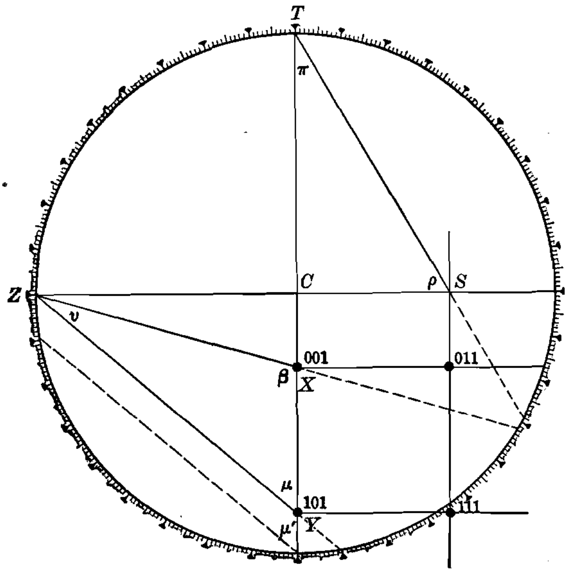
the angle from the center of the projection to 001. This can be measured directly by means of the gnomonic tangent scale. Then construct the triangles CST and XYZ. The angles ρ and π, and υ and ν are measured. This can most easily be done by means of the divided circle and the fact that an angle at the circumference of a circle is measured by one half its subtended arc. The following relations will then yield the axial ratio.

$$\frac{b}{c} = \frac{\sin \rho}{\sin \pi}; \quad \frac{a}{c} = \frac{\sin \upsilon}{\sin \nu}.$$

For the proof of these relations see the explanation of the more general case under the triclinic system, Art. **227**, p. 152.

356



Determination of Axial Elements of Pyroxene from Gnomonic Projection

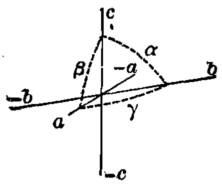
214. To determine, by plotting, the indices of a face on a monoclinic crystal, having given the position of its pole upon the gnomonic projection. There is no essential difference between the orthorhombic and monoclinic systems in the determination of indices from the gnomonic projection. The intercepts of perpendiculars from the poles of the faces upon the front to back and left to right zonal lines running through the pole of $c(001)$ give directly the first two numbers of the indices. The gnomonic projection of the epidote crystal already given (Fig. 348) will serve to illustrate this problem.

VI. TRICLINIC SYSTEM

(Anorthic System)

215. Crystallographic Axes. — The *triclinic system* includes all the forms which are referred to three unequal axes with all their intersections oblique. When orientated in the customary manner one axis has a vertical position and is called the c axis (cf. Fig. 357), a second axis lies in the front-to-back plane, sloping down toward the observer, and is called the a axis. The remaining axis is designated as the b axis. Usually the a and b axes are so chosen that the a axis is the shorter and, like in the orthorhombic system, is sometimes called the brachy-axis. In that case the b axis is longer and is known as the macro-axis. But this is not invariably true; thus with rhodonite the ratio of $a : b = 1{\cdot}073 : 1$. The angle between the axes b and c is called α, that between a and c is β, and that between a and b is γ (Fig. 357).

357



Triclinic Axes

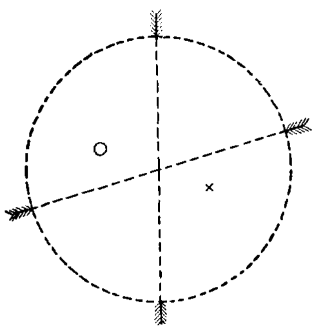
It is to be noted that there is no necessary relation between the values of α, β, and γ, any one may be greater or less than 90°; this is determined by the choice of the fundamental forms.

1. NORMAL CLASS (31). AXINITE TYPE

(Holohedral or Pinacoidal Class)

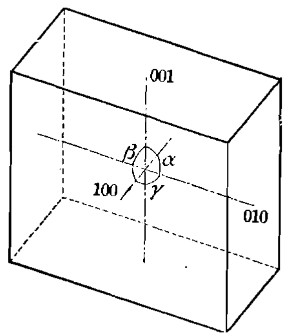
216. Symmetry. — The normal class of the triclinic system is characterized by a center of symmetry, the point of intersection of the three axes, but there is no plane and no axis of symmetry. This symmetry is shown in the accompanying stereographic projection (Fig. 358).

358



Symmetry of Normal Class

359



Triclinic Pinacoids

217. Forms. — Each form of the class includes two faces, parallel to one another and symmetrical with reference to the center of symmetry. This is true as well of the form with the general symbol (hkl) as of one of the special forms, as, for example, the a-pinacoid (100).

Hence, as shown in the following table, the four prismatic faces 110, $\bar{1}10$, $\bar{1}\bar{1}0$, $1\bar{1}0$ include two forms, namely, 110, $\bar{1}\bar{1}0$, and $\bar{1}10$, $1\bar{1}0$. The same is true of the domes. Further, any eight corresponding pyramidal faces, as, for example, 111, $\bar{1}11$, $\bar{1}\bar{1}1$, $1\bar{1}1$, $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$, $1\bar{1}\bar{1}$ belong to four distinct forms, namely, 111, $\bar{1}\bar{1}\bar{1}$; $\bar{1}11$, $1\bar{1}\bar{1}$; $\bar{1}\bar{1}1$, $11\bar{1}$; $1\bar{1}1$, $\bar{1}1\bar{1}$, and similarly in general.

The various types of forms are given in the following table:

	Indices
Macropinacoid or a-pinacoid	(100)
Brachypinacoid or b-pinacoid	(010)
Base or c-pinacoid	(001)
Prisms	$(hk0)$ $(h\bar{k}0)$
Macrodomes	$(h0l)$ $(\bar{h}0l)$
Brachydomes	$(0kl)$ $(0\bar{k}l)$
Pyramids	(hkl) $(\bar{h}kl)$ $(\bar{h}\bar{k}l)$ $(h\bar{k}l)$

In the above table it is assumed that the axial ratio is such that $a < b$. If the opposite were true the names brachy- and macro- would be interchanged.

218. The explanations given under the two preceding systems make it unnecessary to discuss in detail the various forms individually, except as illustrated in the case of crystals belonging to certain typical triclinic species.

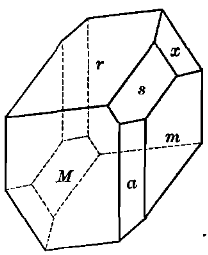
It may be mentioned, however, that Fig. 359 shows the *diametral prism,* which is bounded by three sets of unlike faces, the pinacoids a, b, and c. This is the analogue of the cube of the isometric system, but here the like faces, edges, and solid angles include only a given face, edge, and angle, and that opposite to it.

219. Illustrations. — A typical triclinic crystal is shown in Fig. 360 of axinite. Here $a(100)$ is the macropinacoid; $m(110)$ and $M(1\bar{1}0)$ the two unit prisms; $s(201)$ a macrodome, and $x(111)$ and $r(1\bar{1}1)$ two unit pyramids. The axial ratio is as follows:

$$a : b : c = 0{\cdot}49 : 1 : 0{\cdot}48,\ \alpha = 82^\circ\ 54',\ \beta = 91^\circ\ 52',\ \gamma = 131^\circ\ 32'.$$

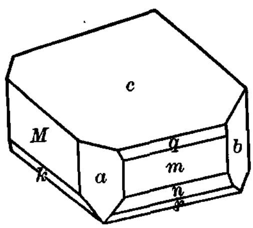
Figs. 361, 362 show two crystals of rhodonite, a species which is allied to pyroxene, and which approximates to it in angle and habit. Here the faces

360

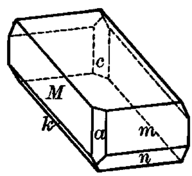


Axinite

361



362

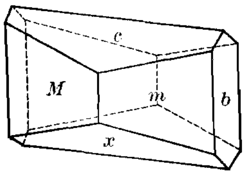


Rhodonite

are: Pinacoids $a(100)$, $b(010)$, $c(001)$; prisms $m(110)$, $M(1\bar{1}0)$; pyramids $q(221)$, $k(\bar{2}21)$, $n(\bar{2}\bar{2}1)$, $r(\bar{1}\bar{1}1)$.

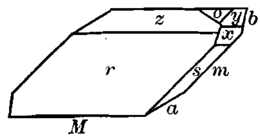
Further illustrations are given by Fig. 363 of albite and Fig. 364 of anorthite. The symbols of the faces, besides the pinacoids and the unit prisms, are as follows: Fig. 363, $x(\bar{1}01)$; Fig. 364, prisms $f(130)$, $z(1\bar{3}0)$; domes $t(207)$, $y(\bar{2}01)$, $e(021)$, $r(061)$, $n(0\bar{2}1)$; pyramids 𝔪(111), $\alpha(1\bar{1}1)$, $o(\bar{1}\bar{1}1)$,

363



Albite

365



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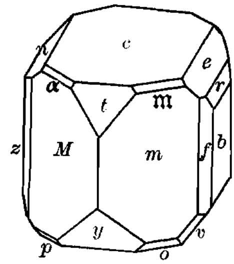
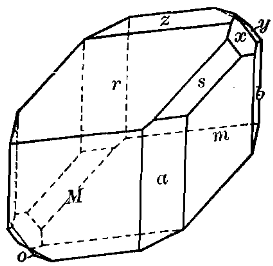
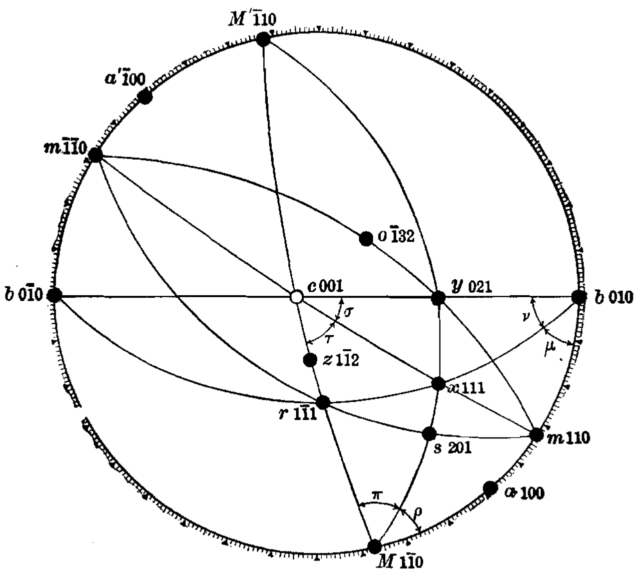


Anorthite



Axinite

366

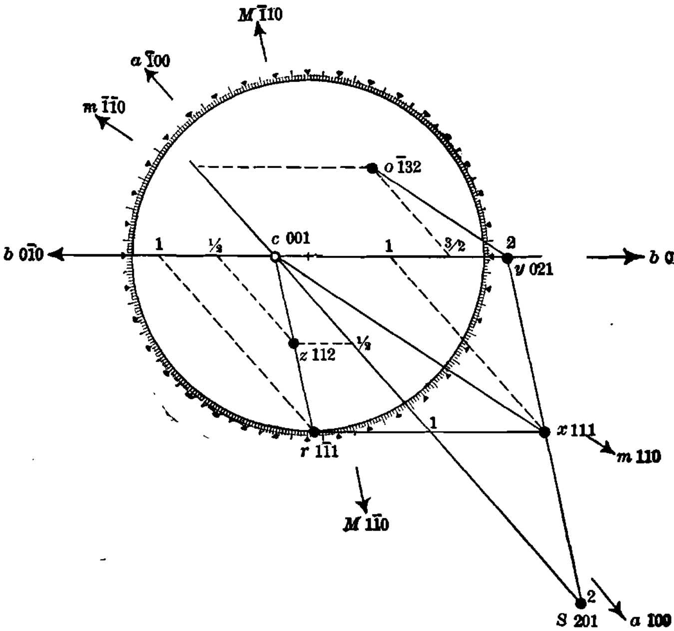


Stereographic Projection of an Axinite Crystal

$p(\bar{2}11)$. In Fig. 364 of anorthite the similarity of the crystal to one of orthoclase is evident on slight examination (cf. Figs. 340, 341), and careful study with the measurement of angles shows that the correspondence is very close. Hence in this case the choice of the fundamental planes is readily made.

Fig. 365 represents a crystal of axinite; Figs. 366 and 367 its stereographic and gnomonic projections.

367



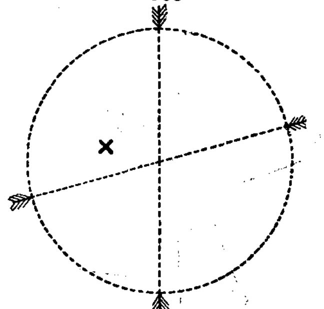
Gnomonic Projection of an Axinite Crystal

2. ASYMMETRIC CLASS (32). CALCIUM THIOSULPHATE TYPE

(*Hemihedral Class*)

220. Besides the normal class of the triclinic system there is another possible class, possessing symmetry neither with respect to a plane, axis nor center; in it a given form has *one face* only. This class finds examples among a number of artificial salts. One of these is calcium thiosulphate ($CaS_2O_3.6H_2O$); as yet no mineral species is known to be included here. This is the most general of all the thirty-two types of forms classified according to their symmetry and comes first, therefore, if the classes are arranged in order according to the degree of symmetry characterizing them. This class is one of those whose crystals may show circular polarization. This is true of eleven of the classes which have been described in the preceding pages.

368



Symmetry of Asymmetric Class

Mathematical Relations of the Triclinic System

221. Choice of Axes. — It is obvious, from what has been said as to the symmetry of this system, that *any* three faces of a triclinic crystal may be chosen as the pinacoids, or the faces which fix the position of the axial planes and the directions of the axes; moreover, there is a like liberty in the choice of the unit prisms, domes or pyramids which further fix the lengths of the axes.

When the crystal in hand is allied in form or composition to other species, whether of the same or different systems, this fact simplifies the problem and makes the choice of the fundamental forms easy. This is well illustrated, as already noted, by the triclinic feldspars (*e.g.*, albite and anorthite, Figs. 363, 364) which are near in angle to the allied monoclinic species orthoclase. Rhodonite (Figs. 361, 362), the triclinic member of the pyroxene group, is another good example.

In other cases, where no such relationship exists, and where varied habit makes different orientations plausible, there is but little to guide the choice. This is illustrated in the case of axinite (Fig. 360), where at least ten distinct positions have been assumed by different authors.

222. Axial and Angular Elements. — The *axial elements* of a triclinic crystal are: (1) the axial ratio, which expresses the lengths of the axes a and c in terms of the third axis, b; and (2) the angles between the axes α, β, γ (Fig. 357). There are here five quantities to be determined which obviously require the measurement of five independent angles between the faces.

The *angular elements* are usually taken as the angles between the pinacoids and, in addition, those between each pinacoid and the unit face lying in the zone of the other pinacoids; that is,

	ab,	$100 \wedge 010$,	*ac,*	$100 \wedge 001$,	*bc,*	$010 \wedge 001$;
also	*am*	$100 \wedge 110$,		$001 \wedge 101$,		$001 \wedge 011$;

or, instead, any one or all of these,

aM,	$100 \wedge 1\bar{1}0$,	$001 \wedge \bar{1}01$,	$001 \wedge 0\bar{1}1$.

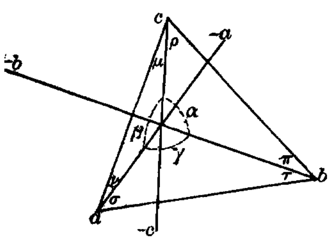
Of these six angles taken, one is determined when the others are known.

223. The mathematical relations existing between the axial angles and axial ratio, on the one hand, and the angles between the faces on the other, admit of being drawn out with great completeness, but they are necessarily complex and in general have little practical value. In fact, most of the problems likely to arise can be solved by means of the triangles of the spherical projection, together with the cotangent formula connecting four planes in the same zone (Art. **49**, p. 49); this will often be laborious and may require some ingenuity, but in general involves no serious difficulty. In connection with the use of the cotangent formula, it is to be noted that in certain commonly occurring cases its form is much simplified; some of these have already been explained under the monoclinic system (Art. **210**). The formulas given there are of course equally applicable here.

224. The first problem may be to find the axial elements from measured angles. Since these elements include five unknown quantities, viz., the three axial angles α, β, γ and the lengths of the axes a and c in terms of b, five measured angles are required, as already stated.

Fig. 369 represents the crystallographic axes of the triclinic mineral rhodonite. The positive ends of the three axes are joined by lines forming three triangles the angles of which are very important. In the triangle, for instance, which has the b and c axes for two of its sides since the length of the b axis is taken as 1·0, it is only necessary to know the angle α and either ρ or π in order to determine the length of the c axis. In the triangle that has the a and b axes for two of its sides it is necessary to know the value of γ and either σ or τ in order to determine the length of the a axis. And lastly in the triangle formed between the a and c axes, if the length of either of these axes is known, the length of the other can be determined from the angle β and either μ or ν. It is assumed that a

369



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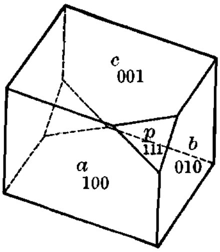
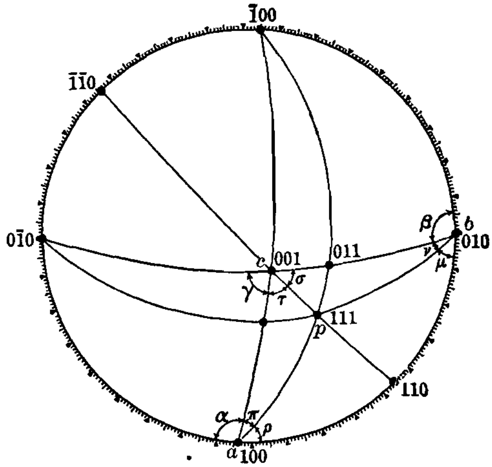


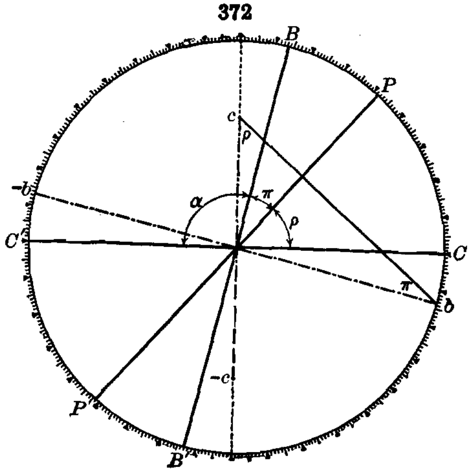
crystal of rhodonite showing the forms $a(100)$, $b(010)$, $c(001)$ and $p(111)$, see Fig. 370, has been measured and the poles of the faces plotted in the stereographic projection, Fig. 371. The angles between the great circles which connect these poles are the same as those shown in the triangles built upon the crystallographic axes, Fig. 369. With the angles between the different crystal faces known by measurement, it is easy, by the formulas of spherical trigonometry, to calculate the value of these other angles and from them obtain the axial ratio.

371



That the angles shown on the stereographic projection, Fig. 371, are identical with those in Fig. 369 may be proved as follows. Let Fig. 372 represent a vertical section cut through the spherical projection of rhodonite in such a way as to include the b and c crystallographic axes. The triangle, which has these axes as two sides and the three angles α, π and ρ, lies therefore in the plane of the figure. The normals to all faces parallel to the c axis, *i.e.* the prism zone, would lie in a plane at right angles to that axis. This plane would intersect the sphere of the spherical projection in a great circle which is represented on the stereographic projection, Fig. 371, by the divided circle. On Fig. 372 this great circle would appear in orthographic projection as the line C–C′ lying at right angles to the c axis. In the same way all faces lying parallel to the b axis, *i.e.* the zone (100)–(101)–(001), would have their normals in a plane which would be foreshortened to the line B–B′ in Fig. 372. Since the lines C–C′ and B–B′ are at right angles respectively to the c and b axes the angle between them must equal the axial angle, α. This same angle will appear therefore on the stereographic projection, Fig. 371, between the great circles of the two zones, the faces of which are parallel respectively to the c and b axes. Further the normals to all faces which intersect the b and c axes at their unit lengths would lie in a plane at right angles to the line b–c, Fig. 372.

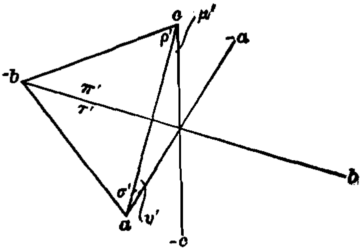
372



This plane would appear in orthographic projection as the line P–P′. On the stereographic projection, Fig. 371, this would be represented as the zonal circle passing through (100), (111), (011), ($\bar{1}$00). The angle between B–B′ and P–P′ will by construction equal π and that between C–C′ and P–P′ will equal ρ. These same angles will appear therefore in the stereographic projection between the corresponding zone circles. In the same way the identity of the angles γ, σ, τ, β, μ and ν in Figs. 369 and 371 can be proved.

With the necessary number of these angles given the formulas required for the calculation of the axial lengths are given below. The angles τ', σ', ν', μ', π' and ρ' are the corresponding angles to τ, σ, etc., in the adjacent quadrants, see Fig. 373.

373



$$\frac{\sin\tau}{\sin\sigma} = \frac{\sin\tau'}{\sin\sigma'} = \frac{a}{b}, \quad \frac{\sin\nu}{\sin\mu} = \frac{\sin\nu'}{\sin\mu'} = \frac{c}{a}, \quad \frac{\sin\pi}{\sin\rho} = \frac{\sin\pi'}{\sin\rho'} = \frac{c}{b}.$$

If the angles given are between the three pinacoids and the pyramid *hkl* (not the unit form) the relations are similar. That is, if for the face *hkl* the corresponding angles be represented by τ_0, σ_0, etc., where τ_0, σ_0 are the angles between the zone circles 100, 001 and 100, 010 respectively and the zone circle 001, *hk*0, these relations may be expressed in the general form

$$\frac{\sin\tau_0}{\sin\sigma_0} = \frac{\sin\tau_0'}{\sin\sigma_0'} = \frac{a}{\frac{h}{k}b} = \frac{k}{h}\cdot\frac{a}{b},$$

$$\frac{\sin\nu_0}{\sin\mu_0} = \frac{\sin\nu_0'}{\sin\mu_0'} = \frac{c}{\frac{l}{h}a} = \frac{h}{l}\cdot\frac{c}{a},$$

$$\frac{\sin\pi_0}{\sin\rho_0} = \frac{\sin\pi_0'}{\sin\rho_0'} = \frac{c}{\frac{l}{k}b} = \frac{k}{l}\cdot\frac{c}{b}$$

Thus for the face 321 the formulas become

$$\frac{\sin\tau_0}{\sin\sigma_0} = \frac{a}{\frac{3}{2}b} = \frac{2\,a}{3\,b}, \quad \frac{\sin\nu_0}{\sin\mu_0} = \frac{3c}{a}, \quad \frac{\sin\pi_0}{\sin\rho_0} = \frac{2c}{b}.$$

It is also to be noted that

$$\alpha = 180° - A, \qquad \beta = 180° - B, \qquad \gamma = 180° - C,$$

where A, B, C are the angles in the pinacoidal spherical triangle 100·010·001 at these poles respectively. That is,

$$A = \pi + \rho = \pi_0 + \rho_0 = (180° - \alpha);$$
$$B = \nu + \mu = \nu_0 + \mu_0 = (180° - \beta);$$
$$C = \tau + \sigma = \tau_0 + \sigma_0 = (180° - \gamma).$$

Also

$$180° - A = \pi' + \rho' = \pi_0' + \rho_0' = \alpha.$$

Hence, having given, by measurement or calculation, the angles between the faces $ab(100 \wedge 010)$, $ac(100 \wedge 001)$ and $bc(010 \wedge 001)$, which are the sides of this triangle, the angles A, B, C are calculated and their supplements are the axial angles α, β, γ respectively.

Still another series of equations are those below, which give the relations of the angles μ, ν, ρ, etc., to the axes and axial angles. By means of them, with the sine formulas given above, the angular elements (and other angles) can be calculated from the axial elements.

$$\tan\mu = \frac{a\sin\beta}{c + a\cos\beta}; \quad \tan\nu = \frac{c\sin\beta}{a + c\cos\beta}.$$

$$\tan\rho = \frac{b\sin\alpha}{c + b\cos\alpha}; \quad \tan\pi = \frac{c\sin\alpha}{b + c\cos\alpha}.$$

$$\tan\tau = \frac{a\sin\gamma}{b + a\cos\gamma}; \quad \tan\sigma = \frac{b\sin\gamma}{a + b\cos\gamma}.$$

These equations apply when $\mu + \nu$, etc., is less than 90°; if their sum is greater than 90° the sign in the denominator is negative.

207. The following equations are also often useful.

$$\tan\alpha = \frac{2\sin\rho\sin\rho'}{\sin(\rho - \rho')} = \frac{2\sin\pi\sin\pi'}{\sin(\pi - \pi')}.$$

$$\tan\beta = \frac{2\sin\mu\sin\mu'}{\sin(\mu - \mu')} = \frac{2\sin\nu\sin\nu'}{\sin(\nu - \nu')}.$$

$$\tan\gamma = \frac{2\sin\tau\sin\tau'}{\sin(\tau - \tau')} = \frac{2\sin\sigma\sin\sigma'}{\sin(\sigma - \sigma')}.$$

Also,

$$\alpha + \pi + \rho = \beta + \mu + \nu = \gamma + \tau + \sigma = 180°.$$

The calculation, from the angular elements or from the assumed fundamental measured angles, either (1) of the angular position of any face whose symbol is given, or (2) of the symbol of an unknown face for which measured angles are at hand, requires no further explanation. The cotangent formula is all that is needed in a single zone, and the solution of spherical triangles on the projection (with the use of the sine formulas) will suffice in addition in all ordinary cases.

225. To determine, by plotting, the axial elements of a triclinic crystal, having given the stereographic projection of its forms. In order to solve this problem it is necessary to have given the position of the poles of the unit forms (100), (010), (001), (111) or to be able to locate them by means of their zonal relations. Through these poles the various zonal circles are drawn as shown in the case of rhodonite, Fig. 371. The angles α, β, γ, π, ρ, etc., are then given upon the projection. These angles can be measured as described in Art. **41**, p. 39. Taking next a certain line as representing the unit length of the b axis and knowing the angles α, π and ρ the triangle that includes the b and c axes, see Fig. 369, can be drawn to scale and the unit length of the c axis determined. In a similar way the length of the a axis can be found.

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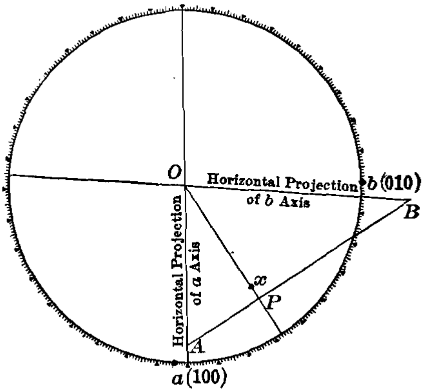
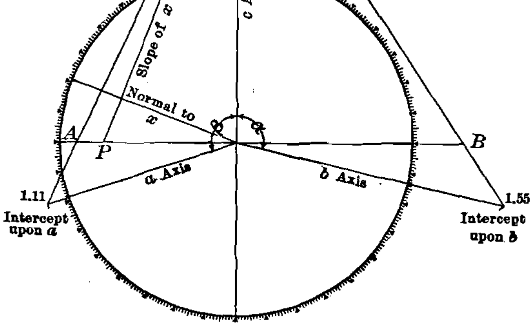


226. To determine, by plotting, the indices of a face upon a triclinic crystal, having given the position of its pole in the stereographic projection and the axial elements of the mineral. To illustrate this problem a possible pyramid face on rhodonite will be used. Its pole is located in the stereographic projection at x, Fig. 374. The position of the poles of the faces $a(100)$ and $b(010)$ must also be known. The directions of the intersections of the planes of the a–c and b–c axes with the plane of the projection can then be drawn. These lines will represent the horizontal projections of the a and b crystallographic axes. A radial line is then drawn from the center of the projection, O, through x. Another line, A–P–B, is drawn perpendicular to this line at any convenient distance from the center, O. The line A–P–B will represent the direction of intersection of the face x with the horizontal plane of the projection. The intercept that the face will make upon the vertical axis can be found by the construction of a right triangle with O–P as its base, a line representing the c axis as its vertical side and the angle between O–x as the angle between the base and the hypothenuse, see Fig. 375. Under the assumed conditions the face will intersect the c axis at a distance of 1·93, the radius of the circle in the figure being 1·0. The face will also pass through the points A and B on the horizontal projections of the a and b axes. With the known angles β and α it is possible to construct the a and b axes with their proper angular relations to the c axis. The intercepts of the face upon these two axes will be given by the extension of the lines from the point 1·93 on the c axis to the points A and B. In this way the intercepts of the face upon the three axes were obtained as 1·11a, 1·55b,

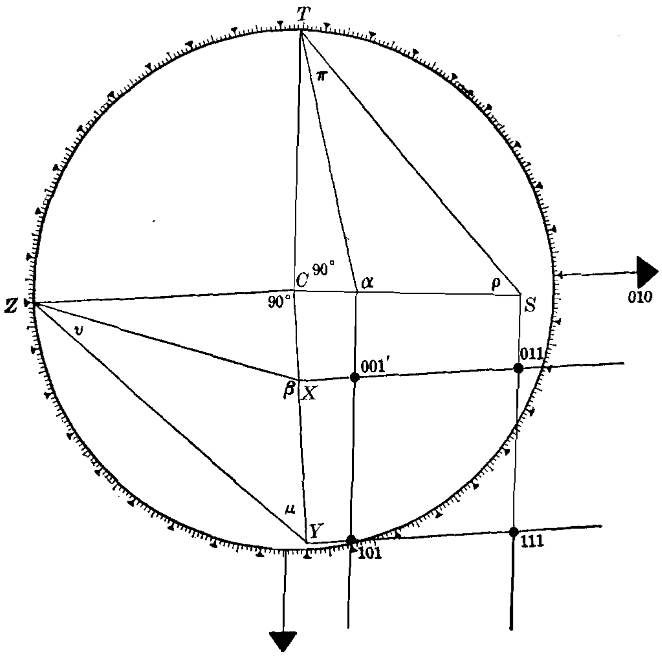
375



1·93c. By dividing these numbers by 1·55 we get the intercepts expressed in terms of the length of the b axis, considering that as 1·0. The intercepts then become 0·71a, 1b, 1·24c. When these are compared with the axial ratio of rhodonite, $a : b : c = 1·114 : 1 : 0·986$, the parameters of the face are found to be $\frac{2}{3}a$, 1b, 2c. The indices of x are therefore 321.

227. To determine, by plotting, the axial elements of a triclinic crystal having given the gnomonic projection of its forms. To illustrate this problem it is assumed that the positions of the poles of the faces, (100), (010), (001), (101), (011) and (111) on rhodonite are known, see Fig. 376. If this figure is compared with the stereographic projection of the same forms given in Fig. 371, it will be seen that the angle between the zones (100)-(101)-(001) and (100)-(111)-(011) is equal to π, that between the zones (100)-(111)-(011) and (100)-(110)-(010) is equal to ρ, between (010)-(011)-(001) and (010)-(111)-(101) is equal to ν and between (010)-(111)-(101) and (010)-(110)-(100) is equal to μ. The method by which the angles between these various zones may be measured was explained in Art. 42, p. 43, and is illustrated by the construction of Fig. 376. From these angles triangles can be readily constructed to give the lengths of the a and c axes in terms of the b axis, with its length taken as equal to 1·0.

376



228. To determine, by plotting, the indices of the forms of a triclinic crystal, having given the position of other poles upon the gnomonic projection. The method for the solution of this problem is similar to that already described under the previous systems. The difference lies in the fact that the lines of reference upon which are plotted the intercepts of the lines drawn to them from the poles of the faces make oblique angles with each other. These reference lines are taken as the zonal lines (001)-(101) and (001)-(011) and the intercepts from which the indices are determined are measured from the pole of (001). A study of the gnomonic projection of axinite, Fig. 367, will illustrate this problem.

MEASUREMENT OF THE ANGLES OF CRYSTALS

229. Contact-Goniometers. — The interfacial angles of crystals are measured by means of instruments which are called *goniometers*.