



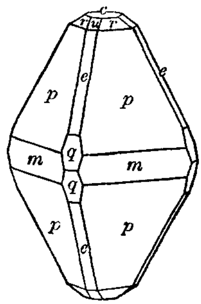
Vesuvianite
First and Second Order Prisms, First Order Pyramid and Base

185



Apophyllite
Second Order Prism, Ditetragonal Prism, First Order Pyramid and Base

186

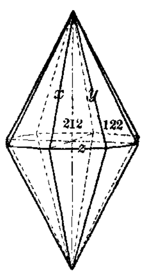


Octahedrite
Two First Order Pyramids, First Order Prism, Three Second Order Pyramids and Base

96. Ditetragonal Pyramid. — The *ditetragonal pyramid*, or double eight-sided pyramid, is the form each of whose sixteen similar faces meets the three axes at unequal distances. This is the most general case of the symbol (hkl), where h, k, l are all unequal and no one is equal to 0. That there are sixteen faces in a single form is evident. Thus, for example, for the form (212) the face 212 is similar to 122, the two lateral axes being equal (not, however, to 221). Hence there are two like faces in each octant. Similarly the indices of all the faces in the successive octants are, therefore, as follows:

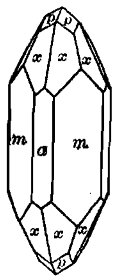
Above	212	122	$\bar{1}22$	$\bar{2}12$	$\bar{2}\bar{1}2$	$\bar{1}\bar{2}2$	$1\bar{2}2$	$2\bar{1}2$
Below	$21\bar{2}$	$12\bar{2}$	$\bar{1}2\bar{2}$	$\bar{2}1\bar{2}$	$\bar{2}\bar{1}\bar{2}$	$\bar{1}\bar{2}\bar{2}$	$1\bar{2}\bar{2}$	$2\bar{1}\bar{2}$

187



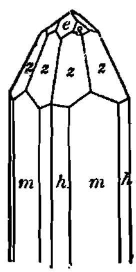
Ditetragonal Pyramid

188



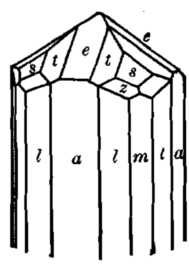
Zircon
First and Second Order Prisms, First Order Pyramid, Ditetragonal Pyramid

189



Cassiterite

190

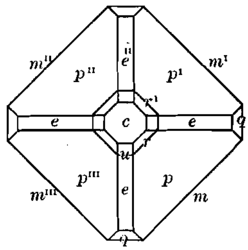


Rutile

This form is common with the species zircon, and is hence often called a *zirconoid*. It is shown in Fig. 187. It is not observed alone, though sometimes, as in Figs. 188 (x = 311) and 189 (z = 321), it is the predominating form. In Fig. 190 two ditetragonal pyramids occur, namely, t(313) and z(321).

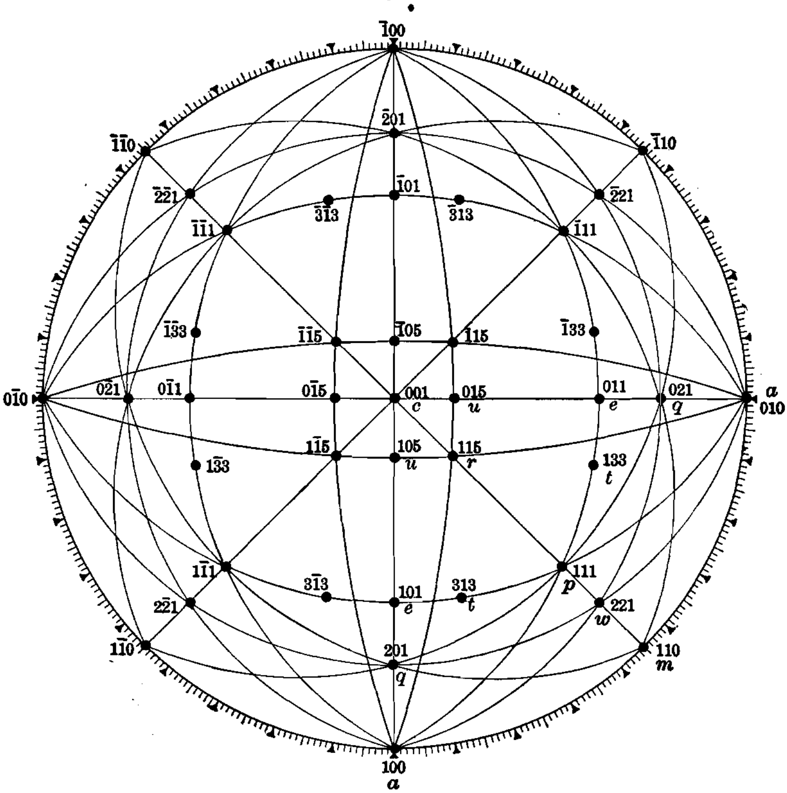
97. In addition to the perspective figures already given, a basal projection (Fig. 191) is added of the crystal of octahedrite already referred to (Fig. 186); also a stereographic (Fig. 192) and gnomonic (Fig. 193) projections of the same with the faces of the forms w(221) and t(313) added. These exhibit well the general relations of this normal class of the tetragonal system. The symmetry here is to be noted, first, with respect to the similar zones 100, 001, $\bar{1}$00 and 010, 001, 0$\bar{1}$0; also, to the other pair of similar zones, 110, 001, $\bar{1}\bar{1}$0, and 1$\bar{1}$0, 001, $\bar{1}$10.

191



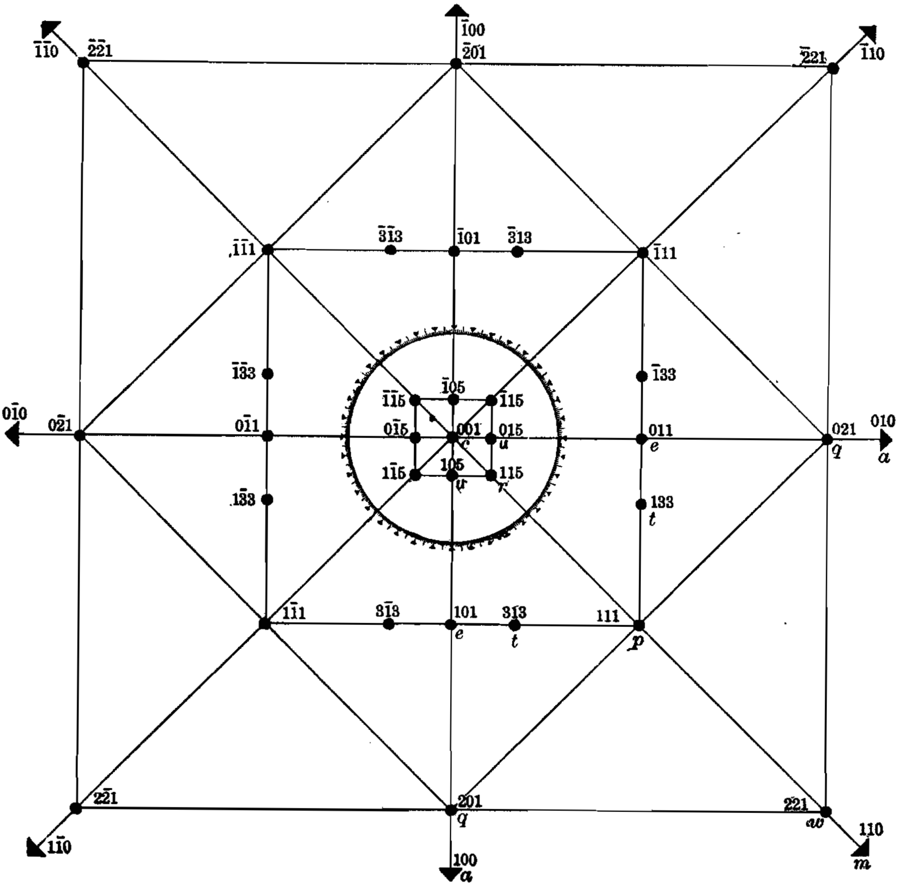
Octahedrite

192



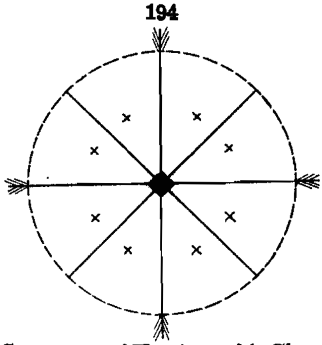
Stereographic Projection of Octahedrite

193



Gnomonic Projection of Octahedrite

194



Symmetry of Hemimorphic Class

2. HEMIMORPHIC CLASS (7). IODOSUCCINIMIDE TYPE

(Ditetragonal Pyramidal or Holohedral Hemimorphic Class)

98. Symmetry. — This class differs from the normal class only in having no horizontal plane of symmetry; hence the forms are hemimorphic as defined in Art. **29.** It is not known to be represented among minerals, but is shown on the crystals of iodosuccinimide. Its symmetry is illustrated by the stereographic projection (Fig. 194). Here the two basal planes are distinct forms, 001 and $00\bar{1}$; the prisms do not differ geometrically from those of the normal class, though distinguished by their molecular structure; further, the pyramids are no longer double pyramids, but each form is represented by one half of Figs. 174, 178, 187 (cf. Fig. 44, p. 22). There are hence six distinct pyramidal forms, corresponding to the upper and lower halves of the first and second order pyramids and the ditetragonal pyramid.

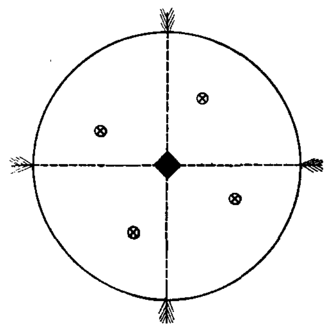
3. TRIPYRAMIDAL CLASS (8). SCHEELITE TYPE.

(Tetragonal Bipyramidal or Pyramidal Hemihedral Class)

99. Typical Forms and Symmetry. — The forms here included have one plane of symmetry only, that of the horizontal crystallographic axes, and one axis of tetragonal symmetry (the vertical crystallographic axis) normal to it. The distinctive forms are the tetragonal prism (*hk*0) and pyramid (*hkl*) of the *third order*, shown in Figs. 196, 197.

The stereographic projection, Fig. 195, exhibits the symmetry of the class and the distribution of the faces of the general form (*hkl*). Comparing this, as well as the figures immediately following, with those of the normal class, it is seen that this class differs from it in the absence of the vertical planes of symmetry and the horizontal axes of symmetry.

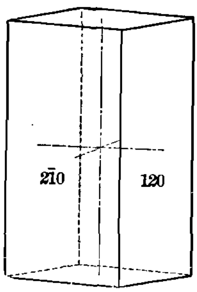
195



Symmetry of Tri-Pyramidal Class

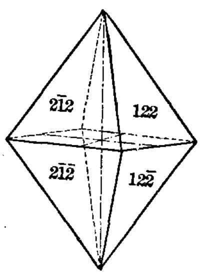
100. Prism and Pyramid of the Third Order. — The typical forms of the class, as above stated, are a square prism and a square pyramid, which are distinguished respectively from the square prisms *a*(100) and *m*(110), shown in Figs. 170 and 171, and from the square pyramids (*h*0*l*) and (*hhl*) of Figs. 174 and 178 by the name "*third order.*"

196



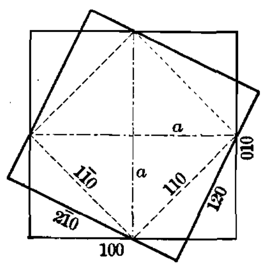
Third Order Prism

197



Third Order Pyramid

198



The third order prism and pyramid may be considered as derived from the ditetragonal forms of the normal class by taking only one half the faces of the latter and the omission of the remaining faces. There are therefore two complementary forms in each case, designated *left* and *right*, which together include all the faces of the ditetragonal prism (Fig. 173) and ditetragonal pyramid (Fig. 187) of the normal class.

The indices of the faces of the two complementary prisms, as (210), are:

Left: 210, $\bar{1}20$, $\bar{2}\bar{1}0$, $1\bar{2}0$.
Right: 120, $\bar{2}10$, $\bar{1}\bar{2}0$, $2\bar{1}0$.

The indices of the faces of the corresponding pyramids, as (212), are:

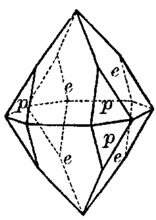
Left: above 212, $\bar{1}22$, $\bar{2}\bar{1}2$, $1\bar{2}2$; below $21\bar{2}$, $\bar{1}2\bar{2}$, $\bar{2}\bar{1}\bar{2}$, $1\bar{2}\bar{2}$.
Right: above 122, $\bar{2}12$, $\bar{1}\bar{2}2$, $2\bar{1}2$; below $12\bar{2}$, $\bar{2}1\bar{2}$, $\bar{1}\bar{2}\bar{2}$, $2\bar{1}\bar{2}$.

Fig. 198 gives a transverse section of the prisms $a(100)$ and $m(110)$, also the prism of the third order (120). Figs. 196, 197 show the right prism (120) and pyramid (122) of the third order.

101. Other Forms. — The other forms of this class, that is, the base $c(001)$; the other square prisms, $a(100)$ and $m(110)$; also the square pyramids $(h0l)$ and (hhl) are geometrically like the corresponding forms of the normal class already described. The class shows therefore three types of square pyramids and hence is called the *tripyramidal class.*

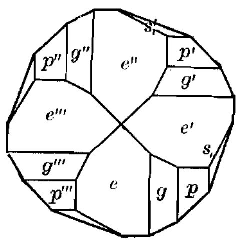
102. To this class belongs the important species scheelite; also the isomorphous species stolzite and powellite, unless it be that they are rather to be classed with wulfenite (p. 87). Fig. 199 shows a typical crystal of

199



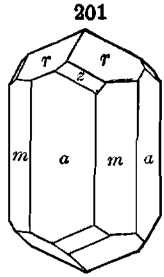
Scheelite

200



Scheelite

201



Meionite

scheelite, and Fig. 200 a basal section of one similar; these illustrate well the characteristics of the class. Here the forms are $e(101)$, $p(111)$, and the third-order pyramids $g(212)$, $s_{,}(131)$. Fig. 201 represents a meionite crystal with $r(111)$, and the third-order pyramid $z(311)$. See also Figs. 203, 204, in which the third-order prism is shown.

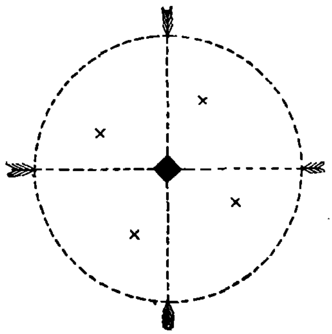
The forms of this class are sometimes described (see Art. **28**) as showing *pyramidal hemihedrism.*

4. PYRAMIDAL-HEMIMORPHIC CLASS (9). WULFENITE TYPE

(Tetragonal Pyramidal or Hemihedral Hemimorphic Class)

103. Symmetry. — The fourth class of the tetragonal system is closely related to the class just described. It has the same vertical axis of tetragonal symmetry, but there is no horizontal plane of symmetry. The forms are, therefore, hemimorphic in the distribution of the faces (cf. Fig. 202). The species wulfenite of the Scheelite Group among mineral species probably belongs here, although the crystals do not always show the difference between the pyramidal faces, above and below, which would characterize distinct complementary forms. Figs. 203, 204 could, therefore, serve as illustrations of the preceding class, but in Fig. 205 a characteristic distinction is exhibited. In these figures the forms are $u(102)$, $e(101)$, $n(111)$; also $f(230)$, $k(210)$. $z(432)$, $x(311)$.

202



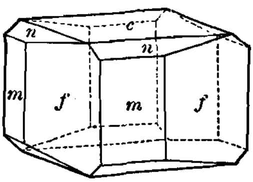
Symmetry of Pyramidal-Hemimorphic Class

5. SPHENOIDAL CLASS (10). CHALCOPYRITE TYPE

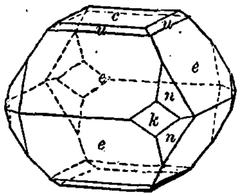
(Tetragonal Sphenoidal, Sphenoidal Hemihedral or Scalenohedral Class)

104. Typical Forms and Symmetry. — The typical forms of this class are the sphenoid (Fig. 207) and the tetragonal scalenohedron (Fig. 208). They and all the combinations of this class show the following symmetry. The three crystallographic axes are axes of binary symmetry and there are two vertical diagonal planes of symmetry.

203

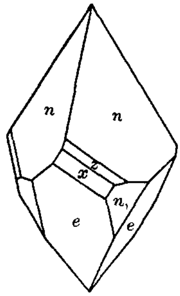


20

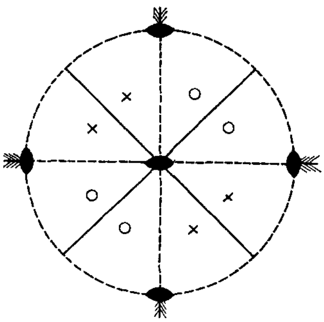


Wulfenite

205



206

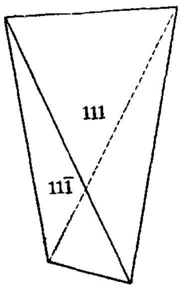


Symmetry of Sphenoidal Class

This symmetry is exhibited in the stereographic projection (Fig. 206), which shows also the distribution of the faces of the general form (hkl). It is seen here that the faces are present in the alternate octants only, and it will be remembered that this same statement was made of the tetrahedral class under the isometric system. There is hence a close analogy between these two classes. The symmetry of this class should be carefully compared with that of the first and third classes of this system already described.

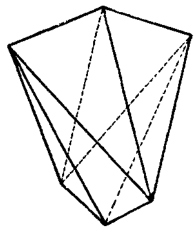
105. Sphenoid. — The *sphenoid*, shown in Fig. 207, is a four-faced solid, resembling a tetrahedron, but each face is an isosceles (not an equilateral) triangle. It may be considered as derived from the first order pyramid of the normal class by the development of only the alternate faces of the latter. There are therefore possible two complementary forms known as the positive and negative sphenoids. The general symbol of the positive unit sphenoid is (111), and its faces have the indices: 111, $\bar{1}\bar{1}1$, $\bar{1}1\bar{1}$, $1\bar{1}\bar{1}$, while the negative sphenoid has the symbol ($1\bar{1}1$). When the complementary forms occur together, if equally developed, the resulting solid, though having two unlike sets of faces, cannot be distinguished geometrically from the first order pyramid (111).

207



Sphenoid

208

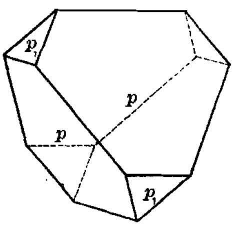


Tetragonal Scalenohedron

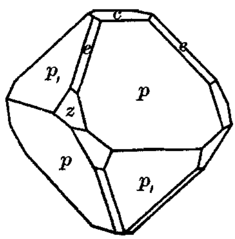
In the species chalcopyrite, which belongs to this class, the deviation in angle and in axial ratio from the isometric system is very small, and hence the unit sphenoid cannot by the eye be distinguished from a tetrahedron (compare Fig. 209 with Fig. 144, p. 68). For this species c = 0·985 (instead of 1, as in the isometric system), and the normal sphenoidal angle is 108° 40′, instead of 109° 28′, the angle of the tetrahedron. Hence a crystal of chalcopyrite with both the positive and negative sphenoids equally developed closely resembles a regular octahedron.

In Fig. 210 the second order pyramids e(101) and z(201) and base c(001) are also present.

209



210



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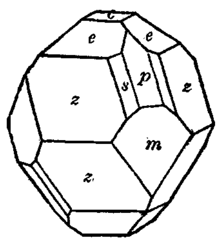


Chalcopyrite

106. Tetragonal Scalenohedron. — The sphenoidal symmetry yields another distinct type of form, that shown in Fig. 208. It is bounded by eight similar scalene triangles, and hence is called a *tetragonal scalenohedron;* the general symbol is (hkl). It may be considered as derived from the ditetragonal pyramid of the normal class by taking the alternate *pairs* of faces of the latter form. The faces of the complementary positive and negative forms therefore embrace all the faces of the ditetragonal pyramid. This form appears in combination in chalcopyrite, but is not observed independently. In Fig. 211 the form *s*(531) is the positive tetragonal scalenohedron.

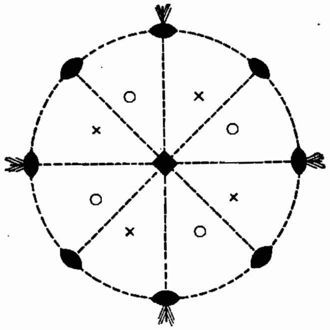
107. Other Forms. — The other forms of the class, namely, the first and second order prisms, the ditetragonal prism, and the first and second order pyramids (*hhl*) and (*h0l*), are geometrically like those of the normal class. The lower symmetry in the molecular structure is onlv revealed by special investigation, as by etching.

6. TRAPEZOHEDRAL CLASS (11). NICKEL SULPHATE TYPE

(*Tetragonal Trapezohedral or Trapezohedral Hemihedral Class*)

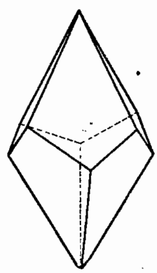
108. The trapezohedral class is analogous to the plagiohedral class under the isometric system; it is characterized by the absence of any plane or center of symmetry; the vertical axis, however, is an axis of tetragonal symmetry, and perpendicular to this there are four axes of binary symmetry. This symmetry and the distribution of the faces of the general form (*hkl*)

212



Symmetry of Trapezohedral Class

213



Tetragonal Trapezohedron

are shown in the stereographic projection, Fig. 212, and Fig. 213 gives the resulting solid, a *tetragonal trapezohedron.* It may be derived from the ditetragonal pyramid of the normal class by the extension of the alternate faces of that form. There are two complementary forms called right- and left-handed which embrace all the faces of the ditetragonal pyramid of the normal class. These two forms are enantiomorphous, and the salts belonging to this class show circular polarization.

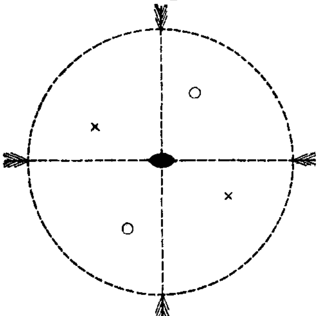
Nickel sulphate and a few other artificial salts belong in this class.

7. TETARTOHEDRAL CLASS (12)

(*Tetragonal Bisphenoidal or Sphenoidal Tetartohedral Class*)

109. Symmetry. — The seventh and last possible class under this system has no plane nor center of symmetry, but the vertical axis is an axis of binary symmetry. The symmetry and the distribution of the faces of the general form (*hkl*) are shown in the stereographic projection (Fig. 214), and the solid resulting is known as a *sphenoid of the third order*. It can be derived from the ditetragonal pyramid of the normal class by taking only one quarter of the faces of that form. There are therefore four complementary forms which are respectively distinguished as right (+ and −) and left (+ and −). These four together embrace all the sixteen faces of the ditetragonal pyramid. The other characteristic forms of this class are the prism of the third order (*hk*0), the positive and negative sphenoids of the first order (111), and also those of the second order (101). It is said that an artificial compound, $2CaO.Al_2O_3.SiO_2$, crystallizes in this class.

214



Symmetry of Tetartohedral Class

MATHEMATICAL RELATIONS OF THE TETRAGONAL SYSTEM

110. Choice of Axes. — It appears from the discussion of the symmetry of the seven classes of this system that with all of them the position of the vertical axis is fixed. In classes 1, 2, however, where there are two sets of vertical planes of symmetry, either set may be made the axial planes and the other the diagonal planes. The choice between these two possible positions of the horizontal axes is guided particularly by the habit of the occurring crystals and the relations of the given species to others of similar form. With a species whose crystal characters have been described it is customary to follow the orientation given in the original description.

111. Determination of the Axial Ratio, etc. — The following relations serve to connect the axial ratio, that is, the length of the vertical axis *c*, when $a = 1$, with the fundamental angles (001 ∧ 101) and (001 ∧ 111):

$$\tan(001 \wedge 101) = c; \qquad \tan(001 \wedge 111) \times \tfrac{1}{2}\sqrt{2} = c.$$

For faces in the same rectangular zone the tangent principle applies. The most important cases (cf. Fig. 214) are:

$$\frac{\tan(001 \wedge h0l)}{\tan(001 \wedge 101)} = \frac{h}{l};$$

$$\frac{\tan(001 \wedge 0kl)}{\tan(001 \wedge 011)} = \frac{k}{l};$$

$$\frac{\tan(001 \wedge hhl)}{\tan(001 \wedge 111)} = \frac{h}{l}.$$

For the prisms

$$\tan(010 \wedge hk0) = \frac{h}{k}, \qquad \text{or} \qquad \tan(100 \wedge hk0) = \frac{k}{h}.$$

112. Other Calculations. — It will be noted that in the stereographic projection (Fig. 214) all those spherical triangles are right-angled which are formed by great circles (diameters) which meet the prismatic zone-circle 100, 010, $\bar{1}00$, $0\bar{1}0$. Again, all those formed by great circles drawn between 100 and $\bar{1}00$, or 010 and $0\bar{1}0$, and crossing respectively the zone-circles 100, 001, $\bar{1}00$, or 010, 001. $0\bar{1}0$. Also, all those formed by great circles drawn between 110 and $\bar{1}\bar{1}0$ and crossing the zone-circle $\bar{1}10$, 001, $1\bar{1}0$, or between $\bar{1}10$ and $1\bar{1}0$ and crossing the zone-circle 110, 001, $\bar{1}\bar{1}0$.

These spherical triangles may hence be readily used to calculate any angles desired; for example, the angles between the pole of any face, as *hkl* (say 321), and the pinacoids 100, 010, 001. The terminal angles (*x* and *z*, Fig. 187) of the ditetragonal pyramid, 212 ∧ $2\bar{1}2$ (or 313 ∧ $3\bar{1}3$, etc.), and 212 ∧ 122 (or 313 ∧ 133, etc.), can also be obtained in the same way. The zonal relations give the symbols of the poles on the zones 001, 100 and 001, 110 for the given case. For example, the zone-circle $1\bar{1}0$, 313, 133, $\bar{1}10$ meets $\bar{1}\bar{1}0$, 001, 110 at the pole 223, and the calculated angle 313 ∧ 223 is half the angle 313 ∧ 133. If a large number of similar angles are to be calculated, it is more convenient to use a formula, as that given below.

113. Formulas. — It is sometimes convenient to have the normal interfacial angles expressed directly in terms of the axis c and the indices h, k, and l. Thus:

(1) The distances of the pole of any face P(hkl) from the pinacoids $a(100) = \mathrm{P}a$, $b(010) = \mathrm{P}b$, $c(001) = \mathrm{P}c$ are given by the following equations:

$$\cos^2 \mathrm{P}a = \frac{h^2c^2}{h^2c^2 + k^2c^2 + l^2}; \quad \cos^2 \mathrm{P}b = \frac{k^2c^2}{h^2c^2 + k^2c^2 + l^2}; \quad \cos^2 \mathrm{P}c = \frac{l^2}{h^2c^2 + k^2c^2 + l^2}$$

These may also be expressed in the form

$$\tan^2 \mathrm{P}a = \frac{k^2c^2 + l^2}{h^2c^2}; \quad \tan^2 \mathrm{P}b = \frac{h^2c^2 + l^2}{k^2c^2}; \quad \tan^2 \mathrm{P}c = \frac{h^2c^2 + k^2c^2}{l^2}.$$

(2) For the distance between the poles of any two faces (hkl), (pqr), we have in general

$$\cos \mathrm{PQ} = \frac{hpc^2 + kqc^2 + lr}{\sqrt{[(h^2 + k^2)c^2 + l^2]\,[(p^2 + q^2)c^2 + r^2]}}$$

The above equations take a simpler form for special cases often occurring; for example, for hkl and the angle of the edge y of Fig. 187.

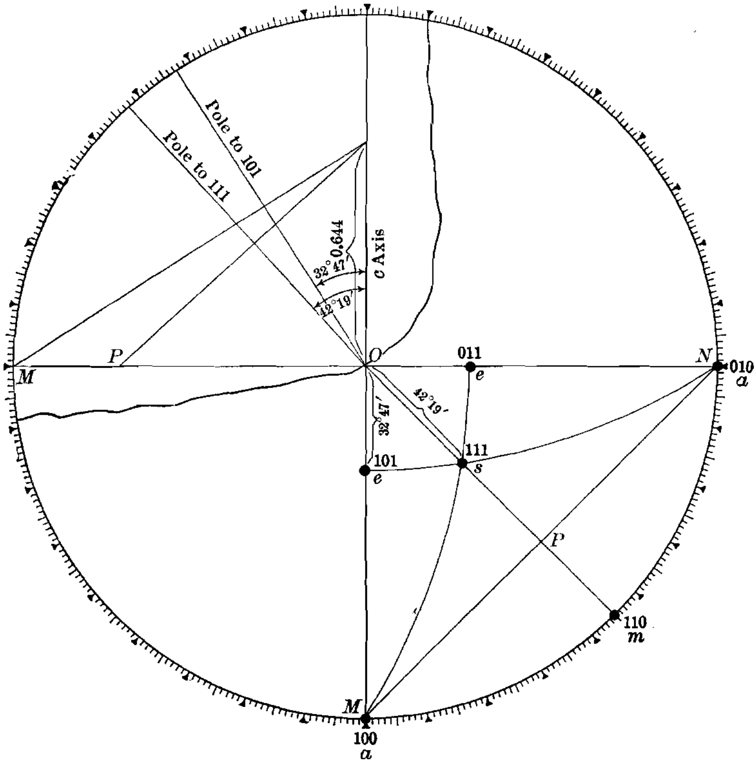
114. Prismatic Angles. — The angles for the commonly occurring ditetragonal prisms are as follows:

	Angle on $a(100)$	Angle on $m(110)$		Angle on $a(100)$	Angle on $m(110)$
410	14° 2¼′	30° 57¾′	530	30° 57¾′	14° 2¼′
310	18 26	26 34	320	33 41¼	11 18¾
210	26 34	18 26	430	36 52¼	8 7¾

115. To determine, by plotting, the axial ratio, $a : c$, of a tetragonal mineral from the stereographic projection of its crystal forms. As an illustrative example it has been assumed that the angles between the faces on the crystal of rutile, represented in Fig 180, have been measured and from these measurements the poles of the faces in one octant located on the stereographic projection, see Fig. 215. In determining the axial ratio of a tetragonal crystal (or what is the same thing, the length of the c axis, since the length of the a axes are always taken as equal to 1) it is necessary to assume the indices of some pyramidal form. It is customary to take a pyramid which is prominent upon the crystals of the mineral and assume that it is the fundamental or unit pyramid of either the first or second order and has as its symbol either (111) or (101). In the example chosen both a first order and a second order pyramid are present and from their zonal relations it is evident that if the symbol assigned to the first order form be (111) that of the second order form must be (101). In order to determine the relative length of the c axis in respect to the length of the a axis for rutile therefore, it is only necessary to plot the intercept of either of these forms upon the axes. In the case of the second order pyramid it is only necessary to construct a right angle triangle (see upper left hand quadrant of Fig. 215) in which the horizontal side shall equal the length of the a axis, (1), the vertical side shall represent the c axis and the hypothenuse shall show the proper angle of slope of the face. The angle between the center of the projection and the pole e(101) is measured by the stereographic protractor and a line drawn making that angle with the line representing the c axis. The hypothenuse of the triangle must then be at right angles to this pole. Its intercept upon the vertical side of the triangle, when expressed in relation to the distance (O–M) which was chosen as representing unity on the a axis, will therefore give the length of the c axis. In rutile this is found to be 0.644.

The same value is obtained when the position of the pyramid of the first order s(111) is used. In this case the line M–P–N is first drawn at right angles to the radial line O–P drawn through the pole s(111). The triangle to be plotted in this case has the distance O–P as the length of its horizontal side. Its hypothenuse must be at right angles to the line representing the pole to (111). The intercept on the c axis is the same as in the first case.

215


Pole to 101
Pole to 111
c Axis
32° 0.644
47'
42°19'
P
M
O
011
e
N
010
a
42°19'
32°47'
101
e
111
s
P
110
m
M
100
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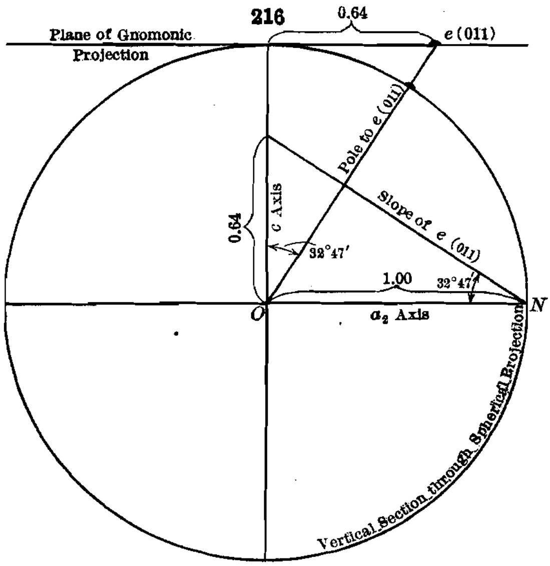
216


0.64
Plane of Gnomonic Projection
e (011)
Pole to e (011)
Slope of e (011)
c Axis
0.64
32°47'
1.00
32°47'
O
a_2 Axis
N
Vertical Section through Spherical Projection

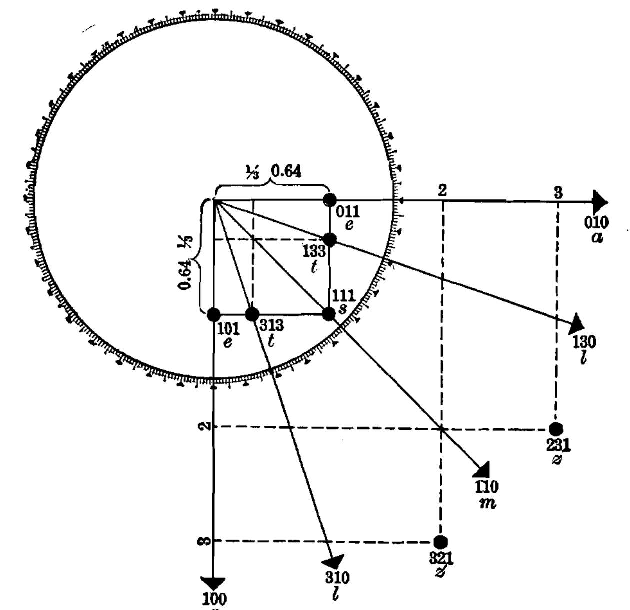
116. To determine, by plotting, the indices of any face (*hkl*) of a tetragonal form from the position of its pole on the stereographic projection. The solution of this problem is like that given in a similar case under the Isometric System, see p. 74, except that the intercept of the face on the vertical axis must be referred to the established unit length of that axis and not to the length of the *a* axis. The method is exactly the reverse of the one used in the problem discussed directly above.

117. To determine, by plotting, the axial ratio, *a* : *c*, of a tetragonal mineral from the gnomonic projection of its crystal forms. As an illustrative example consider the crystal of rutile, Fig. 180, the poles to the faces of which, are shown plotted in gnomonic projection in Fig. 216. The pyramids of the first and second order present are taken as the unit forms with the symbols, *s*(111) and *e*(101). The lines O–M and O–N represent the two horizontal axes a_1 and a_2 and the distance from the center O to the circumference of the fundamental circle is equal to unity on these axes. The intercepts on O–M and O–N made by the poles of *e*(101) or the perpendiculars drawn from the poles of *s*(111) give the unit length of the vertical axis, *c*. In this case this distance, when expressed in terms of the assumed length of the horizontal axes (which in the tetragonal system always equals 1) is equal to 0.64.

That the above relation is true is obvious from a consideration of Fig. 216. This represents a vertical section through the spherical and gnomonic projection including the horizontal axis, a_2. The slope of the face *e*(011) is plotted with its intercepts on the a_2 and *c* axes and the position of its pole in both the spherical and gnomonic projections is shown. It is seen through the two similar triangles in the figure that the distance from the center to the pole *e*(011) in the gnomonic projection must be the same as the intercept of the face *e* upon the vertical axis *c*. And as *e* is a unit form this must represent unity on *c*.

118. To determine, by plotting, the indices of any face of a tetragonal form from the position of its pole on the gnomonic projection. It is assumed that in this case a mineral is being considered whose axial ratio is known. Under these conditions draw perpendiculars from the pole in question to the lines representing the two horizontal axes. Then space off on these lines distances equivalent to the length of the *c* axis, remembering that it must be expressed in terms of the length of the horizontal axes which in turn is equal to the distance from the center of the projection to the circumference of the fundamental circle. Give the intercepts of the lines drawn from the pole of the face to the axes a_1 and a_2 in terms of the length of the vertical axis, add a 1 as the third figure and if necessary clear of fractions and the required indices are the result. This is illustrated in Fig. 217, which is the lower right hand quadrant of the gnomonic projection of the forms shown on the rutile crystal, Fig. 190. Consider first the ditetragonal pyramid *z*(321). Perpendiculars drawn from its pole intersect the lines representing the horizontal axes in distances which are equal to 3 and 2 times the unit length of the *c* axis, 0.64. The indices of the face will therefore be 321. In the case of the ditetragonal pyramid *t*(313), the intercepts are $1a_1$ and $\frac{1}{3}a_2$. This gives the expression $1.\frac{1}{3}.1$ which when cleared of the fraction yields 313,

217



the indices of the face in question. The indices of a prism face like $l(310)$ can be readily obtained in exactly the same manner as described under the Isometric System, Art. 84. p. 75.

III. HEXAGONAL SYSTEM

119. The Hexagonal System includes all the forms which are referred to four axes, three equal horizontal axes in a common plane intersecting at angles of 60°, and a fourth, vertical axis, at right angles to them.

Two sections are here included, each embracing a number of distinct classes related among themselves. They are called the *Hexagonal Division* and the *Trigonal* (or *Rhombohedral*) *Division*. The symmetry of the former, about the vertical axis, belongs to the hexagonal type, that of the latter to the trigonal type.

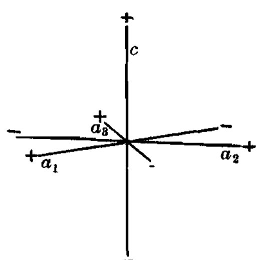
Miller (1852) referred all the forms of the hexagonal system to three equal axes parallel to the faces of the fundamental rhombohedron, and hence intersecting at equal angles, not 90°. This method (further explained in Art. **169**) had the disadvantage of failing to bring out the relationship between the normal hexagonal and tetragonal types, both characterized by a principal axis of symmetry, which (on the system adopted in this book) is the vertical crystallographic axis. It further gave different symbols to faces which are crystallographically identical. It is more natural to employ the three rhombohedral axes for trigonal forms only, as done by Groth (1905), who includes these groups in a *Trigonal System;* but this also has some disadvantages. The indices commonly used in describing hexagonal forms are known as the Miller-Bravais indices, since they were adopted by Bravais for use with the four axes from the scheme used by Miller in the other crystal systems.

120. Symmetry Classes. — There are five possible classes in the Hexagonal Division. Of these the normal class is much the most important, and two others are also of importance among crystallized minerals.

In the Trigonal Division there are seven classes; of these the rhombohedral class or that of the Calcite Type, is by far the most common, and three others are also of importance.

121. Axes and Symbols. — The position of the four axes taken is shown in Fig. 218; the three horizontal axes are called a, since they are equal and interchangeable, and the vertical axis is c, since it has a different length, being either longer or shorter than the horizontal axes. The length of the vertical axis is expressed in terms of that of the horizontal axes which in turn is always taken as unity. Further, when it is desirable to distinguish between the horizontal axes they may be designated a_1, a_2, a_3. When properly orientated one of the horizontal axes (a_2) is parallel to the observer and the other two make angles of 30° either side of the line perpendicular to him. The axis to the left is taken as a_1, the one to the right as a_3. The positive and negative ends of the axes are shown in Fig. 218. The general position of any plane may be expressed in a manner analogous to that applicable in the other systems, viz.

218



Hexagonal Axes

$$\frac{1}{h} a_1 : \frac{1}{k} a_2 : \frac{1}{i} a_3 : \frac{1}{l} c.$$

The corresponding indices for a given plane are then h, k, i, l; these always refer to the axes named in the above scheme. Since it is found convenient