

## Methods of scale-up

### A. Yianatos' Method

This method was described in 2003 (Yianatos et al., 2003). It is quite complicated. It is based on four elements: knowledge of flotation kinetics on laboratory and industrial scale, upgrading curves for both scales of flotation, flotation optimal points of flotation as well as the assumption of equality of product of optimal time and maximum flotation rate constant. Such approach allows to scale-up two parameters: flotation time and flotation recovery. Flotation test in the laboratory scale is carried out in an arbitrary manner, but very specific way. The work of Yianatos and associates (2003) detail the methodology of the flotation were not described precisely. They say only that the flotation tests were conducted in two laboratory Wemco type flotation machines with a 5.3 dm<sup>3</sup> cell (presumably for the main flotation) and a 2.7 dm<sup>3</sup> cell (probably for cleaning flotation or fractionated). The batch flotation were conducted periodically once a day for a month. As a result of flotation 9 points (flotation products) are obtained versus time of flotation. Based on the results obtained the recovery versus flotation time are plotted for creation of a kinetic curve (Figure 3.1.1).

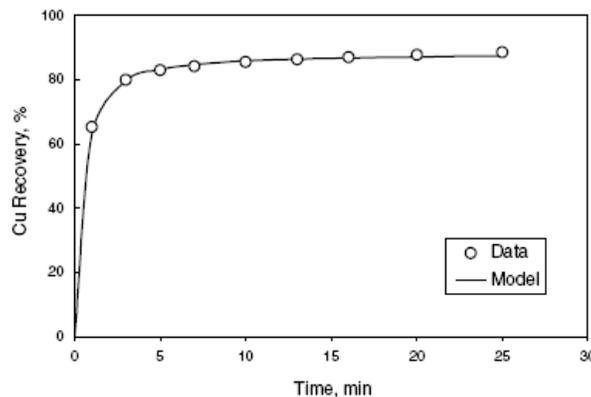


Fig. 3.3.1. Flotation kinetics of laboratory tests (based on Yanatos et al., 2003)

Then the kinetic curve is described mathematically by using kinetic equations:

$$r(t) = r(\infty) \frac{1 - [1 - e^{-k_{\max} t}]}{k_{\max} t} \quad (3.3.1)$$

where  $r(t)$  is the recovery at a given moment of flotation,  $r_{\infty}$  maximum flotation recovery,  $k_{\max}$  is the maximum rate of flotation for the so-called rectangular distribution,  $t$  time of flotation (also called the *residence time*). Rectangular distribution is based on equations

$$F(k) = \frac{1}{k_{\max}} \text{ dla } 0 < k < k_{\max} \quad (3.3.2)$$

$$F(k) = 0 \text{ dla } \infty > k > k_{\max} \quad (3.3.3)$$

In Figure 1. 3.3.1. the results of the flotation, approximated by the formula (1), is shown in the form of a solid line.

Then one draws two enrichment curves. These are the Halbicha and Dell curves. The Dell curve (fig. 3.3.2) shows the dependence of the recovery (cumulative,  $\Sigma \varepsilon$ ) vs cumulative yield  $\Sigma \gamma$  divided by the content of the component in the feed  $\alpha$ .

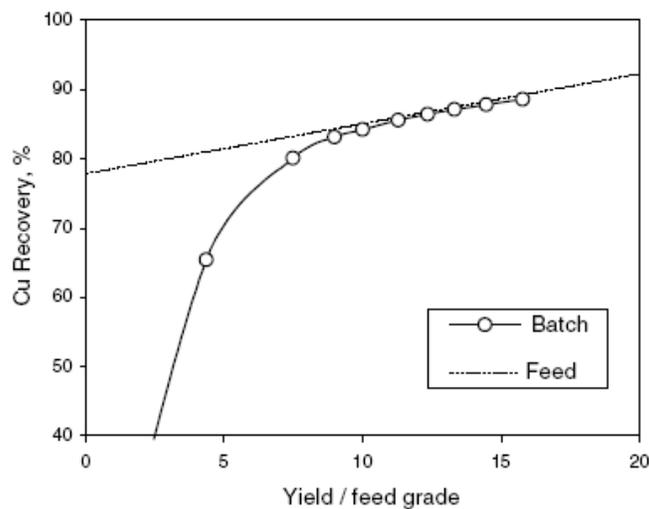


Fig. 3.3.2. Enrichment curve of Dell (Yianatos et al., 2003)

The Dell curve has the advantage that the tangent to the curve  $d(\Sigma \varepsilon)/d(\Sigma \gamma/\alpha)$  gives the noncumulative contents of the component in concentrate ( $\lambda$ ). In Figure 1. 3.3.2 solid straight line gets in contact with the Dell enrichment curve at a point where  $\lambda = \alpha$ .

Calculated from Dell's curve values of ( $\lambda$ ) (named in Figure 3.3.3 as *instantaneous content*) are printed in the form of triangles on the Halbicha enrichment curve in the form of cumulated  $\beta$  vs cumulated recovery  $\Sigma(\varepsilon)$ . In addition, the graph of Halbicha is supplemented with a horizontal line  $\lambda = \alpha$ .

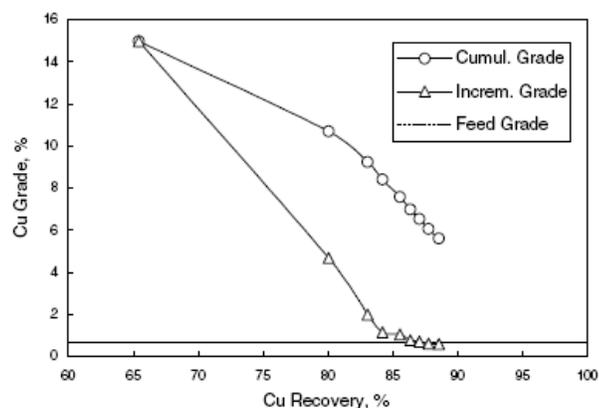


Fig.3.3.3. Halbich's enrichment curve (on the basis of Yianatos et al., 2003)

The procedure with the line  $(\lambda) = \alpha$  allows designation, with the help of kinetic curve (fig. 1), the time of flotation, when flotation concentrate reaches the content useful component of  $\alpha$ . In fact, this point can be read from one enrichment curve, but the use two curves raises the precision of determination of flotation time when the concentrate has the useful component content of  $\alpha$ .

The third element of the methodology of the scale-up is the optimum flotation. This is the time when the concentrate grade reaches  $\alpha$ .

The same procedure must be carried out for the plant flotation. You can also choose one of the nodes of the industrial process. Yianotos and his colleagues chose flotation (initial node *rougher*). This node consisted of 5 parallel banks of flotation machines. Each series contains 9 self areated flotation Wemco cells of capacity of 42.5 m<sup>3</sup> in the 3-3-3-3 system. The work of a node was characterized by using the results of the chemical analysis of samples taken throughout the day by 3 shifts. In addition, they developed a special way of determining the characteristics of the kinetics of the rougher flotation node. These data allow for the plotting the flotation kinetics in this node. The results of the flotation kinetics were described by the formula:

$$R(\tau) = R(\infty) \frac{1 - (1 - [1 + \frac{K_{\max}\tau}{N}]^{1-N})}{\frac{K_{\max}\tau}{N}(N-1)} \quad (3.3.4)$$

where  $R(\tau)$  is the recovery at the moment time flotation,  $R_{\infty}$  maximum recovery of flotation,  $K_{\max}$  is the maximum rate of the flotation having rectangular distribution,  $\tau$  is the total time (flotation *residence time*).

The fourth element of the scale-up is the imposition of yet another criterion. Yianotos and his colleagues (2003) assumed that

$$(K_{\max}\tau)_{Plant} = (k_{\max}t)_{Lab} \quad , \quad (3.3.5)$$

where the subscript *Plant* means the results of the industrial, subscript *Lab* laboratory results. This assumption causes (see kinetic equations) that

$$\left[ \frac{R(\tau)}{R(\infty)} \right]_{Plant} = \left[ \frac{r(t)}{r(\infty)} \right]_{Lab} \quad (3.3.6)$$

and that the

$$\frac{\tau}{t} = \frac{k_{\max}}{K_{\max}} \quad (3.3.7)$$

$$\frac{R(\tau)}{r(t)} = \frac{R_{\infty}}{r_{\infty}} \quad . \quad (3.3.8)$$

Therefore, as a result of flotation tests on the laboratory and industrial scale provide data on the time flotation and technological indicators such as recovery, yield, content (fig.3.3.4). Now one can be made, based on the method of Yianatos and coworkers (2003), scale-up of two

parameters: the optimal time and optimal flotation recovery. As a result of studies of Yianatos and co-workers (2003) for Cu ore processed by Codelco-Chile, Salvador Division, the optimal flotation time of was scaling-up by

$$X = \frac{\tau}{t} = 2.26 \pm 0.35 \quad (3.3.9)$$

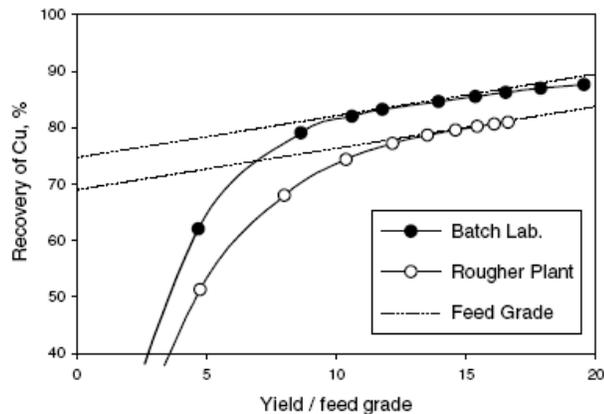


Figure 3.3.4. Curves of Dell for industrial and lab flotation (Yianatos et al., 2003)

It turned out that recovery for the optimal time were scale-up not according to a constant, but the authors were able to determine that % of the quantity of oxidized copper (*% of soluble copper*) gives the relationship

$$\frac{R(\tau)}{r(t)} = 1.045 - 0.00945 (\% \text{ oxidized copper}) . \quad (3.3.10)$$

It should be noted that the laboratory results were better than the plant flotation and on the Dell enrichment curve they assumed the character of Dell curves shown in fig. 3.3.4.

#### B. Method of MFT-FLEET (Dobby and Savassi, 2005)

This method was developed by MinnovEX Technologies followed the order of seven Mining Corporations (Barrick Gold, BHP-Billiton, CVRD, Falconbridge, Grupo Mexico, Phelps Dodge, Rio Tinto). It is based, like the Yianatos method, on determining flotation kinetics during laboratory testing and plant flotation, and then comparing the results to the calibration of the method, but the flotation kinetics results are presented graphically in a completely different way. Also the analysis of the results of the flotation is done differently. Laboratory-scale flotation, also called the procedure of MFT MinnovEX Flotation Test is carried out in a fractionated way to get four separate concentrates (fig 3.3.5). Feed, tailing and combined concentrates are subjected to the sieve analysis to get 4 grain fractions.

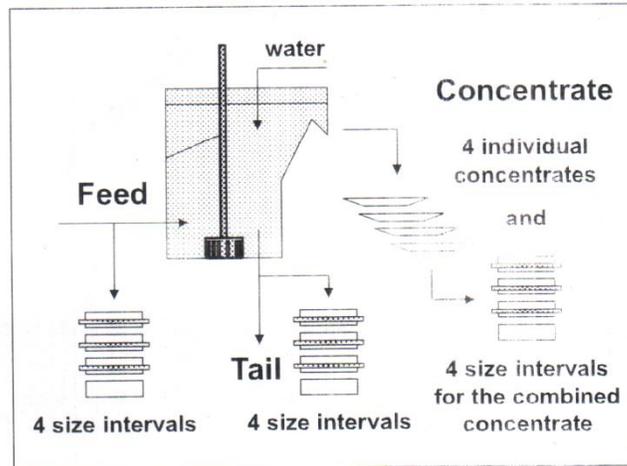


Fig 1 - MFT schematic

Fig. 3.3.5. The method of MinnovEX Laboratory Flotation Test (MFT) (Dobby and Savassi, 2005)

Then, the flotation kinetics of individual ore minerals for various particle size fractions, taking into account only the floating grains and phenomenologically flotation is separated from entrainment of gangue minerals. The results are presented in the form of a yield (weight of a fraction) depending on a rate constant of particles for this class of useful minerals. Non-floating particles useful particles have flotation rate equal to zero.

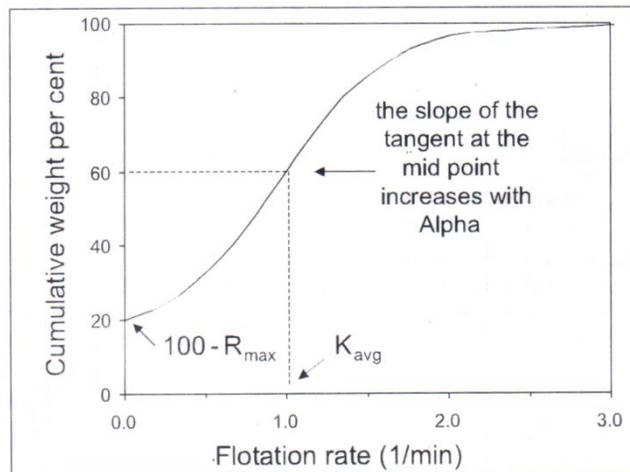


FIG 2 - The k-distribution of a mineral.

Fig. 3.3.6. Distribution of rate constant of flotation of a given mineral (Dobby and Savassi, 2005)

The curve obtained is characterized by three parameters: the maximum recovery of  $R_{max}$ , the weighted average rate constant of flotation  $K_{avg}$  read at accumulated grain yield equal to  $(100 + R_{max})/2$  and the dispersion of rate constant  $K_{avg}$ . Dispersion parameter is indicated by the symbol  $\alpha$ . Each class distribution of  $k$  provides a grain of the same or similar flotation rate and,

therefore, both the free grains and grains of different degrees of liberation. Very fine grain and grain poorly liberated will belong to the same class  $k$ .

The second part of the method is survey of industrial flotation for a wide range of types of ores and the conditions of the process. Each survey should be done at least three times longer time than the flotation *residence time*. Samples shall be taken every half an hour or more. Not described exactly were further actions, but one should guess that the balance sheets and kinetic curves shall be drawn up so as to obtain the same parameters as in the case of the lab flotation. Then one should start the program which calibrate the laboratory data generated and described according to the FLEET procedure for industrial results. Calibration is carried out separately for each type of ore and various working conditions. Based on the FLEET, the plant results for a given ore and way of flotation in the form of concentrate quality  $\beta$  and recovery  $\varepsilon$  are predicted. The creators of the method claim that there is compliance of the data received from the model and the actual results (Figure 4.2.3).

On the basis of the MFT-FLEET method one can also predict the results of flotation for new ores with known properties, by using appropriate parameters for the ore already processed (fig 3.3.7).

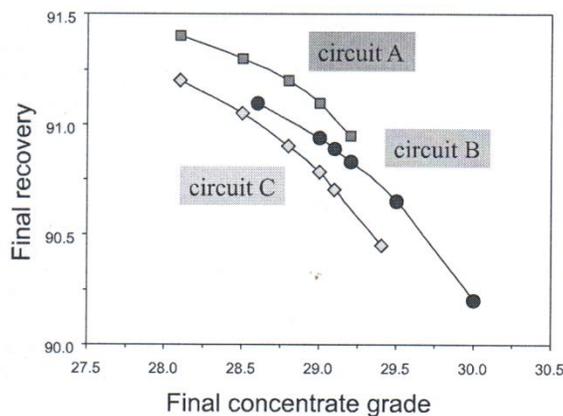


Fig. 3.3.7. The projected, on the basis of the MFT-FLEET, Halbich enrichment curves for new ores depending on the planned scheme (based on Dobby and Savassi, 2005)

### SUPASIM Method

Methodology and SUPASIM program for prediction of results in an enlarged scale was developed by Eurus Mineral Consultants (EMC). Is based on the standard laboratory kinetic tests using identical conditions of flotation, grinding and flotation reagents. Flotation kinetics results are described by the Kelsall two-parameter formula, which characterizes rate and flotation of quickly-and slowly floating particles. The flotation kinetics of both useful and gangue components is determined. Prediction of industrial results is based on scaling-up based on the results of the laboratory and about 30 data sets for industrial flotation PGM. Only recovery of flotation is modelled. Figure 2. 3.3.8 illustrates the scaling-up results.

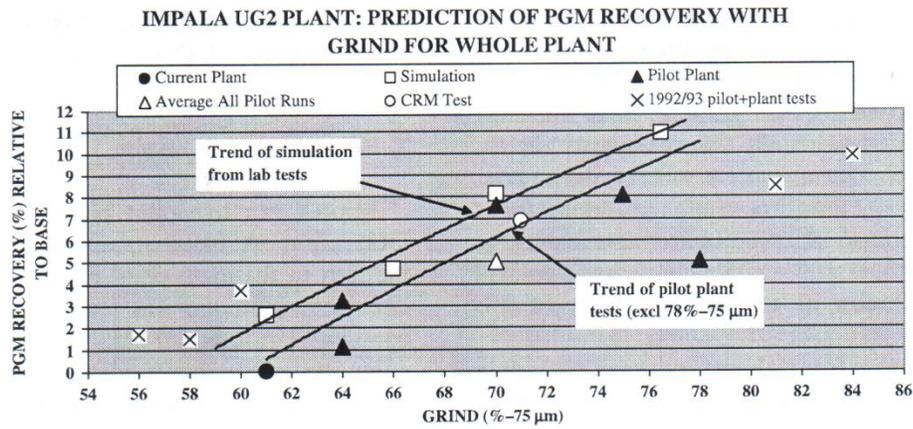


Fig. 3.3.8. A comparison of the predicted industrial flotation results based on laboratory tests (based on Hay and Schröder, 2005)

D. Method Of Outokumpu

Consists in determining indicators of industrial flotation, and then carry out a series of laboratory flotation as long until you receive the same enrichment curve with the industrial enrichment curve (Kalapudas, 1985).

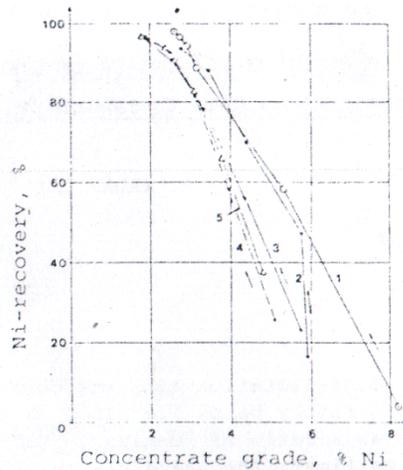


Fig. 3.3.9. Halbich's enrichment curves for continuous flotation (industrial) (graph 1) and laboratory (curves 2-5) received under different conditions (on the basis of Kalapudas, 1985)

Then one compares the two kinetics to determine the kinetics scale-up ratio (*scale-up factor*) defined as:

$$A = \frac{\text{time of continuous flotation in full scale}}{\text{time of laboratory batch flotation}} \quad (3.3.11)$$

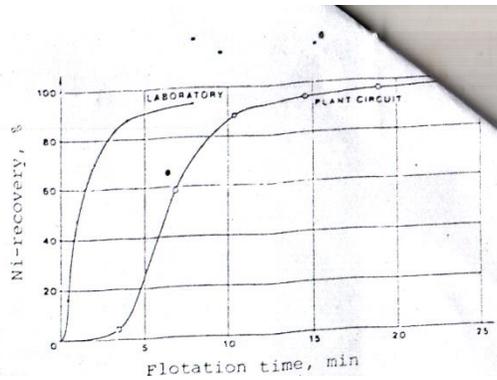


Fig. 3.3.10. A comparison between continuous flotation kinetics with periodic for the designation of A ratio (based on Kalapudas, 1985)

The resulting values of A is dependent on the value of recovery and can be used to predict the retention time (flotation) and volume of flotation cells in new flotation installations.

A values range from 1,5 up to 5 and depend on, as mentioned, the nature of the ore and the intended level of recovery.

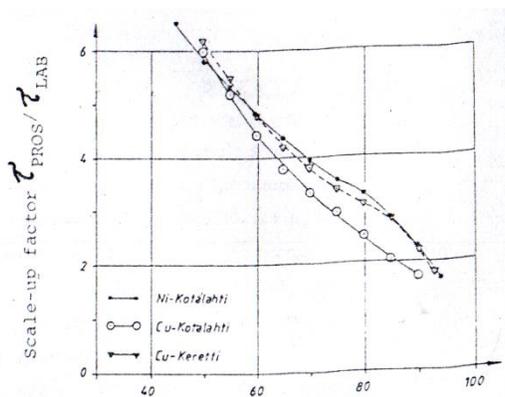


Fig. 3.3.11. Relationship scale indicator and recovery (based on Kalapudas, 1985)

### E. Gorain's and associates Method

Gorain's et al. (1998) method describes the scaling-up of flotation kinetics. Two approaches have been proposed. First, in contrast to the second, is somewhat less precise, but it is easier to perform. For each level of froth, flotation tests shall be carried out at various rates of stirrer and air flow to have different values of surface (per unit area) flow of air bubbles ( $S_b$ ). On this basis, the relationship is a constant rate of flotation  $k$  and  $S_b$  is obtained as described by Gorain and coworkers in another work (Gorain et al., 1997). This dependency, for low foaming, it should be plotted, as shown in figure. 3.3.12. Knowing the value of  $S_{(b)}$  at which you want to predict the rate of flotation ( $k$ ) one shall determine it from Fig. 3.3.12. The value of  $S_{(b)}$  for a large flotation cell is expected to be based on the model described in the work of Gorain (1995).

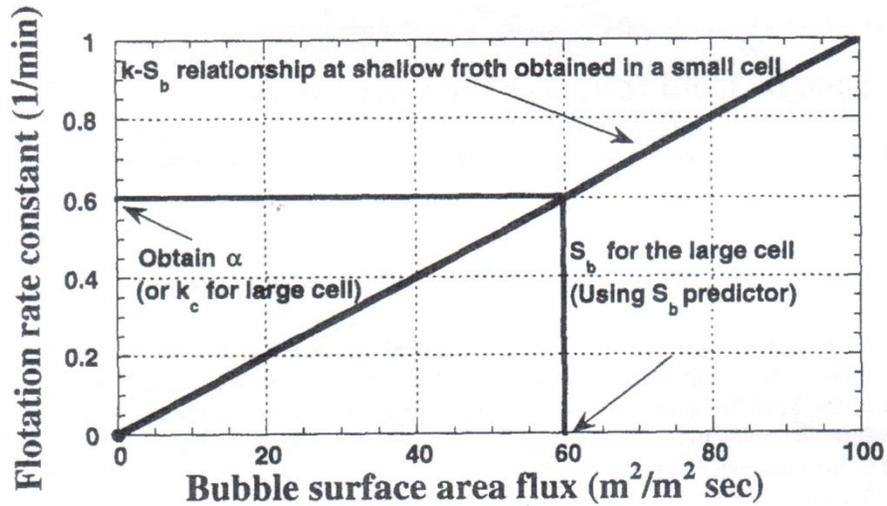


Fig. 3.3.12. Determining the rate constant for a large flotation cell (based on Gorain et al., 1998) ( $k$  vs  $S_b$ )

According to the second method, the tests shall be carried out in the small flotation cell with different levels of froth, and then separately at a high level of froth and different rates of stirring and air surface flow of bubbles having different values (per unit area) ( $S_b$ ) and various flotation rate constant  $k$ . Determination of a constant rate of flotation for a large flotation machine is in this method complex, because it requires the designation and use of multiple parameters. Details are given in the work of Gorain.

#### F. Method Of PWR-AGH

It was developed in the years 2007-2009 as a result of research on Polish copper ore (Tumidajski et al., 2007; Trybalski et al., 2008; Foszcz et al., 2009). Its rules were generally described in work of Drzymala (2010). The method is based on a comparison of the selectivity of flotation set for flotation results in laboratory-scale with the results of the so-called industrial waste and final concentrates fractionated flotation. Preparation of material for laboratory-scale research and flotation is specific (fig 3.3.13).

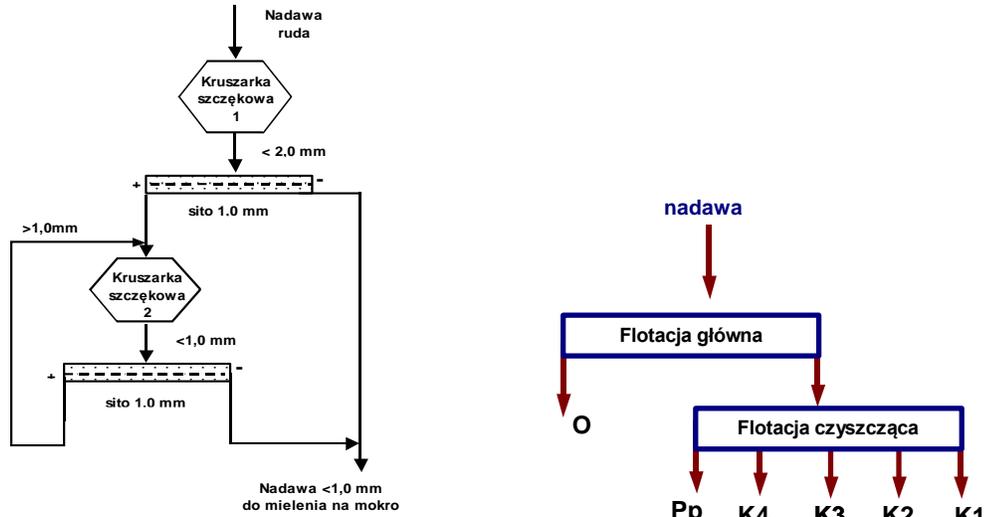


Fig. 3.3.13. General scheme of flotation experiments. a) preparation of the feed flotation, b) flotation scheme

Further details of the flotation are given in the reports (Tumidajski et al., 2007; Trybalski et al., 2008; Foszcz et al., 2009).

The results of the laboratory flotation are plotted in a Fuerstenau plot as recovery of a component and the recovery of other components in the tailing (fig. 3.3.14).

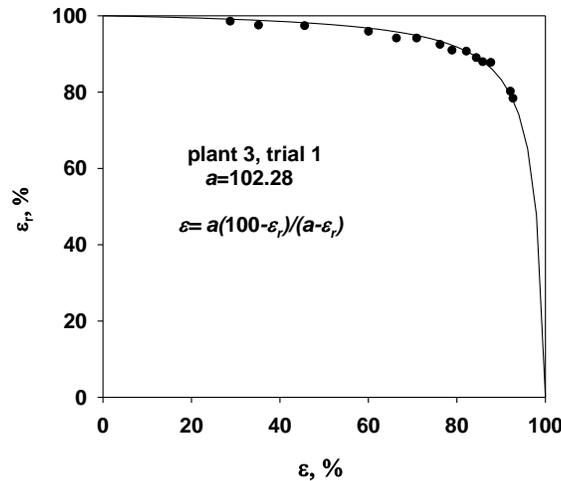


Fig. 3.3.14. Fuerstenau's enrichment curve with laboratory-scale results of flotation

Then, the data points should be described by the formula:

$$\varepsilon = a \frac{(100 - \varepsilon_r)}{(a - \varepsilon_r)} \quad (3.3.12)$$

where the parameter **a** is the selectivity of the process.

In turn, the results of re-flotation of concentrates and industrial flotation tailing, obtained according to the procedure described in the reports (Tumidajski et al., 2007; Trybalski., 2008;

Foszcz et al., 2009) also placed in the Fuerstenau plot, and measuring points are described by two equations

$$\varepsilon = a \frac{(100 - \varepsilon_r)}{(a - \varepsilon_r)} \quad (\text{industrial}) \quad (3.3.13)$$

$$\varepsilon_{r(a')} = a' \frac{(100 - \varepsilon)}{(a' - \varepsilon)} - 0.07(\varepsilon_r - 95.5)^2(100 - \varepsilon_r) \quad (\text{re-flotation}) \quad (3.3.14)$$

as shown in Figure 1. 3.3.15. a characteristic feature of this approach is that  $a = a'$ .

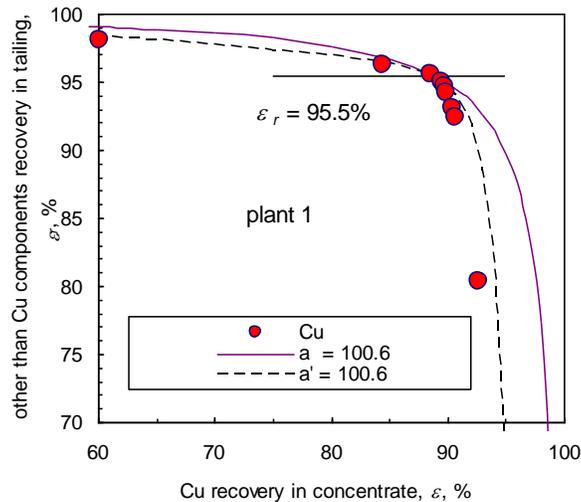


Fig. 3.3.15. Results of industrial flotation based on reflation of separately concentrate and tailing and the connection of data in one enrichment curve (dashed line) and the expected industrial floatability (solid line). The chart also shows the best point of enrichment at  $\varepsilon_r = 95.5\%$

It should be noted that, as a general rule, that the lab flotation results are worse than the industrial flotation data. Therefore, it is necessary to introduce a scale change factor from laboratory-scale to industrial. This is achieved by making the relationship between indicators  $a$  and  $a'$  in both scales. Further research allowed to refine these dependencies (3.3.16).

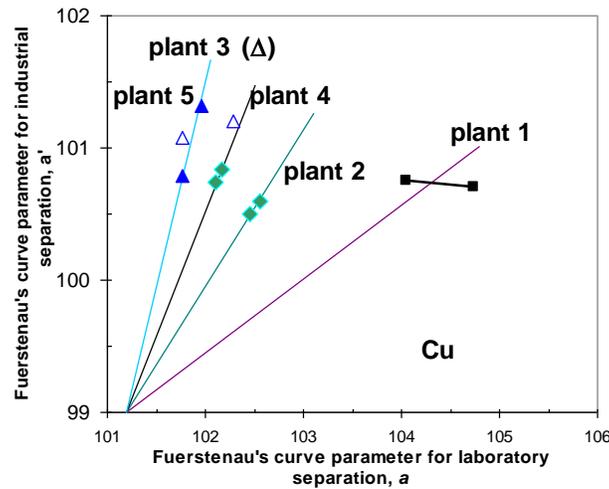


Fig. 3.3.16. Relationship between selectivity of flotation for industrial  $a'$  laboratory and  $a$  specified in the report of Trybalski and others (2008)

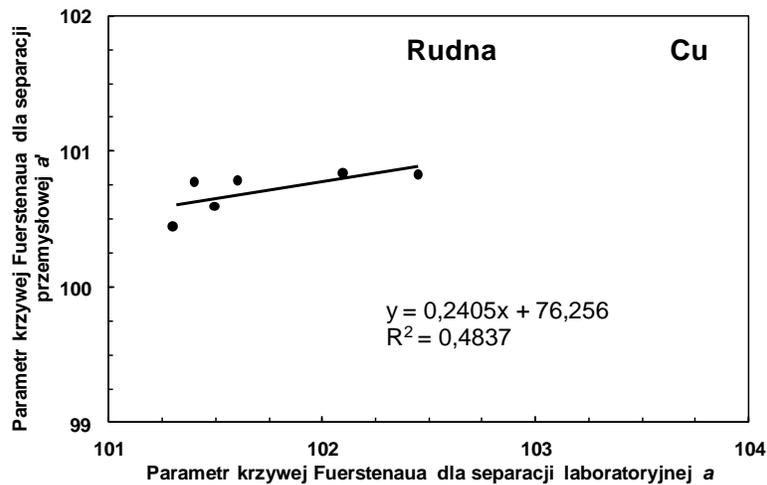


Fig. 3.3.17. Relationship of the selectivity of flotation for industrial  $a$  and laboratory  $a$  for Rudna (Luszczkiewicz, 2012)

The advantage of the method of PWR-AGH is the omission of the stage of setting and plotting of the flotation kinetics (industrial and laboratory). The omission of this step does not mean, however, that the method does not rely on flotation kinetics. This is due to the fact that the combination of flotation kinetics provides the Fuerstenau curve. The advantage of this approach is that it eliminates, to a large extent, kinetics determination errors, since these errors for the useful and gangue cancels. The expected results of industrial flotation based on the results of a laboratory using the method of PWR-AGH can be obtained using a special computer program described in the report of the Foszcz et al. (2009).

The results of laboratory and industrial flotation calibration curves can be used to predict, on the basis of the laboratory flotation, the results of both industrial floatability (based on the parameter  $a$  and  $a'$ ) and the results of re-flotation of concentrates and industrial tailings (for

$a'=a$ ). This is done using the above equations for  $a$  and  $a'$ . The results of the calculations are best represented in the form of enrichment curves. It can be any enrichment curve. One must use suitable curve. For example, to calculate the useful component content in the concentrate and plotting the Halbicha enrichment curve the equation is useful:

$$\beta = \frac{100\alpha(\varepsilon - a)}{100^2 - \alpha(100 - \varepsilon) - 100a} \quad (3.3.15)$$

Research report of Luszczkiewicz (2012) proposed yet another method for predicting the results of industrial flotation (fig 3.3.18). It consists of identification of laboratory floatability based on the procedure Dell and on the basis of the most selective laboratory and industrial products reflation data points.

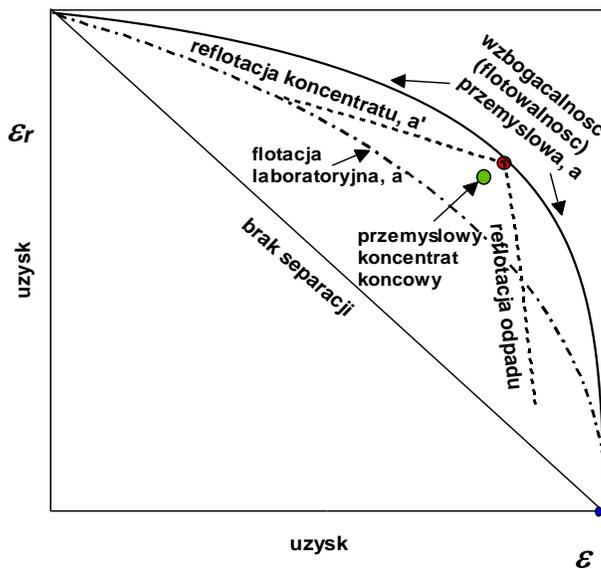


Fig. 3.3.18. Industrial floatability (solid line) based on reflation of concentrate and tailing (dotted line) and the laboratory flotation in accordance with the procedure of Dell (Luszczkiewicz and others, 2012)